NET 2022 Answers

Power Round - Advanced Macroeconomics Division

May 20, 2022

Problem 1: In the Long Run, we are all converging (to steady state) (25 Points)

Part A (2) Cobb Douglas production functions do. Fix $z > 0$ and note

$$f(zk, zn) = (zk)^{\alpha}(nk)^{1-\alpha} = z^{\alpha}z^{1-\alpha}k^{\alpha}n^{(1-\alpha)} = zf(k, n)$$

But constant elasticity of substitution functions do not. Consider for example $c = 1$, $z = 2$, and $\sigma = \frac{1}{2}$. Then

$$u_{\frac{1}{2}}(zc) = \frac{\sqrt{2}}{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \neq 2\frac{1}{\frac{1}{2}} = 1$$

Though of course any other counterexample also works.

One point was given for recognizing cobb-douglas production is CRS. One point was given for noting CES functions are not CRS.

Part B (2) The derivative is linear; note that

$$\frac{\partial \mathcal{U}}{\partial c_t} = \frac{\partial}{\partial c_t}\sum_{t=0}^{\infty}\beta^t u(c_t) = \sum_{t=0}^{\infty}\beta^t\frac{\partial u(c_t)}{\partial c_t} = \beta^t u'(c_t)$$

as all other terms zero.

One point was awarded for distribution the derivative into the sum. One point was awarded for a correct final answer.

Part C (2) $f(k_t, n_t)$ is increasing in n_t and there is no cost of increasing the share of labor today (e.g. no cost of leisure) so it is optimal for everyone to be employed.

Two points were given for a correct answer. No partial credit was awarded for this section.

Part D (2) If $i_t = 0$, then $k_{t+1} = \delta^t k_0$ and simply depreciates over time as there is no new investment. If everything is invested but there is no carry-over in capital, then the entire sequence is just $\{f^n(k_0)\}_{n\in\mathbb{N}}$.

One point was given for each correct answer in the two scenarios.

Part E (2) Given the variables left, the key is to eliminate i_t. Since $i_t = f(k_t, n_t) - c_t$, we can substitute the law of motion to be

$$c_t + k_{t+1} = \delta k_t + f(k_t, n_t)$$

which simply says consumption and capital tomorrow must equal the total resources an individual has on hand today (net depreciation): this is an allocation problem.

One point is given for a good faith attempt. A second point is given for a correct answer.

Part F (3) From Part (E) we have that

$$c_t = -k_{t+1} + \delta k_t + k_t^{\alpha}$$

recalling $n_t = 1$, which gives the desired functional relationship.

Two points are given for finding c_t. One point is for the correct relationship of $f(k_t, 1)$ that does not leave n_t as a variable. One point is for a correct representation of the problem.

Part G (4) The value k_{t+1} shows up at time t and time $t + 1$; in particular, the chained equations are

$$\beta^t u(c_t(k_t, k_{t+1})) + \beta^{t+1}u(c_{t+1}(k_{t+1}, k_{t+2}))$$

Applying the chain rule and differentiating accordingly gives that

$$-\beta^t u'(c_t(k_t, k_{t+1})) + \beta^{t+1} u'(c_{t+1}(k_{t+1}, k_{t+2}))[\alpha k_{t+1}^{\alpha-1} + \delta] = 0$$

Recall $u'(c_t) = c_t^{-\sigma}$; plugging everything in gives

$$\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} = \beta\left(\alpha k_{t+1}^{\alpha-1} + \delta\right)$$

Which is a necessary condition for optimum.

One point was given for isolating the correct (relevant) variables. A second point was given obtaining the correct first order condition. Two points were given for a correct answer.

Part H (4) Using the steady-state value, we know $c_t = c_{t+1}$ so the left hand side of the Euler equation is 1. Rearranging then gives

$$\beta[\alpha k^{\alpha-1} + \delta] = 1$$

Our goal is to solve for k; in particular, we get that

$$k = \left(\frac{1}{\alpha}\left(\frac{1}{\beta} - \delta\right)\right)^{\frac{1}{\alpha-1}}$$

Which is exactly the steady-state level of capital. Note that we can rearrange and simplify this: in particular, by multiplying the α through and combining denominators we obtain

$$k = \left(\frac{1-\delta\beta}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\alpha\beta}{1-\delta\beta}\right)^{\frac{1}{1-\alpha}}$$

One point was given for dropping time indices. One point was given for a correct use of the Euler equation. One point was given for correct algebraic work. A final point was given for the correct answer.

Part I (2) First, the static in β. Differentiate the function to obtain that

$$\frac{\alpha(1-\delta\beta) + \alpha\beta\delta}{(1-\alpha)(1-\delta\beta)^2}\left(\frac{\alpha\beta}{1-\delta\beta}\right)^{\frac{\alpha}{1-\alpha}}$$

is the derivative with respect to β. Clearly, since $\alpha, \beta, \delta \in (0,1)$, all terms are positive and thus this is a positive derivative.

In economic intuition, this argument says that as individuals value consumption in *future* periods more, capital (and thus investment) will increase in steady-state equilibrium.

One point was given for the derivation for β, and one point was given for the intuition.

The part of the question in α was converted to extra credit due to computational intractability.

Consider first $e^{\ln(k)} = k$ to differentiate the function somewhat more easily; in particular, one has

$$k = \exp\left\{\ln\left(\left(\frac{\alpha\beta}{1-\delta\beta}\right)^{\frac{1}{1-\alpha}}\right)\right\} = \exp\left\{\frac{1}{1-\alpha}\ln\left(\frac{\alpha\beta}{1-\delta\beta}\right)\right\}$$

This we can differentiate using the chain rule. In particular, differentiating with respect to α gives

$$\exp\left\{\frac{1}{1-\alpha}\ln\left(\frac{\alpha\beta}{1-\delta\beta}\right)\right\}\left[\frac{-1}{(1-\alpha)^2}\left(\ln\left(\frac{\alpha\beta}{1-\delta\beta}\right)\right) + \frac{\beta}{(1-\alpha)\alpha\beta}\right]$$

Our goal is to sign this derivative. First, note that the exponential is always nonnegative. Second, the term outside the exponential is guaranteed to be positive if

$$\frac{\alpha\beta}{1-\delta\beta} < 1$$

in which case an increase in α would increase the steady-state level of capital. However, if this is not true, the effect is ambiguous, and thus we cannot say much when this fails to hold. (The intuition is that there is a nonmonotonicity in optimal capital, as if capital allocation is perfectly efficient or productive than one may not need to save as much and can consume more instead. How the productivity of capital, α, interacts with depreciation and the discount rate is vitally important to determining which of the two effects dominates.

Two additional extra credit points were given for identifying a necessary condition for a monotone comparative statics attempt. One extra credit point was given for any attempt.

Part J (2) One point was given for each criticism of the neoclassical model that was considered. These include:

(1) The model considers a single representative agent with monotone preferences, which is unrealistic.

(2) The model assumes perfect knowledge of all processes and does not accomodate any potential shocks.

(3) The model assumes consumption is a univariate representative function c_t, and has no heterogenous resource or consumption constraints.

Though other answers were accepted as well.

Problem 2: Search on the Labor Market (25 Points)

Part A (2) Constant returns to scale is a regularity condition; as the number of unemployed people and vacancies increase by some constant amount, so does the number of matches. Consistency requires that no matches occur if either no one is unemployed or three are no vacancies,

One point was given for each explanation.

Part B (2) Using constant returns to scale, we have

$$\frac{m(u,v)}{u} = m\left(\frac{u}{u}, \frac{v}{u}\right) = m(1,\theta)$$

and so $m(u,v) = um(1,\theta)$.

One point was given for applying constant returns to scale. A second was given for obtain the function with $(1,\theta)$ in its arguments.

Part C (1) Some rearranging allows us to write

$$f_t = \frac{m(u_t,v_t)}{u_t} = m(1,\theta) \text{ and } q_t = \frac{m(u_t,v_t)}{v_t} = m(\theta^{-1},1)$$

using the above logic and Part (B). Thus, we have that f_t is the *probability* an unemployed individual is matched at time t and q_t is the *probability* that a firm posting a vacancy at time t finds a worker.

One point was given for each correct interpretation.

Part D (2) WE consider two components; first, your value for being employed includes your payoff from employment today, which is w_t. Second, it includes your expected *continuation payoff*, which is the discounted expected value of being employed tomorrow (note the recursive structure).

One point was given for interpreting w_t, and a second for the term inside the expectation.

Part E (2) This interpretation is similar to the logic we gave above. First, $(y_t - w_t)$ is the value of employment today. The term inside the expectation is the discounted expected value tomorrow given the fact that a worker may be separated from a firm.

One point was given for interpreting $(y_t - w_t)$. A second point was given for interpreting the term inside the expectation.

Part F (3) c_1 is exactly the value b while k is c_2. Clearly, (1) is U_t as the expectation for tomorrow is taken over employment and unemployment, indicating a worker who is looking for a job. (2) is clearly V_t as there is a penalization for posting a job, and then the expected value of finding a job tomorrow (weighted by the relevant probabilities).

One point was given for each successful match. A third point was given for correctly matching c_1 and c_2 to k and b.

Part G (2) From Part (F) and free entry, we know that $V = 0$ always, and so we have that

$$V = -k + \beta(q(\theta)J(w) + (1 - q(\theta)0) \implies 0 = -k + \beta q(\theta)J(w) \implies \frac{k}{\beta q(\theta)} = J(w)$$

From here, note Part (E) gives that

$$J(w) = (y - w) + \beta[\lambda 0 + (1 - \lambda)J(w)] \implies J(w) = (y - w) + \beta(1 - \lambda)J(w) \implies J(w) = \frac{(y - w)}{1 - \beta(1 - \lambda)}$$

Combining these two gives that

$$\frac{k}{\beta q(\theta)} = \frac{(y - w)}{1 - \beta(1 - \lambda)}$$

One point was given for substituting the equation from Part (F) correctly. One point was given for substituting the equation from Part (E) and obtaining a correct final solution.

Part H (2) u individuals are unemployed and matched with probability $f(\theta)$, while $(1 - u)$ individuals are unemployed and fired with probability λ. In steady-state, these two quantities must be the same, and so

$$uf(\theta) = (1 - u)\lambda \implies u = \frac{\lambda}{f(\theta) + \lambda}$$

One point was given for noting correctly the inflows and outflows into unemployment. A second point was given for a correct answer.

Part I (2) We know $V = 0$; thus, we have that $J = (1 - \gamma)S$, so $S = \frac{J}{(1-\gamma)}$. Substituting this into $(E - U)$ gives that $(E - U) = \frac{\gamma}{(1-\gamma)}J$.

One point was given for recognizing $V = 0$ and writing S as a function of (J, γ). A second was given for the correct answer.

Part J (2) Recognize

$$q_t = \frac{m(u_t, v_t)}{v_t} = m(\theta^{-1}, 1) = \theta m(1, \theta) = \theta f_t$$

which finishes the desired argument, using only the definitions of the argument and constant returns to scale.

One point was given for using constant returns to scale in the argument. A second point was given for a correct answer.

Part K (3) There are three endogenous variables and three equations that must hold in steady state: exactly the equations obtained in Parts (G, H, I). First, substitute J from Part (G) as well as explicitly find $(E - U)$ recursively by differencing and taking the representation so that the wage-setting curve is written as a function of endogenous objects and model primitives. Second, use Part (J) to simplify $f(\theta)$ and $q(\theta)$ so that only one of them is present. Third, use standard algebraic tricks to pin down the three variables using the three identified equations. A solution exists, and moreover can be found generically using e.g. MATLAB.

One point was given for each step.

Part L (2) When b increases, we expect that w will increase. Economically, individuals must be paid more as the unemployment benefit increases, as they will no longer accept wages which are below b. In the model, an increase in b will decrease $E - U$ as the unemployment benefit increases. If γ is constant, then J must decrease for the wage setting curve to clear, which, if other model primitives are fixed, would require an increase in w by inspecting the job creation curve.

One point was given for any reasonable economic intuition. A second point was given for reasoning through the aspects of the model.

BONUS I and BONUS II: *These bonuses were not included for brevity and clarity of the exam solutions. For those interested, please email the tournament directly.*

Problem 3: Where are all the Missing Women? (23 Points)

Part A.i In Western Europe, we calculate the newborn male:female ratio as $\frac{5383}{5127} \approx 1.05$. In India, we get $\frac{63743}{58216} \approx 1.095$. The difference is then approximately 0.045. We then want to solve for a general solution for the number of missing women like so: to set the ratios equal, we want the δ that solves:

$$\frac{M_E}{W_E} = \frac{M_I}{W_I + \delta} \implies \frac{W_E}{M_E} = \frac{W_I + \delta}{M_I} \implies \frac{M_I W_E}{M_E} = W_I + \delta$$

$$\implies \delta = \frac{M_I W_E}{M_E} - W_I = \frac{M_I W_E - W_I M_E}{M_E}$$

but there are other ways to solve for this. Plugging in values for M_E, W_E, M_I, W_I, we get $\delta = 2496$ thousand people, or 2496000. One point is awarded for the ratios, one point is awarded for the difference, and one point is awarded for the number of missing women.

Part A.ii It looks like Western Europe lines up pretty well with this expectation, but India's male:female sex ratio is significantly higher among newborns. The unfortunate reality behind this statistic is that, in many developing nations, sex-selective abortions and infanticide take place. This is one of the reasons that the phenomenon is called "missing women" rather than "excessive men" - it is a subtractive process, not an additive one.

Part B Using values from the second row, we get $\frac{M_E}{W_E} \approx 1.045$, $\frac{M_I}{W_I} \approx 1.079$, and the difference is 0.034. The number of missing young adults is $\delta = 1395$ thousand. If we continue to assume that Western Europe is basically the "natural" sex ratio, it looks like the severity of the disparity has decreased: clearly, no new young adults can be introduced into the population (other than possibly through immigration, which one would have to argue being female-dominated in India among these ages), so it must be that boys disproportionately do not survive childhood in India compared to Western Europe. In the presence of child labor in India, girls tend to work in less physically intensive jobs than boys, so this makes intuitive sense. Other answers also acceptable. Two points awarded for the numbers and two points awarded for the explanation.

Part C Using values from the third row, we get $\frac{M_E}{W_E} \approx 0.713$, $\frac{M_I}{W_I} \approx 1.031$, and the difference is 0.318. The number of missing young seniors is $\delta = 3161$ thousand. Again assuming that Western Europe is the "natural" sex ratio, the severity of the disparity has greatly increased. There are a number of valid explanations for this: one might be that, if women have very limited opportunities in the workforce, women who are widowed or unmarried are much less likely to reach advanced age than widowed or unmarried men. Alternatively, one could imagine that, if men are the primary breadwinners and household decision makers, household decisions on health care will favor them over women, leading to disparate death rates among common health problems. There is also the problem of childbirth: especially in with this older cohort (who were giving birth under a less advanced medical system), deaths during childbirth will reduce the number of women reaching these advanced ages. Altogether many good interpretations here - other answers also acceptable. Two points awarded for the numbers and two points awarded for the explanation.

Part D The first problem is geographical. Different problems might affect different genders differently, and the climates of the two regions are very different. Therefore, even if India was "fully developed" in the same way that Western Europe is (however you define "developed"), we might not expect the sex ratios to be perfectly equal, so Western Europe might not be a perfect counterfactual. The second problem is intertemporal. We are comparing cohorts born in different years: the late 80s, the early 70s, and the early 20s. Therefore, when we talk about movements in the differential sex ratio across these generations as we do here, they aren't actually starting at the same point. It might be (and, if you check the data source, it is) the case that the sex ratios at birth among the two older cohorts are different from those born in the early 90s (some teams smartly noted WWII as a reason why the 1920s European cohort might be more female-skewed than the 1920s Indian cohort, *ceteris paribus*), so we are actually seeing differences in age-related death disparities *and* differences in initial conditions! What we really *should* have done is compared the sex ratios over time of people born in the same year, instead of using different birth cohorts. Other answers also acceptable. Two points awarded for each answer.

Part E If we view having a child as an investment, the phenomenon becomes economically obvious. When a family has a child, they at some point observe the gender (either in the womb or at birth). Then, they can decide to either make the investment (i.e. raise the child) or not (see part A.ii). If the value of the investment is equal to the anticipated gains from using the child's labor minus the cost of raising the child (plus any future care they might provide to the parents after retirement), then an increase in the price of tea improve the return on "girl" investment relative to the return on "boy" investment in regions where tea is grown. Therefore, in those regions, we would see comparatively more people invest in girls than in boys, which is exactly what Qian observes. Many respondents also accurately discussed this phenomenon in terms of comparative advantage. However, children are not investments, they are children. If writing the explanation above felt wrong or gross in some way, good! "Sexist baby murder" is somehow obviously wrong - the fact that we can explain it using financial language doesn't change that. However, it is also true that many poor farmers are put in very tough situations. It may be the case that a farm desperately needs more hands to work on it, but would also struggle to feed another mouth - if their farm has a deeper need for male labor than female labor, this could drive such a decision. Points awarded to a well thought out answer. For the example of another instance of this "rational$\neq$moral" phenomenon, there were many very good answers. Three particularly good responses discussed the use of psychologically deceptive marketing practices (Lexington HS), outsourcing to foreign sweatshops in the fast-fashion industry (University HS), and the use of the death penalty (Paul Laurence Dunbar). Two points for each question asked (economics, ethics, and example).