Income Dynamics and Consumption Inequality: Nonlinear Persistence and Partial Insurance

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Short Course, Northwestern University

[*Updated papers and references on my web page*]

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> Recent work incorporates family labor supply and non-separabilities, see *Nemmers Lecture* and recent papers, Blundell, Pistaferri and Saporta, 2016, 2017.

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2. To explore *the nonlinear nature of income shocks and the implications for consumption dynamics and inequality*.

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In particular, the aim is:

1. To consider *alternative ways of modelling persistence*, and
2. To explore *the nonlinear nature of income shocks and the implications for consumption dynamics and inequality*.

⇒ e.g. US Household Panel data and Norwegian Population Register data.

New data on consumption and family income sources

I. Administrative linked data: e.g. Norwegian population register.

- Linked registry databases with unique individual identifiers.
  - Containing records for **every Norwegian from 1967 to 2014.**
  - Detailed demographic and socioeconomic information (market income, cash transfers). Recent links to real estate and assets; and to hours of work. New consumption measurements.
- Family identifiers allow to match spouses and children.
  - see Blundell, Graber and Mogstad (2015).

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II. Newly designed panel surveys: e.g. PSID since 1999.

- Collection of consumption and assets had a major revision in 1999
  - ~70% of consumption expenditures, more since 2004.
  - The sum of food at home, food away from home, gasoline, health, transportation, utilities, clothing etc.
- Earnings and hours for all earners; Assets measured in each wave.
  - see Blundell, Pistaferri and Saporta-Eksten (2016).

- A prototypical "canonical" panel data model of (log) family (earned) income y_{it} is:

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$

where y_{it} is net of a *systematic component*, η_{it} is a *random walk* with innovation v_{it},

$$\eta_{it} = \eta_{it-1} + v_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$

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- Consumption growth is then related to income shocks:

$$\triangle c_{it} = \phi_t v_{it} + \psi_t \varepsilon_{it} + \nu_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$

  where c_{it} is log total consumption net of a systematic component,
  > ϕ_t is the *transmission* of persistence shocks v_{it}, and
  > ψ_t the *transmission* of transitory shocks;
  - the ν_{it} are taste shocks, assumed to be independent across periods.

Covariance Restrictions

Baseline panel data model specification:

$$\triangle c_{it} = \phi v_{it} + \psi \varepsilon_{it} + \nu_{it},$$

$$\triangle y_{it} = v_{it} + \triangle \varepsilon_{it},$$

Implying covariance restrictions:

$$var(\triangle c_{it}) = \phi^2 \sigma_v^2 + \psi^2 \sigma_\varepsilon^2$$

$$var(\triangle y_{it}) = \sigma_\eta^2 + 2\sigma_\varepsilon^2$$

$$cov(\triangle y_{it} \triangle y_{it-1}) = -\sigma_\varepsilon^2$$

$$cov(\triangle c_{it} \triangle y_{it}) = \phi \sigma_v^2 + \psi \sigma_\varepsilon^2$$

$$cov(\triangle c_{it-1} \triangle y_{it}) = \psi \sigma_\varepsilon^2$$

> *For T>3, BPP* include time(age) variation in the σ_*^2 and insurance parameters,
> *BPP* allow for measurement error and extend to MA(1) transitory shocks,
> *BP* develop these covariance restrictions for repeated cross-sections.

Linking Income Dynamics to Consumption Inequality

More specifically, to account for the impact of income shocks on the evolution of consumption inequality we introduce *transmission* or *partial insurance* parameters, writing consumption growth as:

$$\Delta \ln C_{it} \cong \gamma_{it} + \Delta Z'_{it}\varphi + \phi_t v_{it} + \psi_t \varepsilon_{it} + \xi_{it}$$

ϕ_t and ψ_t provide the link between the consumption and income distributions - v_{it} the permanent and ε_{it} the transitory shock to income.

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- For a simple benchmark intertemporal consumption model for consumer of age t, BLP (2013) show

  $$\phi_t = (1 - \pi_{it}) \text{ and } \psi_t = (1 - \pi_{it})\gamma_{Lt}$$

  where

  $$\pi_{it} \approx \frac{\text{Assets}_{it}}{\text{Assets}_{it} + \text{Human Wealth}_{it}}$$

  and γ_{Lt} is the annuity value of a temporary shock to income for an individual aged t retiring at age L.

  [Easily extend to ARMA processes for income.]

• This "*standard*" framework implies a set of extended covariance restrictions for panel data on consumption and income,

▷ allowing the insurance parameters and variances to depend on age and education turns out to be key (analysis of PSID and Norwegian data).

⇒ can show (over-)identification and efficient estimation via nonlinear GMM, see Blundell, Preston and Pistaferri (AER, 2008).

⇒ Blundell, Pistaferri and Saporta (AER, 2016) - develop the nonlinear GMM panel data estimator for wage shocks and family labor supply.

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• Linearity of the income (or wage) process simplifies identification and estimation.

▷ However, by construction, it *rules out the nonlinear transmission of shocks*.

Motivation

• The aim in this lecture is to step back and take a different tack - develop *an alternative approach to modeling persistence* in which the impact of past shocks on current incomes/earnings can be altered by the size and sign of new shocks.

• This new framework draws on a flurry of recent work on nonlinearity and heterogeneity in the dynamics of inequality and income risk (full references in Arellano, Blundell and Bonhomme, 2017).

• The idea is to have a framework allows:

$\Rightarrow$ "*unusual*" shocks to wipe out the memory of past shocks, and

$\Rightarrow$ future persistence of a current shock to depend on the future shocks.

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• The idea is to have a framework allows:

⇒ "*unusual*" shocks to wipe out the memory of past shocks, and

⇒ future persistence of a current shock to depend on the future shocks.

• We will see that the presence of "unusual" shocks matches the data and has a key impact consumption and saving over the life cycle.

Background papers

- Blundell, Pistaferri and Preston [BPP] 'Consumption inequality and partial insurance' (*AER*, 2008)

- Blundell, Low and Preston [BLP] 'Decomposing changes in income risk using consumption data' (*QE*, 2013)

- Blundell, Graber and Mogstad [BGM] 'Labor income dynamics and social insurance' (*JPubE*, 2015; 2017)

- Arellano, Blundell and Bonhomme [ABB] 'Earnings and consumption dynamics: a nonlinear framework' (*Ecta*, 2017)

maybe finding time to look at family labor supply in:

- Blundell, Pistaferri and Saporta-Eksten [BPS1/2] 'Consumption inequality and family labor supply' (*AER*, 2016; JPE, 2017)

-> on my website http://www.ucl.ac.uk/˜uctp39a/pub.html

Nonlinear Persistence

- Consider a cohort of households, $i = 1, ..., N$, and denote age as t. Let y_{it} denote log-labor income, net of age dummies

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$

▷ η_{it} follows a general first-order Markov process (can be generalised).

- Denoting the τth conditional quantile of η_{it} given $\eta_{i,t-1}$ as $Q_t(\eta_{i,t-1}, \tau)$, we specify

$$\eta_{it} = Q_t(\eta_{i,t-1}, u_{it}), \quad \text{where } (u_{it}|\eta_{i,t-1}, \eta_{i,t-2}, ...) \sim \mathit{Uniform}\,(0, 1).$$

▷ ε_{it} has zero mean, independent over time.

▷ The conditional quantile functions $Q_t(\eta_{i,t-1}, u_{it})$ and the marginal distributions F_{ε_t} can all be *age (t) specific*.

A measure of nonlinear persistence

- This framework allows for nonlinear dynamics of income.
- To see this, consider the following measure of persistence

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta}.$$

$\Rightarrow \rho_t(\eta_{i,t-1}, \tau)$ measures the persistence of $\eta_{i,t-1}$ when, at age t, it is hit by a shock u_{it} that has rank τ. Measures the *persistence of histories*.

$\triangleright$ Allows a general form of conditional heteroscedasticity, skewness and kurtosis.

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- In the "canonical model" $\eta_{it} = \eta_{i,t-1} + v_{it}$, with v_{it} independent over time and independent of past $\eta' s$,

$$\eta_{it} = \eta_{i,t-1} + F_{v_t}^{-1}(u_{it}) \quad \Rightarrow \quad \rho_t(\eta_{i,t-1}, \tau) = 1 \text{ for all } (\eta_{i,t-1}, \tau).$$

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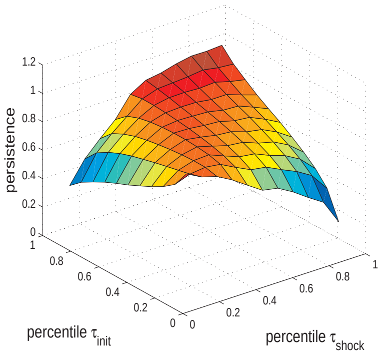
$$\eta_{it}=\eta_{i,t-1}+F_{v_t}^{-1}(u_{it}) \quad \Rightarrow \quad \rho_t(\eta_{i,t-1},\tau)=1 \text{ for all } (\eta_{i,t-1},\tau).$$

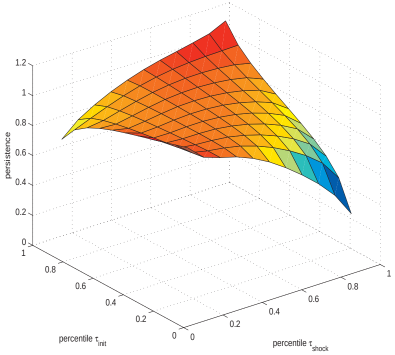
– But what is the evidence for such nonlinearities in persistence?

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$

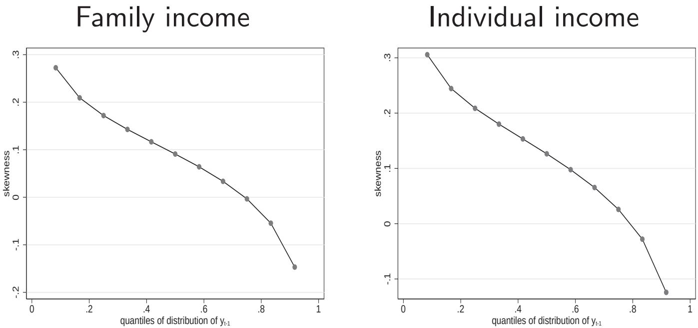
PSID data



Norwegian administrative data



Note: Household labor earnings, Age 30-59, 1999-2009 (US), 2005-2014 (Norway).
Estimates of the average derivative of the conditional quantile function of y_{it} *given* $y_{i,t-1}$



Note: Skewness measured as a nonparametric estimate of

$$\frac{Q_{y_t|y_{t-1}}(y_{i,t-1},.9)+Q_{y_t|y_{t-1}}(y_{i,t-1},.1)-2Q_{y_t|y_{t-1}}(y_{i,t-1},.5)}{Q_{y_t|y_{t-1}}(y_{i,t-1},.9)-Q_{y_t|y_{t-1}}(y_{i,t-1},.1)}.$$

Age 30-59, years 2005-2006.

Layout

- ▷ **Life-cycle model simulations and model specification**
- ▷ **Identification**
- ▷ **Data and estimation strategy**
- ▷ **Empirical results**

Life-cycle model: illustrative simulation

- Calibration based on Kaplan and Violante [KV] (2010). Households enter the labor market at age 25, work until 60, and die with certainty at age 90.

- A single risk-free, one-period bond with return $1 + r$ ($r = .03$),

$$A_t = (1 + r)A_{t-1} + Y_{t-1} - C_{t-1}.$$

- Log-earnings are $\ln Y_t = \kappa_t + \eta_t + \varepsilon_t$, where κ_t is a deterministic age profile. In period t agents know η_t, ϵ_t and their past values, but not η_{t+1} or ε_{t+1} (no advance information).

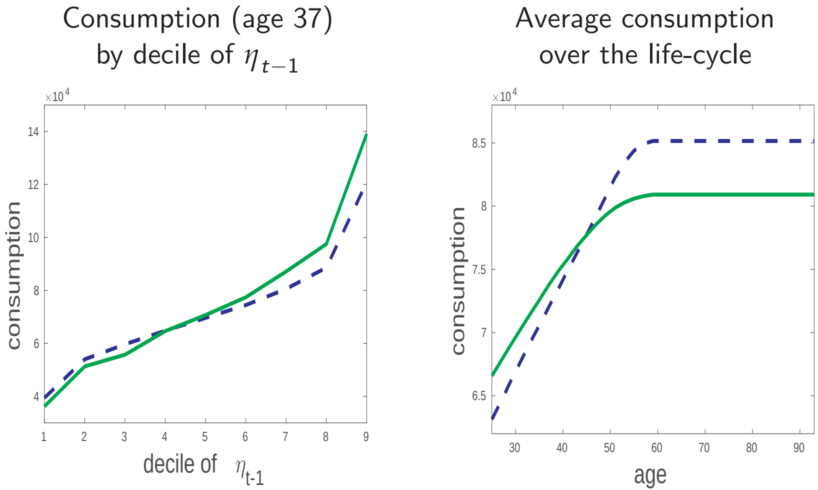
- Period-t optimization

$$V_t(A_t, \eta_t, \varepsilon_t) = \max_{C_t} \; u(C_t) + \beta \mathbb{E}_t \left[V_{t+1} \left(A_{t+1}, \eta_{t+1}, \varepsilon_{t+1} \right) \right],$$

where $u(\cdot)$ is CRRA ($\gamma = 2$), and $\beta = 1/(1 + r) \approx .97$.

- We compare the results for the canonical earnings process used by KV, with our nonlinear process.

Simulation results



Note: Blue is nonlinear earnings process, Green is canonical earnings process.

An Empirical Consumption Rule

- Let c_{it} and a_{it} denote log-consumption and assets (beginning of period) net of age dummies.

- Our empirical specification is based on

$$c_{it} = g_t\left(a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it}\right) \quad t = 1, ..., T,$$

where ν_{it} are independent across periods, and g_t is a nonlinear, age-dependent function, monotone in ν_{it}.

– ν_{it} may be interpreted a taste shifter that increases marginal utility. We normalize its distribution to be standard uniform in each period.

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– ν_{it} may be interpreted a taste shifter that increases marginal utility. We normalize its distribution to be standard uniform in each period.

> This consumption rule is consistent, in particular, with the standard life-cycle model on the earlier slide.

> Can allow for individual heterogeneity, advance information and habits.

Insurance coefficients

- With consumption specification given by

$$c_{it} = g_t\left(a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it}\right), \quad t = 1, ..., T,$$

consumption responses to η and ε are

$$\phi_t(a,\eta,\varepsilon) = \mathbb{E}\left[\frac{\partial g_t\left(a,\eta,\varepsilon,\nu\right)}{\partial \eta}\right], \quad \psi_t(a,\eta,\varepsilon) = \mathbb{E}\left[\frac{\partial g_t\left(a,\eta,\varepsilon,\nu\right)}{\partial \varepsilon}\right].$$

▷ $\phi_t(a,\eta,\varepsilon)$ and $\psi_t(a,\eta,\varepsilon)$ reflect the transmission of the persistent and transitory earnings components, respectively.

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▷ $\phi_t(a,\eta,\varepsilon)$ and $\psi_t(a,\eta,\varepsilon)$ reflect the transmission of the persistent and transitory earnings components, respectively.

- The marginal effect of an earnings shock u on consumption is

$$\mathbb{E}\left[\left.\frac{\partial}{\partial u}\right|_{u=\tau} g_t\Big(a, Q_t(\eta,u), \varepsilon, \nu\Big)\right] = \phi_t\Big(a, Q_t(\eta,\tau), \varepsilon\Big)\frac{\partial Q_t(\eta,\tau)}{\partial u}.$$

Earnings: identification

• For $T = 3$, Wilhelm (2012) gives conditions under which the distribution of ε_{i2} is identified.

– In particular, completeness of the *pdf*s of $(y_{i2}|y_{i1})$ and $(\eta_{i2}|y_{i1})$. This requires η_{i1} and η_{i2} to be dependent.

• In this research we use this result to establish identification of the earnings model.

• Apply the result to each of the three-year sub-panels $t \in \{1, 2, 3\}$ to $t \in \{T-2, T-1, T\}$

$\Rightarrow$ The marginal distribution of ε_{it} are identified for $t \in \{2, 3, ..., T-1\}$.
$\Rightarrow$ By independence the joint distribution of $(\varepsilon_{i2}, \varepsilon_{i3}, ..., \varepsilon_{i,T-1})$ is identified.
$\Rightarrow$ By deconvolution the distribution of $(\eta_{i2}, \eta_{i3}, ..., \eta_{i,T-1})$ is identified.

• The distribution of ε_{i1}, η_{i1}, and ε_{iT}, η_{iT} are not identified in general.

Consumption: assumptions

• u_{it} and ε_{it} are independent of past earnings shocks and past asset holding, for $t \geq 1$, where $\eta_{it} = Q_t(\eta_{i,t-1}, u_{it})$.

• We let η_{i1} and a_{i1} be arbitrarily dependent;

– this is important, because asset accumulation upon entry in the sample may be correlated with past persistent shocks.

• Denoting $\eta_i^t = (\eta_{it}, \eta_{i,t-1}, ..., \eta_{i1})$, we assume (in this talk) that: a_{it} is independent of $(\eta_i^{t-1}, a_i^{t-2}, \varepsilon_i^{t-2})$ given $(a_{i,t-1}, c_{i,t-1}, y_{i,t-1})$;

– consistent with the accumulation rule in the standard life-cycle model with one single risk-less asset.

Consumption: initial assets

- Let $y = (y_1, ..., y_T)$. We have

$$\begin{aligned} f(a_1|y) &= \int f(a_1|\eta_1, y) f(\eta_1|y) d\eta_1 \\ &= \int f(a_1|\eta_1) f(\eta_1|y) d\eta_1, \end{aligned}$$

where we have used that u_{it} and ε_{it} are independent of a_{i1}.

- Note that $f(\eta_1|y)$ is identified from the earnings process alone.

- If $f(\eta_1|y)$ is complete, then $f(a_1|\eta_1)$ is identified.

– Structure is as in the NPIV problem where η_1 is the endogenous regressor and y is the instrument.

Consumption: first period

- We have

$$f(c_1, a_1|y) \equiv \int f(c_1, a_1|\eta_1, y) f(\eta_1|y) d\eta_1$$

and given our assumptions

$$f(c_1, a_1|y) = \int f(c_1|a_1, \eta_1, y_1) f(a_1|\eta_1) f(\eta_1|y) d\eta_1.$$

– $f(a_1|\eta_1)$ can be treated as known.

– Provided we have completeness in $(y_2, ..., y_T)$ of $f(\eta_1|y_1, y_2, ..., y_T)$, then $f(c_1|a_1, \eta_1, y_1)$, is identified.

- Intuition: $y_{i2}, ..., y_{iT}$ are used as "instruments" for η_{i1}.
- Subsequent periods discussed in *ABB* (2017), briefly here...

Consumption: subsequent periods

- We have

$$\begin{aligned} f(a_2|c_1, a_1, y) &= \int f(a_2|c_1, a_1, \eta_1, y_1) f(\eta_1|c_1, a_1, y) d\eta_1 \\ f(c_2|a_2, c_1, a_1, y) &= \int f(c_2|a_2, \eta_2, y_2) f(\eta_2|a_2, c_1, a_1, y) d\eta_2. \end{aligned}$$

- By induction it can be shown that the joint density of η's, consumption, assets, and earnings is identified provided, for all $t \geq 1$, the distributions of $(\eta_{it}|c_i^t, a_i^t, y_i)$ and $(\eta_{it}|c_i^{t-1}, a_i^t, y_i)$ are complete in $(c_i^{t-1}, a_i^{t-1}, y_i^{t-1}, y_{i,t+1}, \ldots, y_{iT})$.

- Intuition: lagged consumption and assets, as well as lags and leads of earnings, are used as instruments for η_{it}.

Identification: extensions

• Similar techniques can be used in the presence of *advance information*, e.g.

$$c_{it} = g_t \left(a_{it}, \eta_{it}, \eta_{i,t+1}, \varepsilon_{it}, \nu_{it} \right),$$

or *consumption habits*, e.g.

$$c_{it} = g_t \left(c_{i,t-1}, a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it} \right).$$

▷ also cases where the consumption rule depends on lagged η, or when η follows a second-order Markov process. (See Section 3 in *ABB*, 2017).

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▷ also cases where the consumption rule depends on lagged η, or when η follows a second-order Markov process. (See Section 3 in *ABB*, 2017).

• Households differ in their initial productivity η_1 and initial assets, the panel data provide opportunities to allow for additional, *unobserved heterogeneity* in earnings and consumption.

▷ For example: heterogeneity ξ_i in discounting or preferences, or heterogeneity $\widetilde{\xi}_i$ in the Markovian transitions of η_{it}

Extensions (cont.)

- Consumption rule with *unobserved heterogeneity:*

$$c_{it} = g_t \left(a_{it}, \eta_{it}, \varepsilon_{it}, \xi_i, \nu_{it} \right).$$

- We assume that u_{it} and ε_{it}, for $t \geq 1$, are independent of (a_{i1}, ξ_i).

- The distribution of $(a_{i1}, \xi_i, \eta_{i1})$ is unrestricted.

- A combination of the above identification arguments and the main result of Hu and Schennach (08) identifies:

– the period-t consumption distribution $f(c_t | a_t, \eta_t, y_t, \xi)$, and

– the distribution of initial conditions $f(\eta_1, \xi, a_1)$.

Data: PSID

- (New) PSID 1999 - 2009, we use 6 waves (every other year), as in *BPS*.

- C_{it}: Information on food expenditures, rents, health expenditures, utilities, car-related expenditures, ...

- A_{it}: Assets holdings are the sum of financial assets, real estate value, pension funds, and car value, net of mortgages and other debt. (Net worth).

- y_{it} are residuals of log total pre-tax household labor earnings on a set of demographics. Note, c_{it} and a_{it} are residuals, using the same set of demographics as for earnings.

$\triangleright$ cohort and calendar time dummies, family size and composition, education, race, and state dummies.

- As in BPS, we select married male heads aged between 25 and 59.
- In this talk we focus on a balanced sub-sample of $N = 792$ households.

Empirical specification: income

- The quantile function of η_{it} given $\eta_{i,t-1}$ is specified as

$$\begin{aligned} Q_t(\eta_{t-1}, \tau) &= Q(\eta_{t-1}, age_t, \tau) \\ &= \sum_{k=0}^{K} a_k^Q(\tau)\varphi_k(\eta_{t-1}, age_t), \end{aligned}$$

where φ_k, $k = 0, 1, ..., K$, are polynomials (Hermite).

- In addition, the quantile functions of ε_{it} and η_{i1} are

$$\begin{aligned} Q_\varepsilon(age_t, \tau) &= \sum_{k=0}^{K} a_k^\varepsilon(\tau)\varphi_k(age_t), \\ Q_{\eta_1}(age_1, \tau) &= \sum_{k=0}^{K} a_k^{\eta_1}(\tau)\varphi_k(age_1). \end{aligned}$$

Empirical specification: consumption

- We specify the (log) consumption function as:

$$
\begin{aligned}
g_t(a_t, \eta_t, \varepsilon_t, \tau) &= g(a_t, \eta_t, \varepsilon_t, age_t, \tau) \\
&= \sum_{k=1}^{K} b_k^g \widetilde{\varphi}_k(a_t, \eta_t, \varepsilon_t, age_t) + b_0^g(\tau)
\end{aligned}
$$

– additivity in the taste shifters, though not essential, is convenient given the sample size.

- In addition, the conditional quantiles of a_{i1} given η_{i1} and age_{i1} are

$$
Q^{(a)}(\eta_1, age_1, \tau) = \sum_{k=0}^{K} b_k^a(\tau) \widetilde{\varphi}_k(\eta_1, age_1).
$$

Implementation choices

• Model $a_k^Q(\tau)$ as piecewise-linear interpolating splines (Wei and Carroll, 2009) on a grid $0 < \tau_1 < \tau_2 < ... < \tau_L < 1$,

– convenient as the likelihood function is available in closed form.

• We extend the specification of the intercept coefficient $a_0^Q(\tau)$ on $(0, \tau_1]$ and $[\tau_L, 1)$ using a parametric model: exponential (λ).

• In practice, for the PSID data, we take $L = 11$ and $\tau_\ell = \ell/L + 1$. φ_k and $\widetilde{\varphi}_k$ are low-dimensional tensor products of Hermite polynomials.

• We set $b_0(\tau) = \alpha + \sigma\Phi^{-1}(\tau)$, where (α, σ) are to be estimated.

Estimation algorithm

- The first estimation step recovers estimates of the income parameters θ.
- The second step recovers estimates of the consumption parameters μ, given a previous estimate of θ.
- Our choice of a sequential estimation strategy, rather than joint estimation of (θ, μ), is motivated by the fact that θ is identified from the income process alone.

Model's restrictions: income

- Let θ be the income-related parameters with true values $\overline{\theta}$.

- Let $\rho_\tau(u) = u(\tau - \mathbf{1}\{u \leq 0\})$ denote the "check" function of quantile regression, and let $\overline{a}_{k\ell}^Q$ denote the value of $a_{k\ell}^Q = a_k^Q(\tau_\ell)$ evaluated at the true $\overline{\theta}$. The model implies

$$\left(\overline{a}_{0\ell}^Q, .., \overline{a}_{K\ell}^Q\right) = \underset{\left(a_{0\ell}^Q, .., a_{K\ell}^Q\right)}{\operatorname{argmin}} \mathbb{E}\left[\int \rho_{\tau_\ell}\left(\eta_{it} - \sum_{k=0}^{K} a_{k\ell}^Q \varphi_k(\eta_{i,t-1}, age_{it})\right) f_i(\eta_i^T; \overline{\theta}) d$$

with additional restrictions involving the other parameters in θ.

- In the above, f_i denotes the posterior density of $(\eta_{i1}, ..., \eta_{iT})$ given the income data

$$f_i(\eta_i^T; \overline{\theta}) = f(\eta_i^T | y_i^T, age_i^T; \overline{\theta}).$$

Model's restrictions: consumption

• Letting μ (true value $\overline{\mu}$) be the consumption-related parameters, the model implies

$$\left(\overline{\alpha},\overline{b}_1^g,..,\overline{b}_K^g\right)=\underset{\left(\alpha,b_1^g,..,b_K^g\right)}{\operatorname{argmin}}\ \mathbb{E}\left[\int\left(c_{it}-\alpha-\sum_{k=1}^{K}b_k^g\widetilde{\varphi}_k(a_{it},\eta_{it},y_{it}-\eta_{it},age_{it})\right)^2 g_i(\right.$$

and

$$\overline{\sigma}^2\ =\ \mathbb{E}\left[\int\left(c_{it}-\overline{\alpha}-\sum_{k=1}^{K}b_k^g\widetilde{\varphi}_k(a_{it},\eta_{it},y_{it}-\eta_{it},age_{it})\right)^2 g_i(\eta_i^T;\overline{\theta},\overline{\mu})d\eta_i^T\right],$$

with additional restrictions involving the other parameters in μ.

• Here g_i denotes the posterior density of $(\eta_{i1},...,\eta_{iT})$ given the earnings, consumption, and asset data

$$g_i(\eta_i^T;\overline{\theta},\overline{\mu})=f(\eta_i^T|c_i^T,a_i^T,y_i^T,age_i^T;\overline{\theta},\overline{\mu}).$$

Overview of estimation

- A compact notation for the restrictions implied by the income model is

$$\overline{\theta} = \underset{\theta}{\text{argmin}} \; \mathbb{E}\left[\int R(y_i, \eta; \theta) f_i(\eta; \overline{\theta}) d\eta\right].$$

- We use a "stochastic EM" algorithm (in a non-likelihood setup).

Starting with $\widehat{\theta}^{(0)}$ we iterate on s=0,1,... the following two steps until convergence of the Markov Chain:

1. *Stochastic E-step:* draw $\eta_i^{(m)} = (\eta_{i1}^{(m)}, ..., \eta_{iT}^{(m)})$ for $m = 1, ..., M$ from $f_i(\cdot; \widehat{\theta}^{(s)})$. *ABB* use a random-walk Metropolis-Hastings sampler.
2. *M-step:* update

$$\widehat{\theta}^{(s+1)} = \underset{\theta}{\text{argmin}} \; \sum_{i=1}^{N} \sum_{m=1}^{M} R(y_i, \eta_i^{(m)}; \theta).$$

Overview of estimation (cont.)

- As the likelihood function is available in closed form, the E-step is straightforward.

- The M-step consists of a number of ordinary regressions and quantile regressions, such as

$$\min_{\left(a^Q_{0\ell},\ldots,a^Q_{K\ell}\right)} \sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{m=1}^{M}\rho_{\tau_\ell}\left(\eta_{it}^{(m)} - \sum_{k=0}^{K} a^Q_{k\ell}\varphi_k(\eta_{i,t-1}^{(m)}, age_{it})\right), \ \ \ell = 1, \ldots, L.$$

- We compute $\widehat{\theta}$ as an average of $\widehat{\theta}^{(s)}$ across S iterations.

- We estimate $\widehat{\theta}$ and $\widehat{\mu}$ sequentially.

Statistical properties

• Nielsen (2000) studies the properties of this algorithm in a likelihood case. He provides conditions for the Markov Chain $\widehat{\theta}^{(s)}$ to be ergodic (for a fixed sample size).

• He also shows that $\sqrt{N}\left(\widehat{\theta}^{(s)} - \overline{\theta}\right)$ converges to a Gaussian autoregressive process as N tends to infinity.

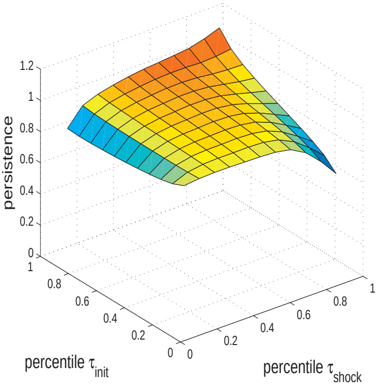
• Arellano and Bonhomme [AB] (2015) adapt Nielsen's arguments to derive the form of the asymptotic variance in a non-likelihood case.

• AB also study consistency as K (number of polynomial terms) and L (number of knots) tend to infinity with N.

Empirical results

Nonlinear persistence of η_{it} (PSID):

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1}, \tau)}{\partial \eta}$$

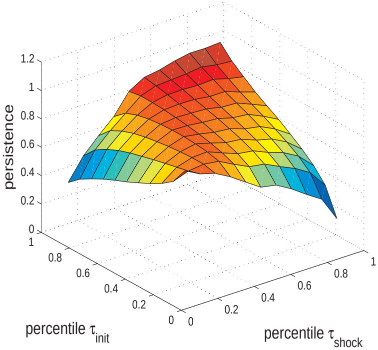


Note: Estimates of the average derivative of the conditional quantile function of η_{it} on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile τ_{shock} and at a value of $\eta_{i,t-1}$ that corresponds to the τ_{init} percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.

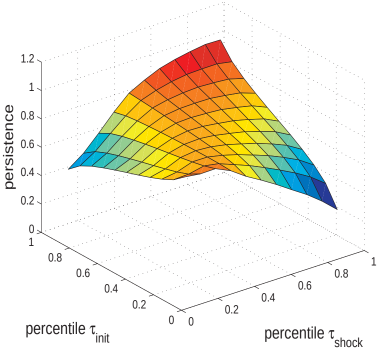
Nonlinear persistence of y_{it}

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$

PSID panel data



Nonlinear model

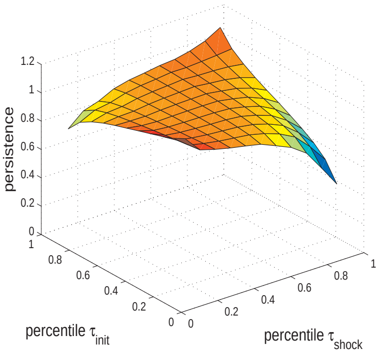


Note: Estimates of the average derivative of the conditional quantile function of y_{it} given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile τ_{shock} and at a value of $y_{i,t-1}$ that corresponds to the τ_{init} percentile of the dist. of $y_{i,t-1}$.

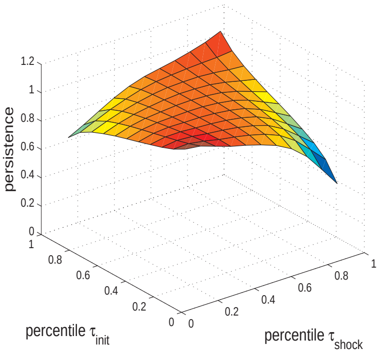
Nonlinear persistence of y_{it}

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$

Norwegian register data



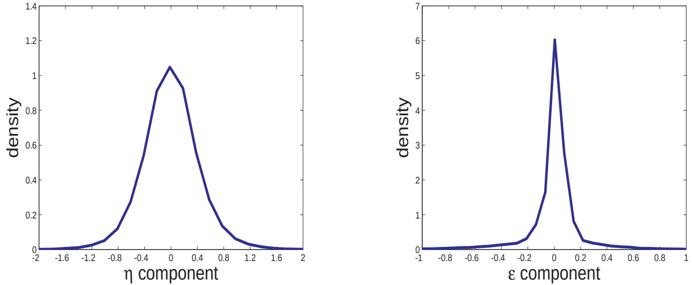
Nonlinear model



Note: Estimates of the average derivative of the conditional quantile function of y_{it} given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile τ_{shock} and at a value of $y_{i,t-1}$ that corresponds to the τ_{init} percentile of the dist. of $y_{i,t-1}$.

Figure: **Densities of persistent and transitory earnings components (PSID)**

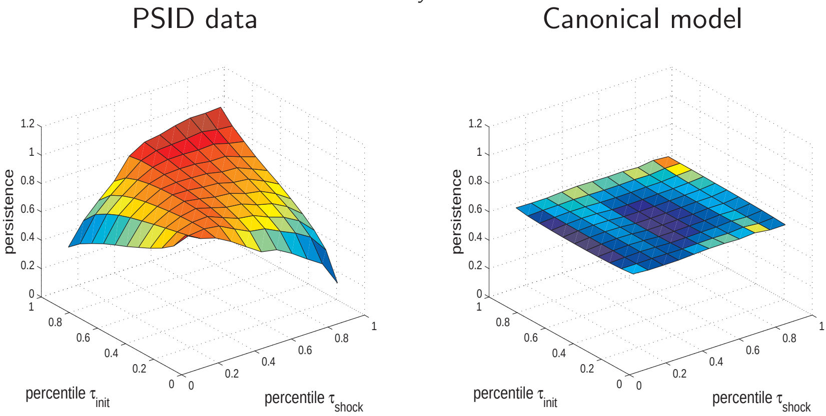
(a) Persistent component η_{it} (b) Transitory component ε_{it}



Note: Nonparametric estimates of densities based on simulated data according to the nonlinear model, using a Gaussian kernel.

Nonlinear persistence of y_{it} (cont.)

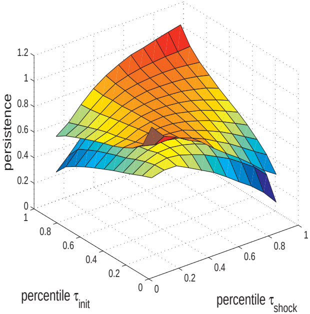
$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$



Note: Estimates of the average derivative of the conditional quantile function of y_{it} given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile τ_{shock} and at a value of $y_{i,t-1}$ that corresponds to the τ_{init} percentile of the dist. of $y_{i,t-1}$.

Nonlinear persistence, 95% confidence bands

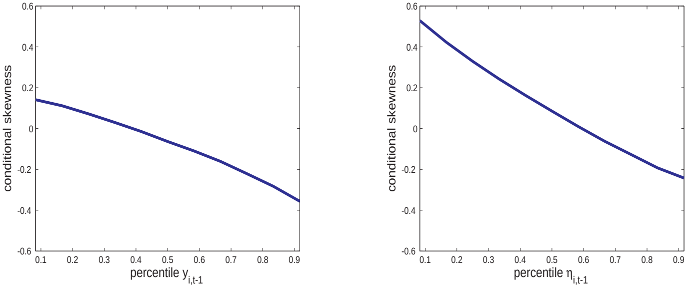
(a) Earnings, PSID data (b) Earnings, nonlinear model



Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.

Figure: **Conditional skewness of log-earnings residuals and η component**

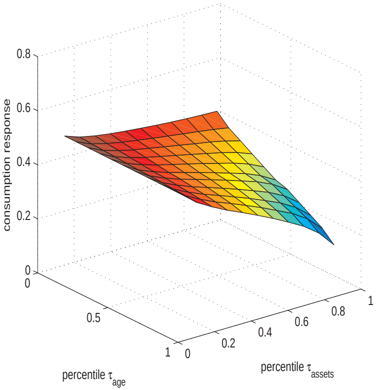
(a) Log-earnings residuals y_{it} (b) Persistent component η_{it}



Note: Conditional skewness $sk(y, \tau)$ and $sk(\eta, \tau)$, for $\tau = 11/12$. Log-earnings residuals (left) and η component (right). The x-axis shows the conditioning variable, the y-axis shows the corresponding value of the conditional skewness measure. Bootstrap confidence intervals in the Appendix.

Consumption response to η_{it}, by assets and age

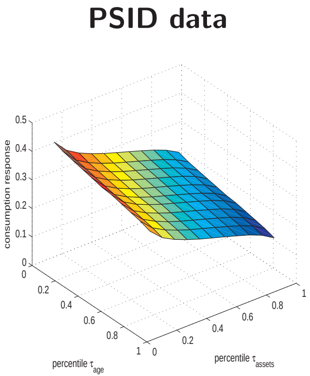
$$\overline{\phi}_t(a) = \mathbb{E}\left[\frac{\partial g_t(a, \eta_{it}, \varepsilon_{it}, \nu_{it})}{\partial \eta}\right], \text{ nonlinear model}$$

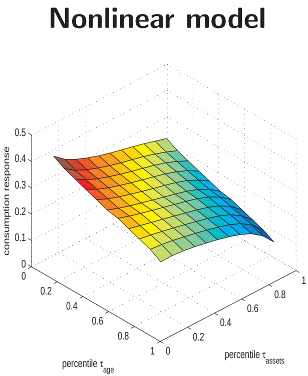


Note: Estimates of the average consumption response $\overline{\phi}_t(a)$ to variations in η_{it}, evaluated at τ_{assets} and τ_{age}.

Consumption responses to y_{it}, by assets and age

$$\mathbb{E}\left[\frac{\partial}{\partial y}\Big|_{y_{it}} \mathbb{E}\left(c_{it} | a_{it} = a, y_{it} = y, age_{it} = age\right)\right]$$

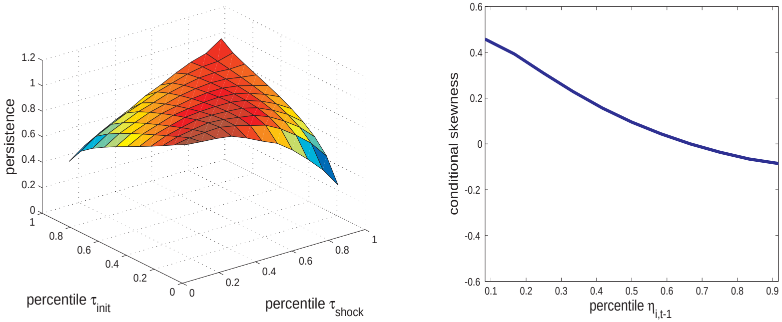




Note: Estimates of the average derivative of the conditional mean of c_{it} given y_{it}, a_{it} & age_{it} with respect to y_{it}, evaluated at values of a_{it} & age_{it} corresponding to their τ_{assets} & τ_{age} percentiles, and averaged over the values of y_{it}.

Figure: **Household heterogeneity in earnings**

(a) Nonlinear persistence of η_{it} (b) Conditional skewness of η_{it}

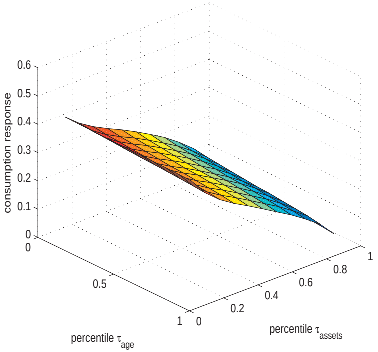


Notes: (a) Estimates of the average derivative of the conditional quantile function of η_{it} on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, based on estimates from the nonlinear earnings model with an additive household-specific effect.

(b) Conditional skewness $sk(\eta,\tau)$, for $\tau = 11/12$, based on the same model.

Consumption response to η_{it}, by assets and age, household heterogeneity

$$\overline{\phi}_t(a) = \mathbb{E}\left[\frac{\partial g_t(a,\eta_{it},\varepsilon_{it},\xi_i,\nu_{it})}{\partial \eta}\right], \text{ nonlinear model}$$



Note: Estimates of the average consumption response $\overline{\phi}_t(a)$ to variations in η_{it}, evaluated at τ_{assets} and τ_{age}.

Model's simulation

- Simulate life-cycle earnings and consumption after a shock to the persistent earnings component (at age 37).

- We report the difference between:

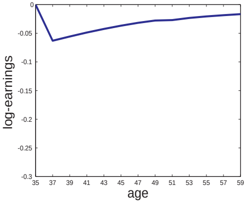
– Households that are hit by a "bad" shock ($\tau_{shock} = .10$) or by a "good" shock ($\tau_{shock} = .90$).
– Households that are hit by a median shock $\tau = .5$.

- Age-specific averages across 100,000 simulations. At age 35 all households have the same persistent component (percentile τ_{init}).

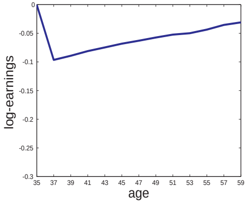
Impulse responses, earnings

Bad shock: $\tau_{shock} = .1$

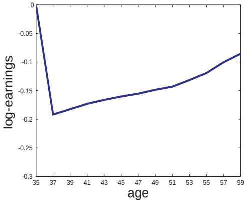
$\tau_{init} = .1$



$\tau_{init} = .5$

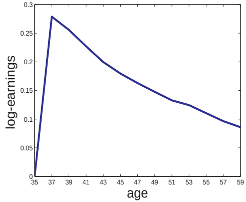


$\tau_{init} = .9$

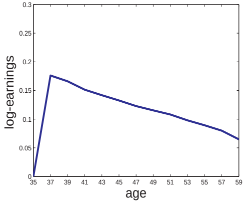


Good shock: $\tau_{shock} = .9$

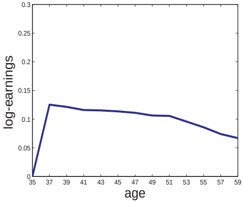
$\tau_{init} = .1$



$\tau_{init} = .5$



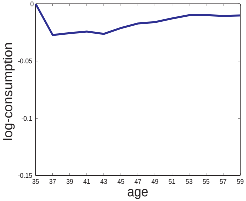
$\tau_{init} = .9$

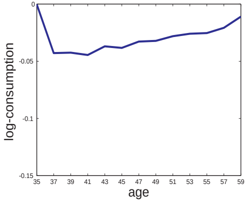


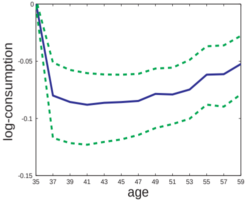
Impulse responses, consumption

Bad shock: $\tau_{shock} = .1$

$\tau_{init} = .1$ | $\tau_{init} = .5$ | $\tau_{init} = .9$

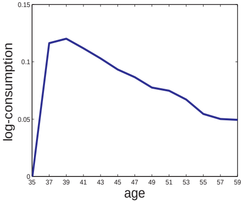


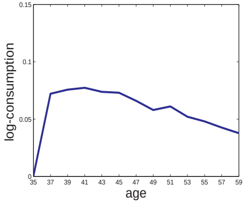


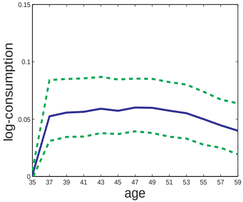


Good shock: $\tau_{shock} = .9$

$\tau_{init} = .1$ | $\tau_{init} = .5$ | $\tau_{init} = .9$



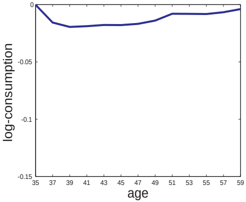




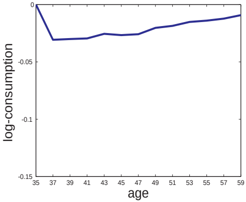
Impulse responses, consumption, household heterogeneity

Bad shock: $\tau_{shock} = .1$

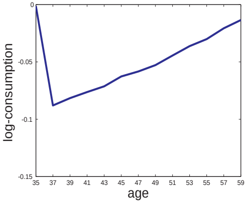
$\tau_{init} = .1$



$\tau_{init} = .5$

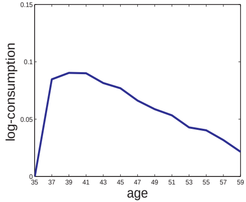


$\tau_{init} = .9$

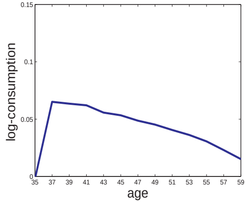


Good shock: $\tau_{shock} = .9$

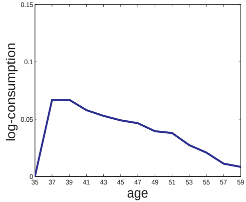
$\tau_{init} = .1$



$\tau_{init} = .5$



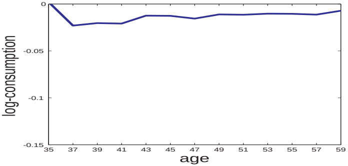
$\tau_{init} = .9$



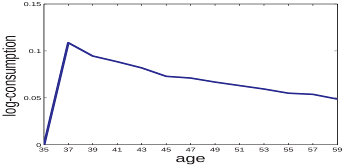
Impulse responses, consumption, linear assets rule

Nonlinear model $\tau_{init} = .1$

(a) $\tau_{shock} = .1$

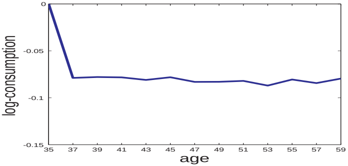


(b) $\tau_{shock} = .9$

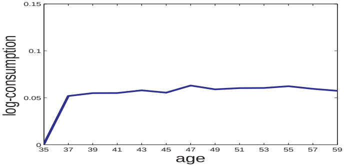


$\tau_{init} = .9$

(e) $\tau_{shock} = .1$



(f) $\tau_{shock} = .9$

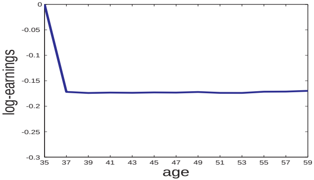


Note: Linear assets accumulation rule. Assets are constrained to be non-negative.

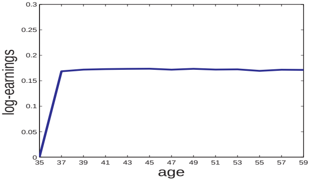
Impulse responses: canonical earnings and linear consumption model

Earnings

$\tau_{shock} = .1$

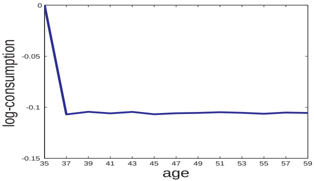


$\tau_{shock} = .9$

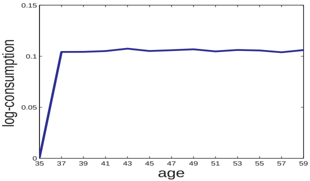


Consumption

$\tau_{shock} = .1$



$\tau_{shock} = .9$

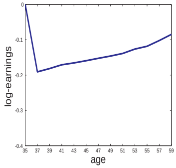


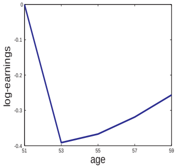
Impulse responses, by age and initial assets

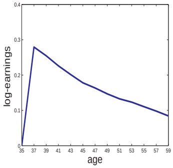
Earnings

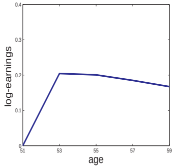
$\tau_{init} = .9, \tau_{shock} = .1$ | $\tau_{init} = .1, \tau_{shock} = .9$

Young | Old | Young | Old





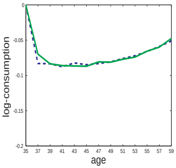


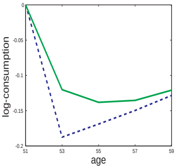


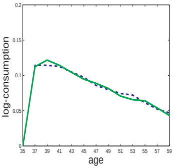
Consumption

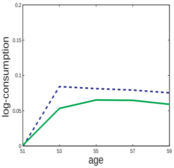
$\tau_{init} = .9, \tau_{shock} = .1$ | $\tau_{init} = .1, \tau_{shock} = .9$

Young | Old | Young | Old









Summary

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▷ A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks in the PSID and in large administrative registers.
▷ An age-dependent nonlinear consumption rule that is a function of assets, permanent income and transitory income.

• Provide conditions for nonparametric identification:
⇒ explain how a simulation-based sequential QR method is feasible.

• This framework leads to new empirical measures of the degree of partial insurance and the link between income and consumption inequality.

• But what about looking inside the family labour income measure ......?

A role for family labour supply?

Families have the possibility of insuring consumption on many margins.

Distinguish four separate mechanisms:

A role for family labour supply?

Families have the possibility of insuring consumption on many margins.

Distinguish four separate mechanisms:

1. Labor supply of other family members,
2. Non-linear taxes and welfare,
3. Self-insurance (i.e., savings through the direct use of net assets),
4. Other informal mechanisms and networks....

- Then examine each step in the distributional dynamics from wages to consumption:

wages->earnings->family earnings->net income->consumption->wealth.

A role for family labour supply?

BPS use data on wage, consumption, income, labor supply and assets from the PSID.

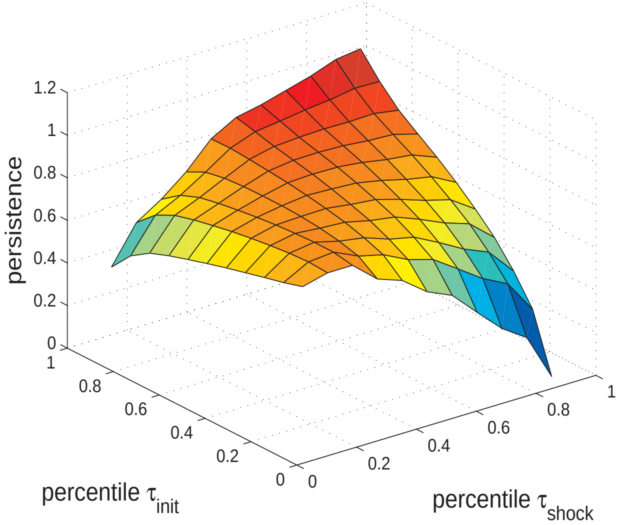
As described in the *Nemmers Lecture*, BPS show that family labor supply can be a key mechanism for 'insuring' unexpected shocks

- especially for younger families and for those with limited access to assets,

- a strong "added-worker" effect as a response to a permanent shock.

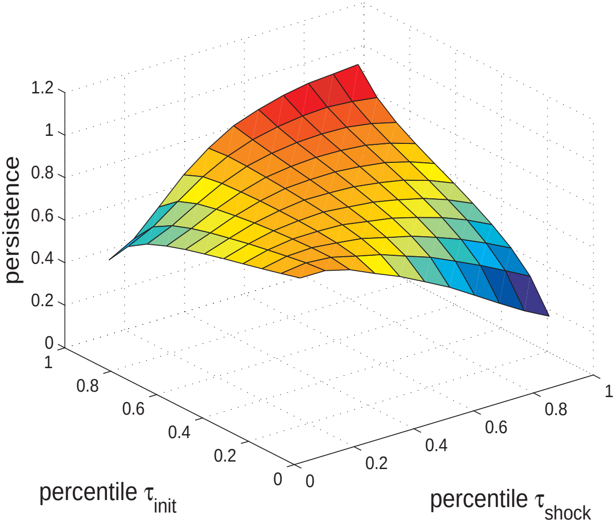
* Find an important role for unusual shocks and nonlinear persistence in the wages......

Measured Nonlinear Persistence in the Male Wage Data: PSID



Notes: Log male wages, Age 30-60 1999-2013 (US). Estimates of the average derivative of the conditional quantile function. Source: Arellano, Blundell and Bonhomme (2017b).

Simulated Nonlinear Persistence in the Male Wage Data: PSID



Notes: Log male wages, Age 30-60 1999-2013 (US). Simulation of the average derivative of the conditional quantile function. Source: Arellano, Blundell and Bonhomme (2017b).

Family labour supply, time-use and consumption smoothing

Recent research (BPS2) combines data on wage, consumption, income, labor supply, assets and *time-use* from the PSID, ATUS and CEX.

- Time-use data from ATUS is used to unpack what's going on in terms of family time allocation responses to to male and female wage shocks

  -> results uncover a tension between the desire of spouses to spend leisure time with each other, and the specialization in care of children.

  -> the presence of young children is found to give rise to Frisch substitutability of hours between spouses, with a switch to Frisch complements as children age and leave home.

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The strong "added-worker" effect from a response to an adverse permanent shock to his earnings is found to induce *a fall in mother's time-use with young children*, especially for low-educated with low assets.

-> **Details of the family labour supply, time-use and consumption smoothing model and results at the end of these lecture slides.**

Next steps

1. Study the implications for child outcomes, currently linking to *CDS*.
2. Separate housing equity and allow a role for local labour markets.
3. Include firm to firm transitions and lay-offs.
4. Include experience/human capital => as in BDMS (*Ecta* 2016).
5. Health and other types of (partially insured) shocks (*HRS, ELSA*).
6. Estimate on the full population (Norwegian) register data.
7. and more......

Additional slides

Identification when $T = 3$: Wilhelm (12)

- We work in L^2-spaces relative to suitable distributions.
- Let $g(y_2, y_3)$ such that there exists a $s(y_2)$ such that

$$\mathbb{E}\left[g(Y_2, Y_3)|Y_1\right] = \mathbb{E}\left[s(Y_2)|Y_1\right].$$

Under completeness of $Y_2|Y_1$, $s(\cdot)$ is unique.

- By conditional independence,

$$\mathbb{E}\left[\mathbb{E}\left(g(Y_2, Y_3)|\eta_2\right)|Y_1\right].$$

- Under completeness of $\eta_2|Y_1$, it follows that

$$\mathbb{E}\left[g(Y_2, Y_3)|\eta_2\right] = \mathbb{E}\left[s(Y_2)|\eta_2\right].$$

The case $T = 3$ (cont.)

• Wilhelm (12) considers the functions $g_1(Y_3) = \mathbf{1}\{Y_3 \leq y_3\}$, and $g_2(Y_2, Y_3) = Y_2\mathbf{1}\{Y_3 \leq y_3\}$, for a given value y_3.

• This yields

$$\begin{aligned} \mathbb{E}\left[\mathbf{1}\{Y_3 \leq y_3\}|\eta_2\right] &\equiv G(\eta_2) = \mathbb{E}\left[s_1(Y_2)|\eta_2\right] \\ \mathbb{E}\left[Y_2\mathbf{1}\{Y_3 \leq y_3\}|\eta_2\right] &= \eta_2 G(\eta_2) = \mathbb{E}\left[s_2(Y_2)|\eta_2\right]. \end{aligned}$$

• Hence, taking Fourier transforms (i.e., $\mathcal{F}(h)(u) = \int h(x)e^{\boldsymbol{i}ux}dx$),

$$\begin{aligned} \mathcal{F}(G)(u) &= \mathcal{F}(s_1)(u)\psi_{\varepsilon_2}(-u) \\ \boldsymbol{i}^{-1}d\mathcal{F}(G)(u)/du &= \mathcal{F}(s_2)(u)\psi_{\varepsilon_2}(-u), \end{aligned}$$

where $\psi_{\varepsilon_2}(u) = \mathcal{F}(f_{\varepsilon_2})(u)$ is the characteristic function of ε_2, and $\boldsymbol{i} = \sqrt{-1}$.

The case $T = 3$ (cont.)

This yields the following first-order differential equation

$$\mathcal{F}_2(u)du = \left[\frac{d\mathcal{F}(s_1)(-u)}{du} - \boldsymbol{i}\mathcal{F}(s_2)(-u)\right]\psi_{\varepsilon_2}(u).$$

• In addition, $\psi_{\varepsilon_2}(0) = 1$.

• This ODE can be solved in closed form for $\psi_{\varepsilon_2}(\cdot)$, provided that $\mathcal{F}(s_1)(u) \neq 0$ for all u (which is another injectivity condition).

• As a result, the distribution of ε_2, and the distribution of Y_3 given η_2, are both nonparametrically identified.

Descriptive statistics PSID (means)

	1999	2001	2003	2005	2007	2009
Earnings	85,001	93,984	100,281	106,684	119,039	122,908
Consumption	30,182	35,846	39,843	47,636	52,175	50,583
Assets	266,958	315,866	376,485	399,901	501,590	460,262

Notes: Balanced subsample from PSID, $N = 749$, $T = 6$.

• Compared to BPS (12), households in our balanced sample have higher assets, and to a less extent higher earnings and consumption.

Consumption response, two-period model

• CRRA utility. The Euler equation is (assuming $\beta(1+r)=1$)

$$C_1^{-\gamma} = \mathbb{E}_1\left[((1+r)A_2+Y_2)^{-\gamma}\right],$$

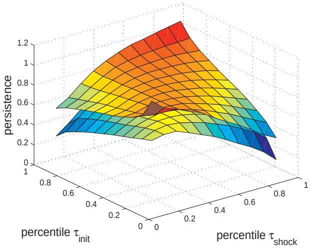
where $\gamma>0$ is risk aversion and we have used the budget constraint $A_3=(1+r)A_2+Y_2-C_2=0$.

• Let $X_1=(1+r)A_1+Y_1$, $R=(1+r)X_1+\mathbb{E}_1(Y_2)$, and $Y_2=\mathbb{E}_1(Y_2)+\sigma W$. Expanding as $\sigma\to 0$ we obtain

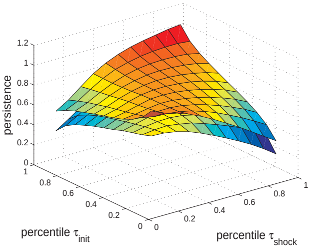
$$C_1 \approx \underbrace{\frac{(1+r)X_1+\mathbb{E}_1(Y_2)}{2+r}}_{\text{certainty equivalent}} \underbrace{-\frac{\gamma+1}{2R}\mathbb{E}_1(W^2)}_{\text{precautionary-variance}} \underbrace{+\frac{(2+r)(\gamma+1)(\gamma+2)}{6R^2}\mathbb{E}_1(W^3)}_{\text{precautionary-skewness}}.$$

Nonlinear persistence, 95% confidence bands

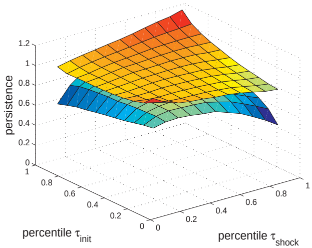
(a) Earnings, PSID data



(b) Earnings, nonlinear model

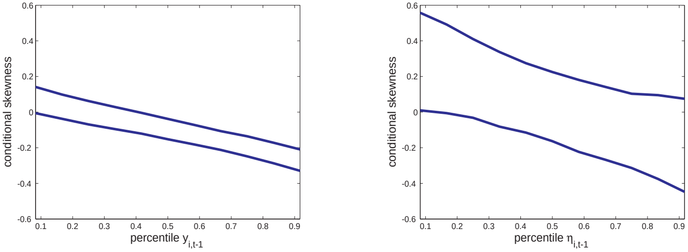


(c) Persistent component η_{it}, nonlinear model



Conditional skewness of log-earnings residuals and η component, 95% confidence bands

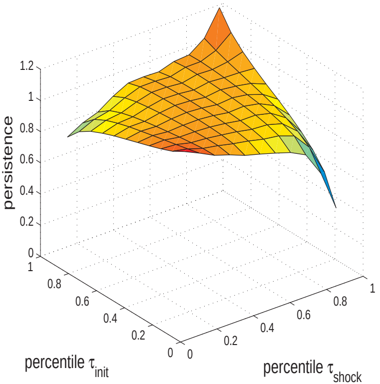
(a) Log-earnings residuals y_{it} (b) Persistent component η_{it}



Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.

Nonlinear persistence of η_{it} (Norwegian register data):

$$\rho_t(\eta_{i,t-1},\tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1},\tau)}{\partial \eta}$$



Note: Estimates of the average derivative of the conditional quantile function of η_{it} on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile τ_{shock} and at a value of $\eta_{i,t-1}$ that corresponds to the τ_{init} percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.

Family Labour Supply, Time-Use and Consumption Smoothing

Modelling Slides

Some related literature

- **Added worker effect**: Lundberg (1985), Hyslop (2001), Stephens (2002), Attanasio, Low and Sanchez-Marcos (2005), Juhn and Potter (2007), Haan and Prowse (2015), Blundell, Pistaferri and Saporta-Eksten (2016), Autor, Kostol, Mogstad and Setzler (2017), ...
- **Time use, time spent with children**: Ghez and Becker (1975), Becker (1976), Aguiar and Hurst (2007, 2013), Guryan, Hurst and Kearney (2008), Ramey and Ramey (2010), Browning, Chiappori and Weiss (2014), Del Boca, Flinn and Wiswall (2014), ...
- **Consumption insurance**: Hall and Mishkin (1982), Blundell and Preston (1998), Krueger and Perri (2006), Guvenen (2007), Blundell, Pistaferri and Preston (2008), Heathcote, Storesletten and Violante (2008), Kaufmann and Pistaferri (2009), Kaplan and Violante (2010), Guvenen and Smith (2013), Heathcote, Storesletten and Violante (2014), Arellano, Blundell and Bonhomme (2017) ....

BPS Model

A life-cycle model of family labour supply, time-use and consumption decisions with:

- Two earners using their time for leisure/input for child production function/work.
- Wage uncertainty for two earners (transitory and persistent).

Model Setup and Solution

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From model to estimation:

Mix semi-structural methods with structural dynamic programming:

1. Semi-structural estimation of a subset of utility and production parameters. Use MRS to derive analytical estimation equations $\Longrightarrow$ estimate a subset of parameters using PSID, CEX and ATUS.

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From model to estimation:
Mix semi-structural methods with structural dynamic programming:

1. Semi-structural estimation of a subset of utility and production parameters. Use MRS to derive analytical estimation equations
$\Longrightarrow$ estimate a subset of parameters using PSID, CEX and ATUS.
2. Structural dynamic model to capture life-cycle dynamics, uncertainty and borrowing constraints. Solve the model numerically given the parameters from stage 1.
$\Longrightarrow$ estimate the remaining parameters using SMM, and provide counterfactual simulations (persistent shock etc.)

Household Lifecycle (Baseline) Model

Household chooses $\{C_{t+s}, L_{1,t+s}, L_{2,t+s}, T_{1,t+s}, T_{2,t+s}\}$ to maximize:

$$\mathbb{E}_t \sum_{s=0}^{T-t} u_{t+s}\left(C_{t+s}, L_{1,t+s}, L_{2,t+s}, T_{1,t+s}, T_{2,t+s}; \mathbf{z}_{t+s}, \boldsymbol{\varepsilon}\right)$$

$$\begin{aligned}
\text{s.t. } A_{t+1} &= (1+r)\left(A_t + \mathcal{T}\left(z_t, H_{1,t}W_{i,1,t} + H_{2,t}W_{i,2,t}\right) - C_t\right) \\
L_{1,t} + H_{1,t} + T_{1,t} &= \bar{L}, \;\; H_{2,t} + L_{2,t} + T_{2,t} = \bar{L} \\
H_{j,t} &\geq 0, L_{j,t} \geq 0, T_{j,t} \geq 0, \;\; A_{t+1} \geq 0, \;\; A_{T+1} = 0
\end{aligned}$$

C: consumption
L_j: leisure time of earner j, T_j: parental time of earner j
$\bar{L}$: maximum time available for work, leisure, childcare
z: demographic characteristics, ε: unobserved heterogeneity
A_t: assets at the beginning of the period
r: nonstochastic interest rate
$W_{j,t}$: hourly wages
$\mathcal{T}(.)$: nonlinear tax function. Details

- Assume the log of real wage of earner $j = \{1, 2\}$ at age t can be written as:

$$\log W_{j,t} = x'_{j,t}\beta^j_W + \eta_{j,t} + u_{j,t} \quad (1)$$
$$\eta_{j,t} = \eta_{j,t-1} + v_{j,t} \quad (2)$$

- Shocks can be correlated across spouses
- $x'_{i,j,t}$: Observed characteristics (e.g. age, state of residence etc.). Assumed to be known to the household.

transitory vs. permanent shocks

Identification (Wage Parameters)

From:

$$\Delta w_{i,j,t} = \Delta u_{i,j,t} + v_{i,j,t}$$

It follows that:

$$\begin{aligned}
\sigma^2_{u_j} &= -E\left(\Delta w_{i,j,t}\Delta w_{i,j,t+1}\right) \\
\sigma^2_{v_j} &= E\left(\Delta w_{i,j,t}\left(\Delta w_{i,j,t+1} + \Delta w_{i,j,t} + \Delta w_{i,j,t-1}\right)\right) \\
\sigma_{u_j u_{-j}} &= -E\left(\Delta w_{i,j,t}\Delta w_{i,-j,t+1}\right) \\
\sigma_{v_j v_{-j}} &= E\left(\Delta w_{i,j,t}\left(\Delta w_{i,-j,t+1} + \Delta w_{i,-j,t} + \Delta w_{i,-j,t-1}\right)\right)
\end{aligned}$$

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\end{aligned}$$

- Easily adapted for dynamic quantile model with nonlinear persistence is particularly well suited to our mixed quasi-structural/dynamic programming approach.

- **Two earner household utility within period** t (baseline spec)**:**

$$
\begin{aligned}
u(.) &= \phi_{C,i}\frac{\tilde{C}_{i,t}^{1-1/\eta_{c,p}}}{1-1/\eta_{c,p}} - \frac{1}{1-\rho_L}\left(\phi_{L_1,i}L_{1,i}^{1-1/\varphi_{L_1}} + \phi_{L_2,i}L_{2,i}^{1-1/\varphi_{L_2}}\right)^{1-\rho_L} \\
&\quad - \frac{1}{1-\rho_T}\left(\phi_{T_1,i}T_{1,i}^{1-1/\varphi_{T_1}} + \phi_{T_2,i}T_{2,i}^{1-1/\varphi_{T_2}}\right)^{1-\rho_T} \\
0 &< \phi_{0,i}, \phi_{L_1,i}, \phi_{L_2,i}, \phi_{T_1,i}, \phi_{T_2,i},\ \ 0 < \varphi_{L_1}, \varphi_{L_2} < 1,\ \ \rho_L, \rho_T < 1
\end{aligned}
$$

- Allow marginal utility of consumption to shift with employment (nonseparability), let $\tilde{C} = e^{\gamma E_2} C_{t+s}$ where $E_2 = 1\{H_2 > 0\}$.

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&\quad -\frac{1}{1-\rho_T}\left(\phi_{T_1,i}T_{1,i}^{1-1/\varphi_{T_1}} + \phi_{T_2,i}T_{2,i}^{1-1/\varphi_{T_2}}\right)^{1-\rho_T} \\
0 &< \phi_{0,i}, \phi_{L_1,i}, \phi_{L_2,i}, \phi_{T_1,i}, \phi_{T_2,i}, \;\; 0 < \varphi_{L_1}, \varphi_{L_2} < 1, \;\; \rho_L, \rho_T < 1
\end{aligned}
$$

- Allow marginal utility of consumption to shift with employment (nonseparability), let $\tilde{C} = e^{\gamma E_2} C_{t+s}$ where $E_2 = 1\{H_2 > 0\}$.
- Utility shifters for good x**:** $\phi_{x,i} = f_x\left(z_{i,t}, \varepsilon_{x,i,t}, \zeta_{x,i}\right)$

z: includes children characteristics,
$\varepsilon, \zeta_{x,i}$: unobserved stochastic heterogeneity components.

From MRS to (Semi-structural) Estimation Equations

- For interior solutions:

$$
\begin{aligned}
L_2 &= \left(\frac{W_{1,t}}{W_{2,t}}\frac{\tilde{\phi}_{L_1}}{\tilde{\phi}_{L_1}} L_1^{1/\varphi_{L_1}}\right)^{\varphi_{L_2}} \\
L_2 &= \left[W_{2,t}\frac{\phi_C}{\tilde{\phi}_{L_1}} C^{-1/\eta_{c,p}}\left(\phi_{L_1} L_1^{1-1/\varphi_{L_1}} + \phi_{L_2} L_2^{1-1/\varphi_{L_2}}\right)^{\rho_L}\right]^{-\varphi_{L_2}} \\
\text{where} &: \quad \tilde{\phi}_x \equiv \phi_x \left(1/\varphi_x - 1\right)
\end{aligned}
$$

- Note: similar relation for parental time use T_1, T_2.
- $\rho_L > 0$ implies *Frisch complement.*
- Assume preference heterogeneity and shifts in marginal utility can be written as:

$$
\log\left(\tilde{\phi}_{x,i,t}\right) = \overline{\tilde{\phi}_x}\left(z_{i,t}\right) + \varepsilon_{x,i,t} + \zeta_{x,i}
$$

- Imply log-linear quasi-structural estimation equations:

$$
\begin{aligned}
l_2 &= K_0 + \varphi_{L_2}\left(w_1 - w_2\right) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \nu_1 \\
l_2 &= K_1 - \varphi_{L_2} w_2 + \frac{\varphi_{L_2}}{\eta_{c,p}} c_t + \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_L \left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2} \rho_L M + \nu_3
\end{aligned}
$$

where: $M = \frac{\varphi_{L_2}\left(1-\varphi_{L_1}\right)}{\varphi_{L_1}\left(1-\varphi_{L_2}\right)} \frac{W_2 L_2}{W_1 L_1}$

(lower case for log(), and omitting time and household subscripts).

- K's are deterministic, and ν's are linear combinations of $\varepsilon' s$ and $\zeta' s$.
- *Analogous equations for child time use inputs* t_2.

Using MRS to Recover Preference Parameters

- Imply log-linear quasi-structural estimation equations:

$$\begin{aligned} l_2 &= K_0 + \varphi_{L_2}(w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \nu_1 \\ l_2 &= K_1 - \varphi_{L_2} w_2 + \frac{\varphi_{L_2}}{\eta_{c,p}} c_t + \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_L \left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2} \rho_L M + \nu_3 \end{aligned}$$

where: $M = \frac{\varphi_{L_2}\left(1-\varphi_{L_1}\right)}{\varphi_{L_1}\left(1-\varphi_{L_2}\right)} \frac{W_2 L_2}{W_1 L_1}$

(lower case for log(), and omitting time and household subscripts).

- K's are deterministic, and ν's are linear combinations of $\varepsilon' s$ and $\zeta' s$.
- *Analogous equations for child time use inputs t_2.*
- Imply nonlinear panel data moment conditions (with appropriate instruments, and participation condition), to consistently estimate φ_{L_1}, φ_{L_2}, ρ_L, $\eta_{c,p}$, φ_{T_1}, φ_{T_2}, and ρ_T by nonlinear GMM.

Using MRS to Recover Preference Parameters

- Imply log-linear quasi-structural estimation equations:

$$
\begin{aligned}
l_2 &= K_0 + \varphi_{L_2}(w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \nu_1 \\
l_2 &= K_1 - \varphi_{L_2} w_2 + \frac{\varphi_{L_2}}{\eta_{c,p}} c_t + \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_L \left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2} \rho_L M + \nu_3
\end{aligned}
$$

where: $M = \frac{\varphi_{L_2}\left(1-\varphi_{L_1}\right)}{\varphi_{L_1}\left(1-\varphi_{L_2}\right)} \frac{W_2 L_2}{W_1 L_1}$

(lower case for log(), and omitting time and household subscripts).

- K's are deterministic, and ν's are linear combinations of $\varepsilon's$ and $\zeta's$.
- *Analogous equations for child time use inputs t_2.*
- Imply nonlinear panel data moment conditions (with appropriate instruments, and participation condition), to consistently estimate φ_{L_1}, φ_{L_2}, ρ_L, $\eta_{c,p}$, φ_{T_1}, φ_{T_2}, and ρ_T by nonlinear GMM.
- But need to recover, fixed cost parameter(s) γ, $\overline{\tilde{\phi}_x}\left(z_{i,t}\right)$ and the distribution of unobserved heterogeneity and taste shocks.

Structural Model SMM Estimation for Remaining Parameters

- Structural model is also required for counterfactual simulations.

Method:

- Solve the stochastic life cycle problem given φ_{L_1}, φ_{L_2}, ρ_L, $\eta_{c,p}$, φ_{T_1}, φ_{T_2}, and ρ_T, and use SMM to complete the estimation.
- Moments to target include:
  - Distribution of hours and time spent with children of each earner at different points over the life-cycle.
  - Levels of employment and employment/non-employment transitions.
  - Consumption changes with children.
- How does this 'mixed' structural approach this compare with the 'partial insurance' approximations?

Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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$$\kappa_{h_j,u_j} \quad = \quad \eta_{h_j,w_j} \rightarrow \text{[Frisch]}$$

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$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

$$\kappa_{h_j,u_j} = \eta_{h_j,w_j} \rightarrow \text{[Frisch]} \qquad \kappa_{h_j,v_j} \rightarrow \text{[Marshall]}$$

Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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$${\color{red}\pi_{i,t}} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}$$

Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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$$\kappa_{c,v_j} = (1-\pi_{i,t})\, s_{i,j,t} \frac{\eta_{c,p}\left(1+\eta_{h_j,w_j}\right)}{\eta_{c,p} + (1-\pi_{i,t})\,\overline{\eta_{h,w}}}$$

$$s_{i,j,t} \approx \frac{\text{Human Wealth}_{i,j,t}}{\text{Human Wealth}_{i,t}}$$

Comparison with partial insurance approach

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$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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$$\eta_{c,p} \rightarrow \text{Consumption EIS}$$

- Use approximation of FOCs (under separability) and of lifetime budget constraint

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & 0 & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ 0 & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

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$$\overline{\eta_{h,w}} = s_{i,j,t}\eta_{h_j,w_j} + s_{i,-j,t}\eta_{h_{-j},w_{-j}}$$

Comparison with partial insurance approach

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Comparison with partial insurance approach

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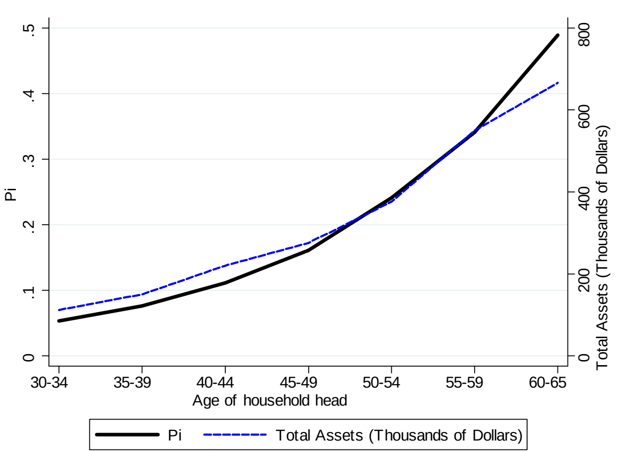
$$\beta \rightarrow \text{External insurance (networks, etc.)}$$

The share of assets to human wealth by age

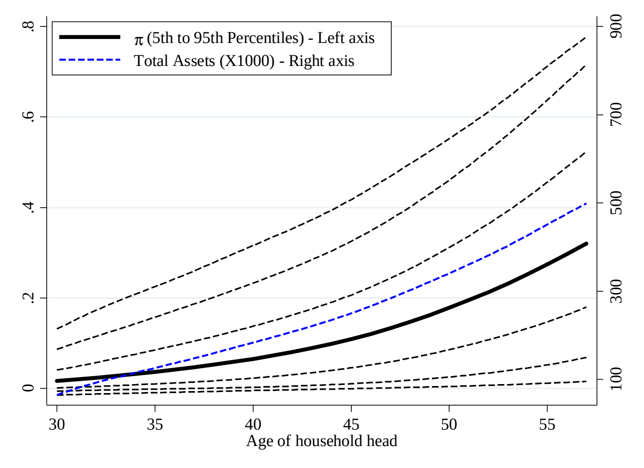
$$\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}$$



Source: Blundell, Pistaferri and Saporta-Eksten (2016)

The distribution of shares of assets to human wealth by age

$$\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}$$



Source: Blundell, Pistaferri and Saporta-Eksten (2016)

- When preferences are non-separable:

$$\begin{pmatrix} \Delta h_{1,t} \\ \Delta h_{2,t} \\ \Delta c_t \end{pmatrix} \simeq \Theta X + \begin{pmatrix} \kappa_{h_1,u_1} & \kappa_{h_1,u_2} & \kappa_{h_1,v_1} & \kappa_{h_1,v_2} \\ \kappa_{h_2,u_1} & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\ \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

- $\kappa_{c,u_j} \rightarrow$ non-separability between consumption and leisure of member j
    - Identified by response of consumption to transitory shock having no wealth effects
- $\kappa_{h_j,u_{-j}} \rightarrow$ non-separability between spouses' leisures
    - Identified by response of member j's labor supply to transitory shock faced by spouse
- BPS estimates suggest Frisch substitutes for families with younger children.
- But, as with other similar semi-structural methods, insufficient to identify intertemporal/life-cycle counterfactuals.

$$\begin{pmatrix} \Delta c_\tau \\ \Delta l_{1,\tau} \\ \Delta l_{2,\tau} \\ \Delta t_{1,\tau} \\ \Delta t_{2,\tau} \end{pmatrix} \simeq \begin{pmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{t^l_1,u_1} & \kappa_{t^l_1,u_2} & \kappa_{t^l_1,v_1} & \kappa_{t^l_1,v_2} \\ \kappa_{t^l_2,u_1} & \kappa_{t^l_2,u_2} & \kappa_{t^l_2,v_1} & \kappa_{t^l_2,v_2} \\ \kappa_{t^c_1,u_1} & \kappa_{t^c_1,u_2} & \kappa_{t^c_1,v_1} & \kappa_{t^c_1,v_2} \\ \kappa_{t^c_2,u_1} & \kappa_{t^c_2,u_2} & \kappa_{t^c_2,v_1} & \kappa_{t^c_2,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

- where $\kappa_{m,v_j} = \kappa^{m,v_j}\left(\pi_t, s_t, \eta, \mathcal{T}()\right)$,
- and $s_t \approx \frac{\text{Human Wealth}_{1,t}}{\text{Human Wealth}_t}$, $\pi_t \approx \frac{\text{Assets}_t}{\text{Assets}_t + \text{Human Wealth}_t}$.
- This quasi-structural approach performs less well near/at corners.
- It cannot recover fixed cost/extensive margin parameters and cannot simulate counterfactuals.
- However, useful ‘moments’ to simulate in the structural model to examine ‘Frisch’ and ‘Marshallian’ responses.

Data and Estimation

Estimation in Practice

Estimating the MRS equations requires data on:

- Leisure, parental time and hourly wages of both earners.
- Household consumption and assets.
- Family composition.
- Valid instruments for endogenous variables (consumption, leisure etc.), and wages (due to measurement error and selection).

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- Family composition.
- Valid instruments for endogenous variables (consumption, leisure etc.), and wages (due to measurement error and selection).

Where can we find such data?...

- PSID:
  - Unique panel data on consumption, assets, hours of work and hourly wages of both earners (biennial since 1999).
  - Very noisy parental time use measures in CDS diary (used in previous work for this paper).
- ATUS: Detailed time use data; (annual since 2003).
- CEX: Detailed consumption data (to match annual ATUS).

Combine the Multiple Sources

1. Use PSID (1999-2009/2013) to estimate the MRS equations for leisure by GMM for families **with no young children** $(\Rightarrow \varphi_{L_1}, \varphi_{L_2}, \rho_L, \eta_{cp})$.
2. For the moment estimator of the MRS equations for parental time:
   - Use ATUS (2003-2014) parental time of married women **with young children** combined with hourly wages of both spouses.
     Combine with cohort-education-year aggregate of husband parental time (ATUS), and consumption (CEX).
     $(\Rightarrow \varphi_{T_1}, \varphi_{T_2}, \rho_T)$.

- Note: apply similar sample selection in all datasets:
  - Married couples, wife aged 25-64.
  - In GMM, condition on employment of both earners (and apply correction).

Descriptives of Time Use data in the ATUS

	(1)	(2)	(3)	(4)
	mean	p25	median	p75
Non-zero childcare time (head)	0.69			
Non-zero childcare time (wife)	0.91			
childcare annual hours (head) inc. 0s	320	0	195	498
childcare annual hours (wife) inc. 0s	709	260	585	1,023
childcare annual hours (head) exc. 0s	466	182	355	628
childcare annual hours (wife) exc. 0s	778	347	650	1,070

Notes: ATUS data from 2003-2014 for the sample of married couples, wife aged 25-65 with youngest child aged 10 or less.

Descriptives of Consumption, Leisure and Wages in the PSID

	(1)	(2)	(3)	(4)
	mean	p25	median	p75
Total Consumption (exc. durables)	40,997	26,237	35,654	49,307
Hours of husband	2,011	1,835	2,080	2,500
Hours of wife	1,349	347	1,645	2,016
Hourly wage of husband	31.3	15.2	22.6	34.8
Hourly wage of wife	21.3	11.4	17.3	26.3

Notes: PSID data from 1999-2013 PSID waves, for the sample of married couples, wife aged 25-65 with youngest child aged 10 or less. Consumption and wages in 2010 prices. In computations leisure time is calculated assuming total hours is 4160 (5*16*52).

Results

MRS Estimates

(1)		(2)	
PSID		ATUS	
φ_{L_1}	0.161***	φ_{T_1}	0.115**
	(0.044)		(0.049)
φ_{L_2}	0.115***	φ_{T_2}	0.505***
	(0.027)		(0.191)
ρ_L	0.646***	ρ_T	-0.192**
	(0.092)		(0.084)
η_{cp}	0.807***		
	(0.069)		
Obs.	8,443		2,901

Notes: In Columnl 1 the parameters are estimated by GMM on PSID. Standard errors clustered by household in parenthesis. Parameter estimates reported in Column 2 use matched moments from ATUS and CEX data. *, **, *** = Significant at 10%, 5%, and 1%.

Parameterization

- Estimated wage process follows from BPS (2016):
  - $\sigma^2_{u_1} = 0.0275, \sigma^2_{u_2} = 0.0125, \sigma^2_{v_1} = 0.0303, \sigma^2_{v_2} = 0.0382,$
  - cross wage correlations are small and positive, see BPS.
  - No insurance here!
- Wages of both earners (transitory and permanent) discretized.
- Assets discretized, assuming net worth positive constraint.
- Discrete unobserved preference heterogeneity/types.

Moment match

Comparing Time Use Responses for Low and High Assets Cases

		L_1		L_2		T_1	T_2
		With kids	W.o. kids	With kids	W.o. kids	With kids	
Low (lowest quartile) assets at age 25							
Trans.	Δu_1	-0.15	-0.17	0.01	-0.01	-0.08	0.16
	Δu_2	0.02	~0	-0.10	-0.12	0.04	-0.52
Perm.	v_1	-0.07	-0.06	0.07	0.07	-0.06	0.26
	v_2	0.10	0.10	-0.04	-0.04	0.07	-0.42
High (top quartile) assets at age 25							
Trans.	Δu_1	-0.22	-0.24	-0.05	-0.06	-0.10	0.08
	Δu_2	-0.03	-0.06	-0.14	-0.16	0.03	-0.40
Perm.	v_1	-0.10	-0.08	0.04	0.06	-0.07	0.23
	v_2	0.09	0.09	-0.05	-0.05	0.06	-0.35

Relating Frisch Time Use Elasticities to Labor Supply Elasticities (Wife's Example)

- Own elasticity:

$$\eta_{h_2,w_2} = -\eta_{l_2,w_2}\frac{L_2}{H_2} - \eta_{t_2,w_2}\frac{T_2}{H_2}$$

where we expect $\eta_{l_2,w_2} < 0, \eta_{t_2,w_2} < 0$

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  where we expect $\eta_{l_2,w_2} < 0, \eta_{t_2,w_2} < 0$
- Cross elasticity:

$$\eta_{h_2,w_1} = -\eta_{l_2,w_1}\frac{L_2}{H_2} - \eta_{t_2,w_1}\frac{T_2}{H_2}$$

  where signs are unrestricted, but:
  - Complementarity of leisure consistent with $\eta_{l_2,w_1} < 0$
  - Specialization in caring for children consistent with $\eta_{t_2,w_1} > 0$

Comparing Consumption and Hours Responses for Low and High Assets Cases

		C		H_1		H_2	
		With kids	W.o. kids	With kids	W.o. kids	With kids	W.o. kids
Low (lowest quartile) assets at age 25							
Trans.	Δu_1	0.22	0.17	0.19	0.18	-0.20	-0.10
	Δu_2	0.21	0.15	-0.04	~0	0.50	0.39
Perm.	v_1	0.40	0.42	0.11	0.09	-0.40	-0.31
	v_2	0.38	0.39	-0.13	-0.13	0.34	0.25
High (top quartile) assets at age 25							
Trans.	Δu_1	0.07	0.01	0.34	0.32	0	0.07
	Δu_2	0.06	0.02	0.03	0.07	0.80	0.51
Perm.	v_1	0.34	0.38	0.18	0.13	-0.29	-0.18
	v_2	0.34	0.35	-0.16	-0.14	0.46	0.29

- Counterfactual consumption response to a male's permanent wage shock in two key components:
  - insurance via family labour supply, and
  - insurance through savings.
- Wife's response to husband's permanent wage:
  - leisure complementarity,
  - specialization,
  - wealth effect.
- We illustrate these channels by decomposing the average simulated counterfactual response to a permanent shock.

What Does a 10% Permanent Reduction in Husband's Hourly Wage Look Like?

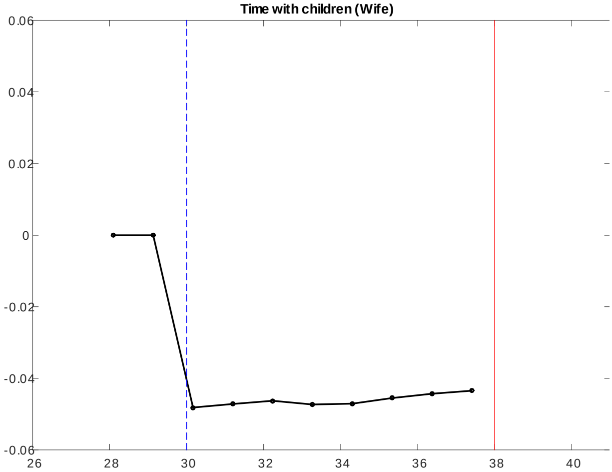
Consumption:	-4.2%
After-tax household earnings:	-5.1%
pre-tax household earnings:	-5.6%

	Husband	Wife
Earner's average share of pre-tax earnings:	0.66	0.34
Earner's pre-tax earnings response:	-10.4%	+3.3%
Hours	-1.0%	+4.2%
Leisure	+1.3%	-1.4%
Parental time	+0.7%	-5.1%

Notes: for a sample of working husbands and wives, working at least 80 hours per year. Based on the regressions run at age 35 in the model

Mother's labor supply response to a persistent adverse shock (10%) to husband's earnings

Mother's time with children response to a persistent adverse shock to husband's earnings



	(1) C	(2) H_1	(3) H_2	(4) E_2	(5) L_1	(6) L_2	(7) T_1	(8) T_2
A: Exp 1: Unconditional Subsidy for Families with Young Children (yk)								
Total	0.6%	-0.4%	-0.7%	-0.4%	0.4%	0.3%		
Before yk	0.9%	-0.4%	-0.5%	-0.2%	0.4%	0.4%		
With yk	1.3%	-0.6%	-1.8%	-1.0%	0.8%	0.7%	0.2%	1.0%
After yk	0.1%	-0.1%	-0.1%	-0.1%	0.1%	0.1%		
Consumption equivalent utility value: 0.95%								
B: Exp 2: Employment Subsidy for Wives with Young Children (yk)								
Total	0.1%	-0.2%	1.9%	4.6%	0.2%	-0.5%		
Before yk	0.9%	-0.4%	-0.5%	-0.2%	0.4%	0.4%		
With yk	-0.3%	-0.3%	6.5%	13.0%	0.3%	-1.7%	0.3%	-5.7%
After yk	0.1%	-0.1%	-0.1%	-0.1%	0.1%	0.1%		
Consumption equivalent utility value: 0.17%								

IMPLICATIONS....

- This research implies that family labor supply can be a key mechanism for 'insuring' unexpected shocks
  - especially for younger families and for those with limited access to assets,
  - leisure time turns out to be a Frisch complement but a Marshallian substitute.

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- It is mother’s time with children that takes a hit.

Summary and next steps

- Study the interaction between time spent with children, labor supply responses and consumption insurance.
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- Natural next steps:
  - study the implications for child outcomes, currently linking to CDS
  - experience/human capital => as in BDMS (*Ecta* 2016),
  - other types of (partially insured) shocks,
  - allow for unusual shocks and nonlinear persistence in the wages as in ABB (2017).

Taxation

- Nonlinear progressive taxation (including EITC, child tax credits, SNAP and TANF) is approximated by:

$$\mathcal{T}\left(\sum_{j=\{1,2\}} H_{j,t} W_{j,t}; \mathbf{z}_t\right) \approx \left(1-\chi_t\left(\mathbf{z}_t\right)\right)\left(b_t\left(\mathbf{z}_t\right)+\sum_{j=\{1,2\}} H_{j,t} W_{j,t}\right)^{1-\mu_t\left(\mathbf{z}_t\right)}$$

- χ_t, μ_t and b_t chosen to match the tax scheme and can depend on year and family composition.
- Advantages of this function:
    - Performs well for the US tax system (see next slide).
    - Allows for extensive margin.

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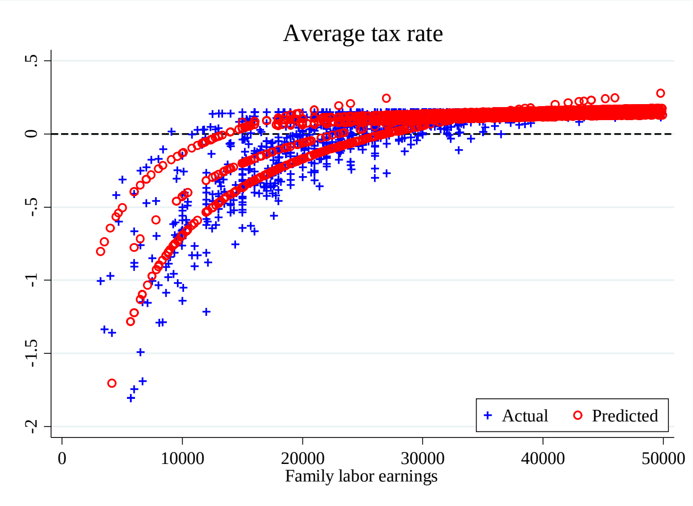
Performance of the Tax and Benefit Function



Source: Blundell, Pistaferri and Saporta-Eksten, 2016

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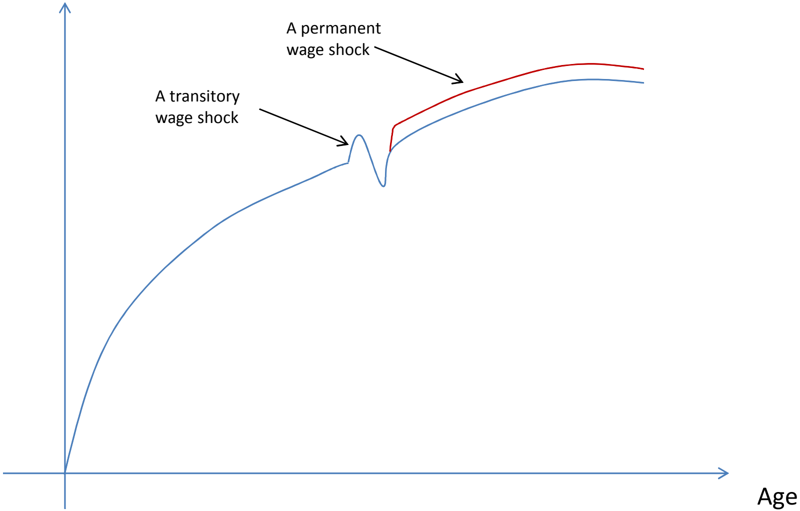
Transitory vs. Permanent Wage Shock



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- Continue with the wife's cross elasticity:

$$\eta_{h_2,w_1} = -\eta_{t_2^l,w_1}\frac{T_2^l}{H_2} - \eta_{t_2^c,w_1}\frac{T_2^c}{H_2}$$

Where do children show up?

- $\frac{T_2^l}{H_2}, \frac{T_2^c}{H_2}$ (and similarly for the husband $\frac{T_1^l}{H_1}, \frac{T_1^c}{H_1}$):
  - The case of no children: $\frac{T_2^c}{H_2} = 0$. With leisure complementarity: $\eta_{t_2^l,w_1} < 0$, $\eta_{h_2,w_1} > 0$.
  - The case of very young children: $\frac{T_2^c}{H_2} >> 0$. If $\eta_{t_2^c,w_1} > 0$, η_{h_2,w_1} might become negative. graphical illustration

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- Without separability between sub-aggregates, the Frisch elasticities of time use (e.g. $\eta_{t_2^l,w_1}$ and $\eta_{t_2^c,w_1}$) might depend on children presence and ages.

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Moments Match in SMM Estimation

	Data	Model
Hours of work: wife with young kids	1,251	1,248
Hours of work: wife without young kids	1,814	1,816
Hours of work: husband with young kids	2,218	2,225
Hours of work: husband without young kids	2,126	2,121
Hours of parental time: wife with young kids	784	778
Hours of parental time: husband with young kids	346	337
Interquartile range hours: wife with young kids	1,818	1,957
Interquartile range hours: wife without young kids	576	605
Employment probability of wife with young kids	0.77	0.76
Employment probability of wife without young kids	0.90	0.90
Change in consumption when kid is born	0.075	0.073
B. Non-targeted Moment (Wife 50-55, no kids)		
Hours of work: wife (aged 50-55, no kids)	1,411	1,633
Hours of work: husband (aged 50-55, no kids)	1,910	2,036
Employment probability of wife (aged 50-55, no kids)	0.78	0.83
Interquartile range hours of wife	1,485	1,311

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