# NET 2022 Power Round

Advanced Division: Macroeconomics

April 2022

### Instructions

This is the **macroeconomics portion** of the advanced division of the 2022 Northwestern Economics Tournament Power Round. There are three questions of *unequal* weight, accounting for a weighted *half* your score for the Power Round. You are encouraged to work together on these questions. Answer each question as clearly and succinctly as possible. You may write on a blank sheet of paper where you *clearly indicate* where your answer to each part is. It will be useful to note that a question's point value is *not* informative of its difficulty; to ensure a fair test, some (longer) easy questions are worth more points, while some (shorter) hard questions are worth less points, and vice versa. If you are unsure of your answer, take your best guess: there is no penalty for incorrect answers. If you find yourself stuck on a question, skip it and return to it at the end if necessary. It is recommended you spend approximately seventy five (75) minutes of the total exam time on this portion. Remember, we do *not* share your answers or scores with Northwestern admissions, nor do we keep them for ourselves. You are not expected to know how to answer each question on the exam; rather, this test is designed to assess your economic and formal reasoning skills. Have fun, and good luck!

## Problem 1: In the Long Run, we are all converging (to steady state) (25 points)

*This problem considers the neoclassical model of economic growth to deliver a striking prediction about the steady-state equilibrium. This problem requires substantial use of tools from calculus. Do not be intimidated! The problem is broken up into steps with the goal of being particularly tractable.*

We will start by setting up and proving some basic facts about our model economy. In particular, we consider a setting where there is an infinitely-lived representative consumer, some production function, and a central planner seeking to maximize the utility of the representative consumer.

**Part A (2)** The class of constant elasticity of substitution functions are the functions

$$u_\sigma(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $0 < \sigma < 1$. The class of Cobb-Douglas production functions are $f(k,n) = k^\alpha n^{1-\alpha}$ for some $0 \leq \alpha \leq 1$. Do Cobb-Douglas functions exhibit constant returns to scale? What about constant elasticity of substitution functions?[1]

**Part B (2)** Suppose our representative consumer's consumption in time $t$ is $c_t$ and derives utility from there; so time $t$ utility is given by some $u : \mathbb{R} \to \mathbb{R}$. Aggregate utility is the discounted-sum-of-utility; that is, for a stream of consumption $\{c_t\}_{t=0}^\infty$, the consumer's utility is

$$\mathcal{U}(c_1, \ldots c_n \ldots) = \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ where } u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\mathcal{U} : \mathbb{R}^\infty \to \mathbb{R}$ takes a consumption stream and returns the discounted lifetime utility given that stream $\{c_t\}$, and where $\beta, \sigma \in (0,1)$. $\beta$ is a measure of how much one discounts the future, e.g. how much one "unit" of consumption would be worth tomorrow relative to today. $\sigma$ is a measure of relative risk aversion (how sensitive you are to changes across time in the consumption stream). Find $\frac{\partial \mathcal{U}}{\partial c_t}$ for any arbitrary $t$. *(Hint: Only 1 term in the infinite sum depends on $c_t$, the rest depend on $c_i$ for $i \neq t$. Therefore, the partial derivative $\frac{\partial \mathcal{U}}{\partial c_t}$ is simply equal to the derivative of this term with respect to $c_t$.)*

**Part C (2)** The country faces an *aggregate* resource constraint; in particular, the total resources they can split between consumption $c_t$ and investment $i_t$ is a function of the total amount of capital and labor in the economy; that is, $c_t + i_t \leq f(k_t, n_t)$ where $f(k_t, n_t)$ is the Cobb-Douglas function from Part (A). $n_t$ is the *proportion* of the labor force in use, so $n_t = \frac{1}{2}$ would imply 50% of workers are employed at time $t$. Explain why it is optimal for $n_t = 1$.

**Part D (2)** Aside from the *intratemporal* resource constraint (i.e. the static constraint in each time period), we also have a *dynamic* constraint that relates resources in *each* period, which is government by the law of motion: $k_{t+1} = i_t + \delta k_t$ for some $\delta \in [0,1]$. That is, the amount of capital available in each future period is equal to the investment made in the previous period plus the amount of undepreciated capital (where $(1-\delta)k_t$ amount of capital depreciates in each period). Assume that there is some initial stock of capital $k_0$ at time $t = 0$. Describe how the path of capital $\{k_t\}$ changes over time in each of the following mechanical investment scenarios:

(1) $i_t = 0$ for all time periods $t$.

(2) $i_t = f(k_t, n_t)$ in each time period $t$ and $\delta = 0$.

**Part E (2)** With these basic preliminary facts in hand, we can write our planner's macroeconomic problem in the standard economic way: as a utility-maximization problem with respect to some constraints. In particular, the planner maximizes choices of consumption and investment subject to their discounted stream of utility:

$$\max_{\{c_t, i_t\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \text{ with constraint } c_t + i_t = f(k_t, n_t) \text{ and } k_{t+1} = \delta k_t + i_t$$

[1] Reminder: A function $f : \mathbb{R} \to \mathbb{R}$ exhibits CRS if $f(cx) = cf(x)$ for all positive constants $c$; $f : \mathbb{R}^2 \to \mathbb{R}$ exhibits CRS if $f(cx, cy) = cf(x, y)$ for all positive constants $c$.

However, this leaves us with *two* constraints, which is unnecessarily complicated. Combine the two constrains in the problem to eliminate $i_t$. so there is a *single* budget constraint as a function of $(c_t, k_t, k_{t+1})$.

**Part F (3)** Furthermore, we can also rewrite the maximization problem so that $c_t$ is a function of $k_t$ and $k_{t+1}$ using the remaining constraint. Substitute this into the utility function so that your problem simplifies to something of form

$$\max_{\{k_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t(k_t, k_{t+1})^{1-\sigma}}{1-\sigma} \text{ with constraint } \textbf{(blank)}$$

where **(blank)** is the answer from Part (E) and $c_t(k_t, k_{t+1})$ is derived in this part. *(Hint: $f(k_t, n_t)$ has a functional form given in Part (A); use this to explicit substitute as a function of $k_t$ using a fact about $n_t$ derived in Part (C)).*

**Part G (4)** We can solve this problem using standard tools from calculus[2]. To see this, note the problem essentially looks "the same" at every time period. Thus, we can differentiate for any one $t$ and obtain the *same* first order condition in each time period. Find this derivative in some variable and set it to 0 to obtain a *necessary* condition for an optimum. (Note this is called the *Euler equation* formally in discrete-time dynamic programming, which you are doing!). *(Hint: $k_{t+1}$ only shows up in two time periods: period $t$ and period $t+1$. It may be easier to abstract your argument into $c_t(k_t, k_{t+1})$ and apply the chain rule!)*

**Part H (4)** A *steady-state equilibrium* is one where in each period, the problem looks the same; that is, $c_t = c$, $k_t = k$, $n_t = n$ for all $t$. Using the Euler equation you found above, find the steady-state equilibrium of this economy in terms of the parameters $(A, \delta, \beta, \alpha)$.

**Part I (2)** Take your steady-state value for capital, $k$ in equilibrium. What happens to this value when $\beta$ increases? When $\alpha$ increases? Interpret these results using economic intuition.

**Part J (2)** Suppose you were a policymaker criticizing the neoclassical model as presented in this problem. Isolate two specific shortcomings of the neoclassical model.

[2] A useful hint here: the *partial derivative* $\frac{df}{dk}$ of a multivariate function $f$ with respect to a variable $k$ is just the derivative with respect to $k$ treating all of the other variables as constants.

## Problem 2: Search on the Labor Market (25 points)

*In this problem, we consider the Nobel-prize winning work of Northwestern professor emeritus Dale Mortensen with coauthors Peter Diamond and Chris Pissarides, investigating a model of matching in labor markets. This problem will require some tools from calculus. We do not recommend attempting the bonuses unless you find yourself with a significant surplus of time.*

**Part A (2)** Consider a labor market with people and firms. At any one time, there exist some proportion $0 \leq u \leq 1$ of unemployed people looking for jobs and some proportion $0 \leq v \leq 1$ vacancies posted by firms looking to hire. Define a *matching function* $m : [0,1]^2 \to \mathbb{R}$ to take in the total quantity of *successful* matches found in an economy. We will put the following parametric assumptions on the matching function:

(1) Constant returns to scale, so $\alpha m(u,v) = m(\alpha u, \alpha v)$.

(2) The matching function is "consistent":

$$m(u,v) \in [0, \min\{u,v\}] \text{ for all } (u,v) \text{ and } m(0,v) = 0, m(u,0) = 0$$

Interpret these conditions intuitively.

**Part B (2)** Write the matching problem as a function of one variable, $\theta = \frac{v}{u}$ and interpret what this ratio is economically. *(Hint: Use the fact that $m(u,v)$ is constant returns to scale and consistent, as per Part (A)).*

**Part C (1)** Let

$$f_t = m(1,\theta) \text{ and } q_t = m\left(\frac{1}{\theta}, 1\right)$$

Interpret $f_t$ and $q_t$ economically as a *probabilities*.

**Part D (2)** The DMP model's key insight is that any model of employment must consider both *labor inflows* and *labor outflows* on both the supply and demand side of the market. Let $U_t$ be the value of unemployment to some worker at time $t$. Then the payoff they get for being employed today at wage $w_t$ is

$$E(w_t) = w_t + \beta \mathbb{E}[\lambda U_{t+1} + (1-\lambda)E(w_{t+1})]$$

where $\mathbb{E}$ is an expectations operator, e.g. denoted what they expect to happen in future periods, $0 < \lambda < 1$ is some probability that they are fired in the next period, and $0 < \beta < 1$ is some discount term (e.g. how much you "discount" the future). Using economic logic, explain why this is a reasonable way to define the value of being employed today.

**Part E (2)** Similarly, the firm retains some value to having a worker. Assume that in every period they produce some output that gives payout $y_t$. Then their utility function for a worker at wage $w_t$ is

$$J_t(w_t) = (y_t - w_t) + \beta \mathbb{E}[\lambda V_t + (1-\lambda)J(w_{t+1})]$$

where $V_t$ is the value of a firm who has not hired a worker. Economically, explain why this makes sense.

**Part F (3)** In parts (D) and (E), we assumed that the values of unemployment and unmatched firms were "fixed" at $U_t$ and $V_t$, though of course we can define them analogously as well, in particular by pulling on our answer from Part (C). Consider the following two questions:

(1) $c_1 + \beta \mathbb{E}[f_t E(w_{t+1}) + (1-f_t)U_{t+1}]$

(2) $-c_2 + \beta \mathbb{E}[q_t J(w_{t+1}) + (1-q_t)V_{t+1}]$.

where $c_1$ and $c_2$ are constants that differ based on the equation in question. Let $b \in \mathbb{R}$ be a constant that represents the "unemployment benefit" at time $t$ and let $k \in \mathbb{R}$ represent the cost to a firm to post a vacancy (e.g. search for a worker) at time $t$. Given the two equations above and the constants $(k,b)$, (1) explain which one is $U_t$ and which one is $V_t$ and (2) match $c_1$ and $c_2$ with $k$ and $b$. Justify your answer.

**Part G (2)** From here, assume that we are in a steady-state: that is, $w_t = w_{t+1}$ (and there is no uncertainty), $V_t = V_{t+1}$, $U_t = U_{t+1}$, etc. Moreover, assume *free entry*: that is, $V = 0$. Using your answer from Part (F) first and subsequently your answer from Part (E) for $J$, complete the equation for the *job creation curve*:

$$\frac{k}{\beta q(\theta)} =$$

*(Hint(s): Since there is no uncertainty, the expectations operators disappear; for example, the firms' equation is now $J(w) = (y - w) + \beta\left[\lambda V + (1-\lambda)J(w)\right]$.*

**Part H (2)** In steady-state, we impose an additional restriction: the percentage of unemployment individuals in every period must remain constant. Recalling $u$ is the percentage of unemployed individuals and the definitions of $f(\theta)$ and $\lambda$, write $u$ a a function of $f(\theta)$ and $\lambda$ to obtain the *Beveridge curve (Hint: in equilibrium,* $(1-u)$ *percent of workers are employed, and there is an equality between firings and hirings in steady state).*

**Part I (2)** Assume there is some abstract "surplus" that being matched delivers for the firm and the worker, $S$. In steady state, there is some parameter $\gamma$[3] that determines how much of the surplus is divded between the worker and the firm; that is, $E - U = \gamma S$ while $J - V = (1-\gamma)S$. Write $(E - U)$ as a function of $J$ and $\gamma$ to obtain the *wage-setting curve. (Hint: The free-entry condition may be useful here!)*

**Part J (2)** We want to eliminate either $q$ or $f$ in our model for algebraic simplification. Using (1) the fact that $m(u, v)$ is constant returns to scale and (2) the definitions of $\theta$ as well as $f(\theta)$ and $q(\theta)$, complete

$$q(\theta) =$$

where your answer should only include $\theta$ and $f(\theta)$.

**Part K (3)** There are three unknowns that are "endogenous" to the model, e.g. they are set by agents and not a parameter that we input: $(w, \theta, u)$. Explain how (do not algebraically solve!) one would go about solving for these unknowns.

**Part L (2)** The wage $w$ is partially determined by the unemployment benefit $b$. Conjecture *(do NOT prove)* how $w$ will change as a response to an increase in $b$ based on the (1) your economic intuition and (2) the partial solution to the model you've worked through. *(One point is awarded for each part of the explanation).*

**Bonus I (4)** Algebraically solve for $(w, \theta, u)$ as a function of model primitives $(k, \gamma, \beta, \lambda, f(\theta))$. Your answer should *not* include $\{E, U, J, V\}$ (which are endogenous objects, e.g. ones we derived inside the model itself).

**Bonus II (2)** Prove your conjecture in Part (L) using your answer in the first bonus, e.g. show that the wage (increases/decreases) as a function of the unemployment benefit, in the case where $m(u, v) = Au^{\alpha}v^{1-\alpha}$ for some $\alpha \in (0, 1)$.

[3] This can be microfounded as the unique equilibrium in a model of Nash bargaining.

## Where are all the Missing Women? (23 points)

*In 1990, Nobel laureate Amartya Sen coined the term "missing women" to describe the fact that the ratio of men to women is higher in many developing nations (particularly India and China) than in developed nations. Sen uses this observation to claim that approximately 200 million women in 1990 were "missing", demographically speaking, in the developing world. In this question, we dig deeper into this phenomenon using Sen's methods.*

Here is a table of populations (in thousands of people) by gender in 1990 at different age ranges in Western Europe (Austria, Belguim, France, Germany, Luxembourg, the Netherlands, and Switzerland) and in India (source: UN World Population Prospects 2019).

| Age Range | W. Europe, Male | W. Europe, Female | India, Male | India, Female |
|---|---|---|---|---|
| 0-4 | 5383 | 5127 | 63743 | 58216 |
| 15-19 | 5837 | 5588 | 45709 | 42364 |
| 65-69 | 3486 | 4886 | 7320 | 7099 |

**Part A.i (3)** Find the male:female sex ratios in W. Europe and in India among newborns (age 0-4) and the difference between them (for consistency let's say India's male:female sex ratio minus W. Europe's). Then, calculate how many additional girls at ages 0-4 would need to be added to India's population for India's male:female sex ratio in this age range to equal W. Europe's. This is Sen's estimate for the number of "missing women" in India in 1990 in the ages 0-4. While not necessary to receive full credit, we would suggest solving for a general solution for the number of missing women (call this $\delta$) in terms of $M_E, W_E, M_I, W_I$ to make your work easier to check and apply to later parts. There are a number of valid ways to derive this formula.

**Part A.ii (2)** For reference, the "natural" male:female sex ration at birth is usually estimated to be between 1.03 and 1.06. What did you find here? How do you explain any potential differences between the sex ratios in the two regions?

**Part B (4)** Repeat the same tasks among young adults (age 15-19). Namely, in this age group, calculate the male:female sex ratios in the two regions, the difference between these two sex ratios, and the number of missing women in India. What do you find now? How has the difference in sex ratios changed compared to your result in part A? How do you explain any potential changes?

**Part C (4)** Repeat the same task again among young seniors (age 65-69). For reference, at a certain age, the "natural" male:female sex ration is expected to start decreasing steadily (as women are typically better biologically suited to longevity). How has the difference in sex ratios changed compared to your result in parts A and B? How do you explain any potential changes?

**Part D (4)** Detail 2 potential problems with this analysis. As a hint, consider the two types of comparison we are making: geographical and intertemporal. Are these valid? Why or why not?

**Part E (6)** The subject of missing women is of great interest to modern development researchers. Northwestern economist Nancy Qian has researched a unique phenomenon related to missing women in China. Qian reports that, across Asia, women are comparatively advantaged in and the primary laborers in the farming of tea, whilst men are typically comparatively advantaged and the primary laborers in farming orchard fruits (apples, oranges, etc.). When a set of agricultural reforms led to an exogenous increase in price of Chinese tea, the male:female sex ratio at birth in regions with more tea crops decreased relative to the male:female sex ratio at birth in regions with more orchard fruits (some will recognize this from last year's Power Round as a "difference-in-differences" estimate). Here, Qian has demonstrated a "rational reason" for a change in the sex ratio. First, briefly describe the economic reasoning behind this based on this evidence in your own words (note: describe the economics behind the phenomenon Qian observed, not the statistical methods she used to observe it). Then, react to this finding ethically. We as social scientists often unconsciously make normative assumptions about rationality: if it is "rational" according to our models of the world, it should also be "good" (or at least acceptable) in some moral sense. How does this finding make you more aware of and skeptical of this assumption? Finally, describe somewhere else in the social sciences (economics, sociology, political science, etc.) where you think this assumption is violated.