NET 2022 Answers

Power Round - Advanced Microeconomics Division

May 20, 2022

Problem 1: Oligopoly Types (24 Points)

Part (A) (I) $(144 - q_1 - q_2)q_1 = \pi_1$. (II) $(144 - q_1 - q_2)q_2 = \pi_2$.

Part B: Take derivatives and arrive at 2: $q_1 = \frac{144-q_2}{2}$ 3: $q_2 = \frac{144-q_1}{2}$

Part C 1: Arrive at a formula with only $q_1$or q_2 or some method to solve for them. For example $q_1 = \frac{144-\frac{144-q_1}{2}}{2}$ 2: $q_1 = 48$ $q_2 = 48$ 3: 2304 is the profit for both firms

Part D 1: Monopoly 2: $(144 - Q)Q = \pi$

Part E 1: take derivative with respect to Q to arrive at $Q = 72$ 2:$P = 77$ 3: $\pi = 5184$ 4: Quality mark for full mathematical work being shown in ALL parts

Part F 1: Firm 1 has higher profit 2: Firm 2 has lower profit (3): If either 1 or 2 is explained further intuitively then a bonus point is awarded but not to exceed 2 total points on this question. Essentially if 1 or 2 is wrong but this is right, the response is still awarded 2 points but if 1 and 2 are correct, 2 points are also awarded.

Part G 1: We know that firm 2 still maximizes using $q_2 = \frac{144-q_1}{2}$ since they still depend on what firm 1 does before them. 2: Considering firm 1, they know firm 2 acts in the way decribed previously and thus maximize profit already knowing this value of q_2. $\pi_1 = (144 - \frac{144-q_1}{2} - q_1)q_1$. 3: Thus maximizing π_2, we get that $q_1 = 72$ and therefore $q_2 = 36$. 4: $P = 41$. 5: $\pi_1 = 2592$ $\pi_2 = 1296$

Part H 1: Stackleberg, Cournot, Monopoly for CS. 2: Monopoly, Cornout, Stackleberg for Profit. 3: Quality mark for an answer that logially follows from pervious answers, even if previous answers were wrong.

Problem 2: Bargaining, Fairness, & Money on the Table (24 Points)

Part A (2) The best offer that can be made is for R to offer G 0 dollars. To see why, note that if R makes any other offer, they can do strictly better by offering some $\varepsilon > 0$ less and profit. If they offer 0, G is indifferent between accepting and rejecting, and thus accepts the offer by our tie-breaking condition. R cannot do better as they cannot offer any amount smaller than 0.

One point was awarded for the answer. One point was awarded for the justification.

Part BI (1) $\frac{1}{\delta} \times 0.7$ in the second period. We solve the equation that $\delta x = 0.7$ where x is the offer in the second period and δx is the present value of the offer today.

One point was awarded for the correct answer.

Part BII (2) In the second stage G is the one offering, and they will in particular offer 0 because of the same logic as in Part (A).

One point was awarded for the answer. A second point was awarded for the justification (e.g. through referencing Part A).

Part BIII (2) To make G indifferent, they know that in the second period G can get 1, so they must offer $\delta \times 1 = \delta$.

One point was given for the answer. A second point was awarded for the justification.

Part BIV (2) Clearly R will not offer more, and if R offers less then G will reject to claim all of the surplus in the next period which makes them better off from today's perspective. However when R offers G exactly δ, the only possible deviation (reject) will not make them better off. At indifference, they accept, and the game ends.

Both points were awarded for explanation: one for showing R cannot profitably deviate, and one for showing G cannot. One point was deducted if no intertemporal considerations were mentioned (e.g. invoking G's ability to offer in the second period) which was necessary to show that G could not profitably deviate.

Part C (3) When there are three players we use the same logic as before. In the last period, R will offer 0 and thus get 1. In the penultimate period, G will offer R the discounted value they would obtain if they rejected (henceforth the *continuation value*) and thus offer δ and receiver $(1 - \delta)$. Finally, in the first period R will offer $\delta(1 - \delta)$ to G to make them indifferent between accepting and rejecting.

One point is awarded for the answer. One point is given for reasoning backwards. One point is given for correct continuation values at each point in the game.

Part D (4) This is a one period game where R offers and G gets outside option $(1 - x)$. Thus, R must offer $\delta(1-x)$. Given this formula, we know the 4 period game is just a one period game attached to a three period game where in the second part of the game (the one with three periods), (R, G) thus get $(\delta(1-\delta), 1-\delta(1-\delta))$, respectively (note that G here is the one offering at the beginning of the second portion of the game). R must then offer $\delta(1 - \delta(1 - \delta))$ to match G's continuation value. By a similar logic, with 5 periods, R will have to offer G $\delta(1 - \delta(1 - \delta(1 - \delta)))$ in order to keep them indifferent. Note alternative this can be seen as the alternating partial sum $\delta \sum_{k=0}^{t} (-\delta)^k$ as the payoff, and so for example a solution would take the form

$$\delta - \delta^2 + \delta^3 - \delta^4 \dots$$

One point was given for the first solution. For closed form solutions of 4 and 5 periods, 1 point in total was given. 2 points were given for logic and reasoning.

Part E (4) The key trick here is to note that we need only consider offers. Let N denote the total number of periods and let $\{y_t\}_{t=0}^{N}$ be *reverse* indexed offers from each period (so y_0 is the last period). Note y_0 is 0 as you offer nothing in the last period. If the offer in some period is y_t, then the individual offering in period $t + 1$ knows, by Part (D), that the must offer $\delta(1 - y_t) = \delta - \delta y_t$ to satisfy the fact the person who is not offering is indifferent to accepting and rejecting in the next period. Thus, $y_{t+1} = \delta - \delta y_t$. Using the hint with $a = \delta$ and $b = -\delta$ we obtain that

$$y_t = \sum_{k=0}^{t-1} \delta(-\delta)^k \implies \lim_{t \to \infty} y_t = \frac{\delta}{1 + \delta}$$

since $y_0 = 0$ for all t.

Any faithful attempt was given one point. One point was given for indexing offers from the last period. One point was given for recognizing a and b in the linear difference equation. One point was given for a correct final answer.

Incorrect methodological attempts which demonstrated understanding of the underlying economic reasoning were eligible to receive up to two points.

Part F (3) As $n \to \infty$, the first term tends to 0, and the other term is simply a geometric series with coefficient δ^2. Hence the equilibrium is exactly

$$\frac{\delta}{1+\delta}$$

as the equilibrium offer given this logic.

One point was given for a good faith attempt. Two points were given for a correct solution (by recognizing an infinite geometric sum). Solutions which were correct given the typo in the hint for Part (E) received full credit provided their solution was consistent.

Part G (1) A point was given for any potential application. One example is two companies who were seeking to divide a piece of land.

Bonus (3) The logic is similar as before but now we have to group individuals by considering *two* periods. Without loss start in a period with an offer given by individual G in period y_{t-2}. In period y_{t-1}, their continuation value is exactly $\delta_G(1-y_{t-2}) = \delta_G - \delta_G y_{t-2}$. From here, person R obtains $1-y_{t-1}$ but discounts at R_A, and so person G at y_t must offer $\delta_R(1-y_{t-1})$. Expanding, we have that

$$y_t = \delta_R - \delta_R\delta_G + \delta_R\delta_G y_{t-2}$$

which is a second order linear difference equation, but which can be solved in a similar way. In particular, taking $a = \delta_R - \delta_R\delta_A$ and $b = \delta_R\delta_G$ and only considering every *other* time and reindexing these as s instead, then when G offers gives that

$$y_s = (\delta_R - \delta_R\delta_G)\sum_{k=0}^{s-1}(\delta_R\delta_G)^k \implies \lim_{s\to\infty} y_s = \frac{\delta_R(1-\delta_G)}{1-\delta_R\delta_G}$$

So person G, if they go first, will offer the amount on the right hand side.

One point was given for noting that time could be collapsed to every other period. One point was given for setting up the finite-time difference equation. One point was given for a correct solution.

Problem 3: Winners Win: When More is Better

Part A (3) Note that $u(x,t)$ is (weakly) negative for any values of x and t, so this function is maximized if it outputs 0. This happens exactly when $x = t$ is chosen for all t. Thus, as t increases, x must increase as well.

Two points were given for recognizing $x^*(t) = t$. One point was given for noting this is an increasing relationship.

Part B (2) The key is that as the type increases, (e.g. $t' > t$), the differences in x increase: quite literally increasing differences: bigger types lead to bigger marginal gains.

If the word marginal or anything to that effect is used, one point was awarded. A second point was awarded for giving economic meaning to t and x.

Part C (7) We prove the theorem without giving subparts. Assume not, so that $x^*(t)$ is not monotone increasing in t. then there exists $t_1 < t_2$ s.t. $x^*(t_1) > x^*(t_2)$. By definition of $x^*(t)$ and the fact the optimum is unique, it must be that

$$u(x^*(t_1), t_1) > u(x^*(t_2), t_1) \text{ and } u(x^*(t_2), t_2) > u(x^*(t_1), t_2)$$

Adding together elements on their respective sides of the inequality gives

$$u(x^*(t_1), t_1) + u(x^*(t_2), t_2) > u(x^*(t_2), t_1) + u(x^*(t_1), t_2)$$

Taking differences to match types to actions gives that this implies

$$u(x^*(t_1), t_1) - u(x^*(t_2), t_1) > u(x^*(t_1), t_2) - u(x^*(t_2), t_1)$$

which contradicts the increasing differences assumption as $t_1 < t_2$ but $x^*(t_1) > x^*(t_2)$, and we are done.

In the two parts with multiple points, one point was given in (II) for writing each of the two inequalities out. In (III), one point was given for adding the inequalities together, one for matching types across the inequality, and one for noting this contradicts increasing differences.

Part D (4) First, we use the differentability argument. Differentiating in x gives $-2t(x-t)$; differentiating in t gives $-2x+4t$. Since we know the optimal solution occurs around $x=t$, this is increasing in t *in the domain of the optimal solution*, which is sufficient for the desired comparative statics result. Intuitively, note that $u(x',t') - u(x,t')$ is *almost* a derivative: if this is increasing, then $\frac{u(x',t')-u(x,t')}{x'-x}$ as $x' \to x$ will be increasing as well, but this is just the partial derivative with respect to x.

One point was awarded for using the differential definition. A second point was awarded for noting the need to only consider "local" behavior. Two points were awarded for either a mathematical or economic intuition for the equivalence.

Part E (2) If the derivative with respect to x and then t is increasing, then the first function, e.g. the derivative of u with respect to x must be increasing by the equivalent definition from Part (D).

Two points were given for a correct answer; one for noting $u_x(x,t)$ is increasing in t and one for using the equivalent definition.

Part F (4) The choice of the firm is the quantity and the parameter is the cost. Taking the second cross partial gives $\frac{\partial^2 \pi}{\partial q \partial c} = -1 < 0$ everywhere. However, if we instead consider $-c$, then the partial is 1; so the function is increasing differences in *negative cost*: thus, as $-c$ increases, (or c decreases), q will increase. Thus, $q^*(c)$ is decreasing in c, as desired.

One point is given for showing the function is *submodular* in cost (e.g. decreasing differences) or increasing in negative cost. One point is given for letting $t=-c$. A third point is given for invoking Part (E) to achieve the monotone comparative statics result.

Part G (3) The firm now has a subsidy. We first solve for the first order condition: this is univariate unconstrained optimization. The FONC is

$$\frac{\partial \pi}{\partial q} = \left(100 - 5q - c + \frac{1}{4}cq\right) + \left(-5 + \frac{1}{4}c\right) q = 0$$

Grouping terms and rearranging gives that

$$2q\left(5 - \frac{1}{4}c\right) = 100 - c \implies q = \frac{100-c}{10 - \frac{1}{2}c}$$

Quickly verifying the second order condition to ensure we are at a maximum gives that we require

$$-10 + \frac{1}{4}c < 0$$

which gives that we are at a local maximum for any $c < 40$, and so we consider only this case (otherwise, no interior solution exists!) Over this range, we have $q^*(c)$. Clearly this function is not monotone decreasing; for example, when $c=0$, $q^*(c)=10$, while if $c=10$, $q^*(c)=45$. Moreover, it is not monotone increasing either: if $c=20$, then the optimal $q^*(c)=-8$, even though this is a local maximum. This contradicts our argument in Part (F) for any parametricization of the type space, though this is consistent with our monotone comparative statics result as the profit function in this scenario is not increasing differences.

One point was given for finding $q^*(c)$. One point was given for the desired example. One point was given for noting that the function is not increasing differences.

Bonus (3) In some sense this proof is simpler. Assume not, so that there exist $x^*(t_1)$ and $x^*(t_2)$ s.t. $x^*(t_1) > x^*(t_2)$ but $t_1 < t_2$, and $u(x,t)$ is single crossing. The same set of inequalities hold:

$$u(x^*(t_1), t_1) > u(x^*(t_2), t_1) \text{ and } u(x^*(t_2), t_2) > u(x^*(t_1), t_2)$$

Difference the two inequalities by subtracting t

$$u(x^*(t_1), t_1) - u^*(x^*(t_2), t_1) > 0 \text{ but } 0 > u(x^*(t_1), t_2) - u(x^*(t_2), t_2)$$

Setting $x^*(t_1) = x' > x = x^*(t_2)$ and $t_1 = t < t' = t_2$ makes it clear this contradicts single crossing.

This bonus was all or nothing; all three points were awarded for a correct answer and zero points were given for an incorrect answer.