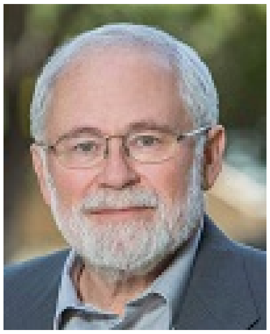
Northwestern | Department of Economics

Short Course for Faculty and Graduate Students



From Discrete- to Continuous-Time Models in Economics, and Back Again

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Stanford University
Holder of the Erwin Plein Nemmers Prize in Economics

INTRODUCTION AS PART OF THEORY "BAG LUNCH" SEMINAR
THURSDAY OCTOBER 24, 12:15-1:15, KELLOGG GLOBAL HUB, ROOM 4302
SHORT COURSE
MONDAYS AND WEDNESDAYS, OCTOBER 28, 30, NOVEMBER 4, 6, 11, 13, NOON-1:30PM
KELLOGG GLOBAL HUB, 2211 CAMPUS DRIVE, ROOM 3301

Over the past decade, a literature has developed in which continuous-time models, employing Brownian motion in one form or another, are advanced. These models provide considerable advantages over discrete-time models, most notably the ability to give closed-form solutions. However, continuous time and, more particularly, Brownian motion, are tricky. One would like to know that the "answers" obtained in "nearby" discrete-time models "approximate" the answers obtained in the "limiting" continuous-time model, where the scare-quotes around answers, nearby, approximate, and limiting indicate that the research agenda here is to give precise and formal meaning to each of these terms.

The short course will undertake a case study of these issues: For the limiting continuous-time model of financial markets studied by Black and Scholes and by Merton, which discrete-time process models give asymptotically the same economic answers? The classic paper by Cox, Ross, and Rubinstein suggests that one answer is a model based on a discrete-time, binomial random walk. But is that true? (For the most part, yes, but you have to be precise in what does and doesn't work.) And is that all? (Not hardly.)

TEXTBOOK: The text for the short course is the just-published Econometric Society Monograph, *The Black-Scholes-Merton Model as an Idealization of Discrete-time Economies* (Cambridge University Press, 2019). (I'm not sure of the release date for this book in the US. Students should check on Amazon.com and email me at kreps@stanford.edu if it doesn't seem to be available by the start of the course.) The monograph will be supplemented by some research papers, to be supplied.

PREREQUISITES: The course will cover the necessary basic theory (chapters 2 and 3 from the monograph), but participants should ideally have a good working knowledge of Radner's equilibria of plans, prices, and price expectations, with a focus on Chapter 16 in Kreps, *Microeconomic Foundations, I: Choice and Competitive Markets* (Princeton UP, 2012), especially Proposition 16.8, as well as an acquaintance with Brownian motion and stochastic integration.