Leverage Stacks
and the Financial System

John Moore

Edinburgh University and
London School of Economics

Nemmers Conference
Northwestern University

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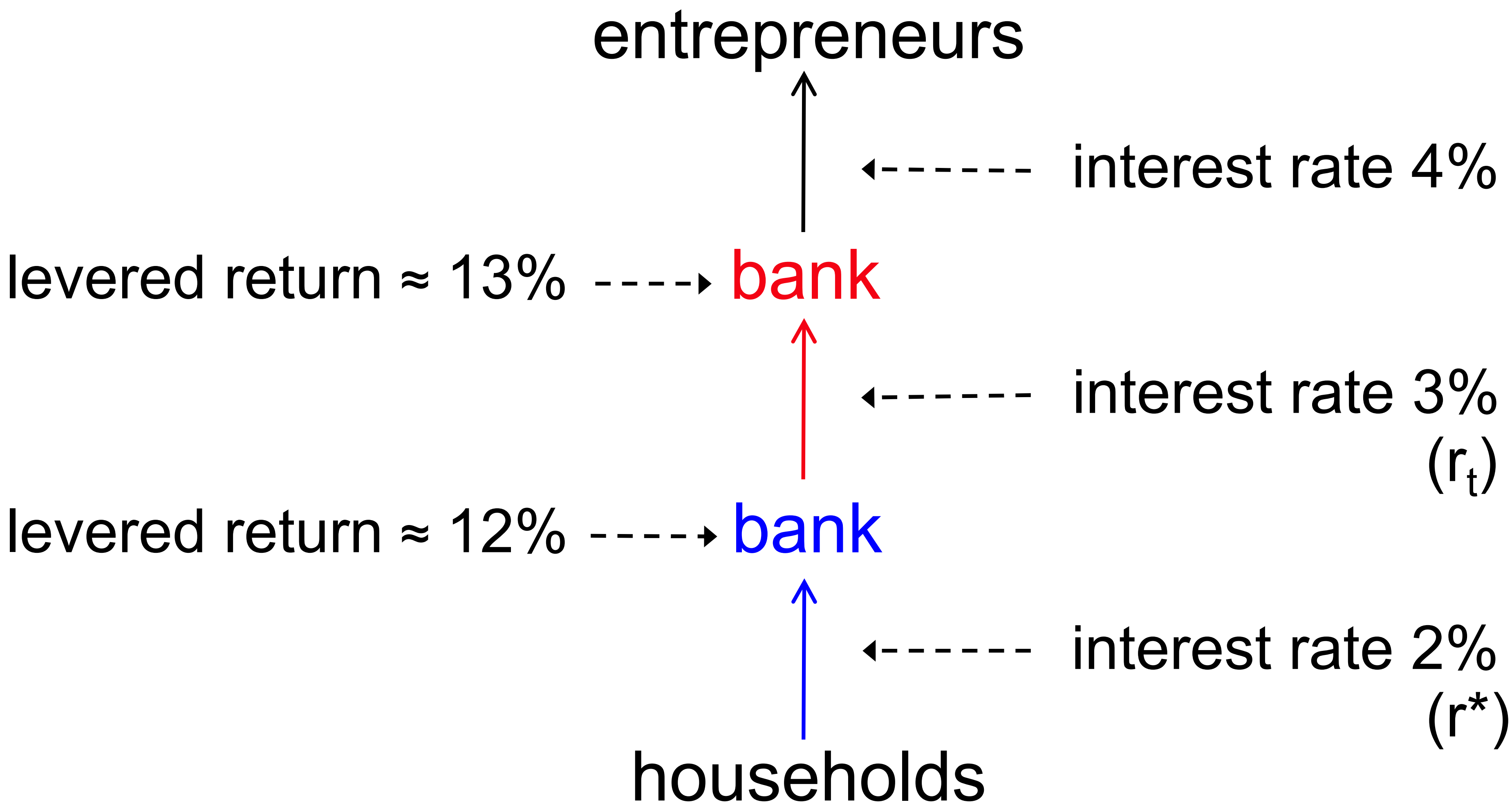
Leverage Stack:

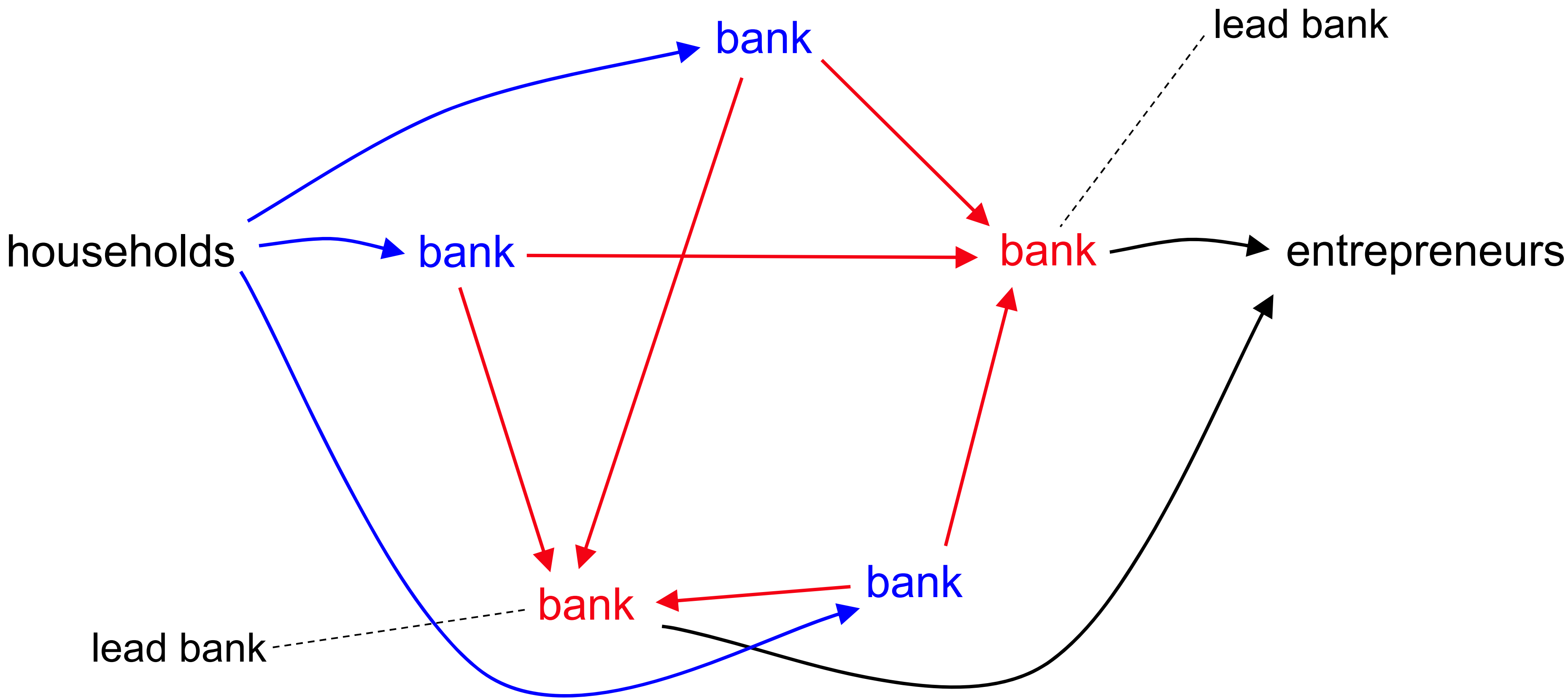


at each level, borrower can pledge $\leq \frac{9}{10}$ of return

(θ, θ^*)

Entrepreneurial lending opportunities are i.i.d. (prob π)

e.g. five banks and $\pi = 2/5$:



Note: no mutual gross positions yet

To allow for mutual gross positions, suppose

loans to entrepreneurs are long-term

$$\Downarrow$$

every bank (even one of today's non-lead banks) has some of these old assets on its b/sheet

– from when, in the past, it was a lead bank

typical bank’s balance sheet

assets		liabilities
capital investment holdings (long-term)	*secured against*	interbank bonds issued (short-term)
interbank bond holdings (short-term)	*secured against*	household bonds issued (short-term)
		own equity

Should non-lead bank spend its marginal dollar

on paying down ($\equiv$ not rolling over) old interbank debt secured against these old assets

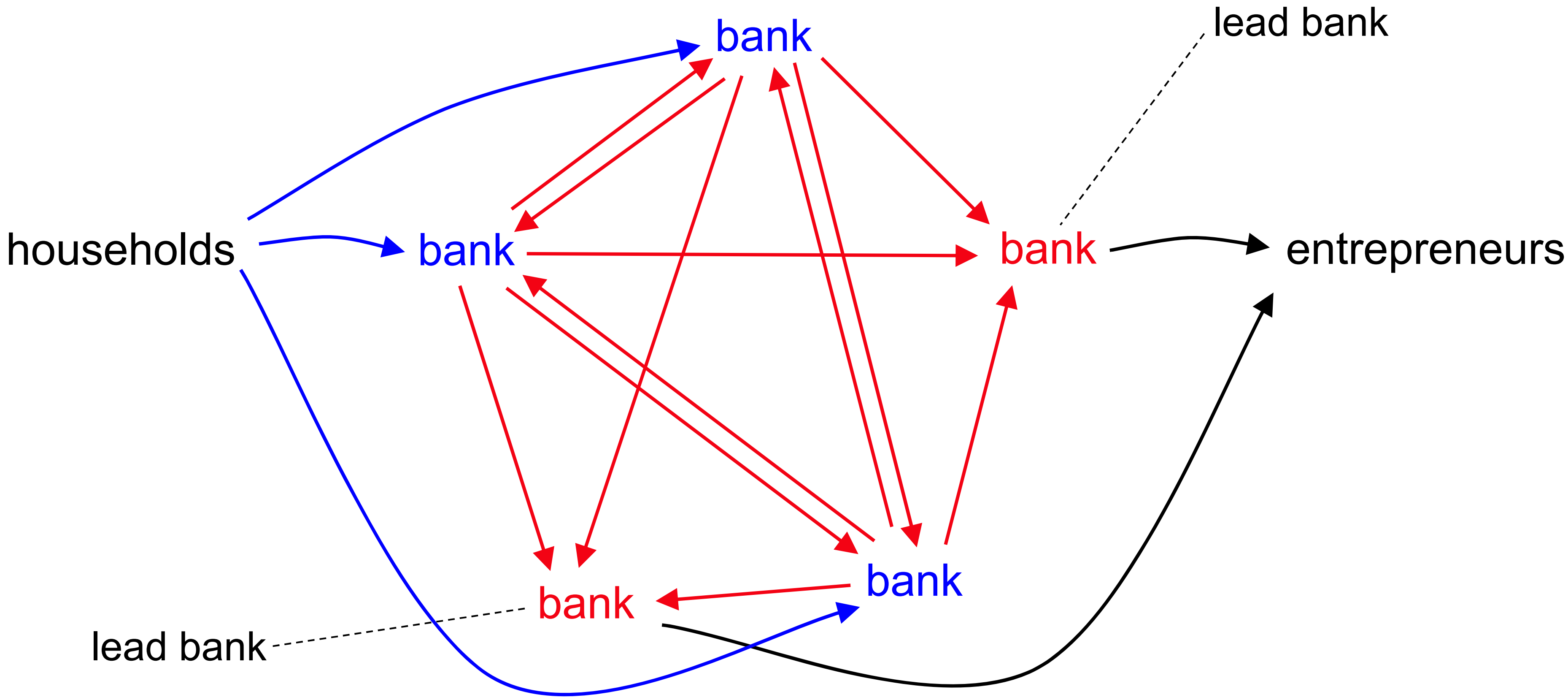
$\Rightarrow$ return of 3%

or on buying new interbank debt @ 3%, levered by borrowing from households @ 2%

$\Rightarrow$ effective return of $\approx$ 12% ✓

That is, non-lead banks should "max out"

Hence there are mutual gross positions among non-lead banks:



Mutual gross positions among non-lead banks "certify" each others' entrepreneurial loans and thus offer additional security to households

$\Rightarrow$ more funds flow in to the banking system, from households

$\Rightarrow$ more funds flow out of the banking system, to entrepreneurs

$\Rightarrow$ greater investment & aggregate activity

But though the economy operates at a higher average level, it is susceptible to systemic failure

MODEL

discrete time, dates $t = 0, 1, 2, \ldots$

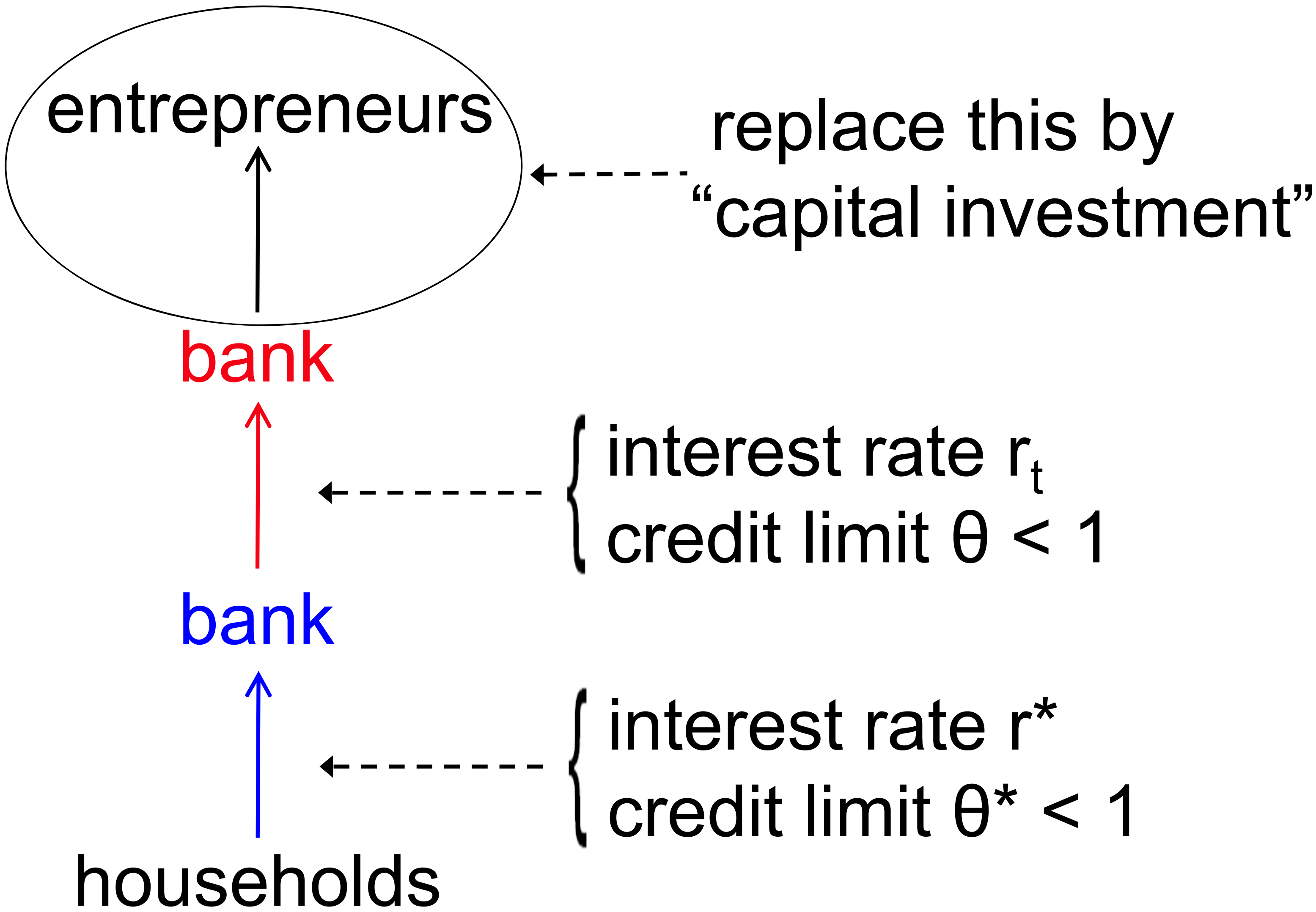
at each date, single good (numeraire)

fixed set of agents (banks), who derive utility from their scale of investment

$\Rightarrow$ a bank invests maximally if opportunity arises

in background: outside suppliers of funds (e.g. households)

Remove top of leverage stack:



Capital investment

constant returns to scale; per unit of project:

date t	date $t+1$	date $t+2$	date $t+3$	$\ldots$
-1	a_{t+1}	λa_{t+2}	$\lambda^2 a_{t+3}$	$\ldots$
unit cost		depreciation factor $\lambda < 1$		

where the economy-wide productivities $\{a_{t+s}\}$ follow two-point i.i.d. process: a_{high} / a_{low}

Capital investment is illiquid: projects are specific to the investing bank

However, the bank can issue *"interbank bonds"* (i.e. borrow from other banks) against its capital investment:

per unit of project, bank can issue

$\theta < 1$ interbank bonds

price path of interbank bonds: $\{q_t, q_{t+1}, q_{t+2}, \ldots\}$

an interbank bond issued at date t-1 promises

$$[\, E_{t-1}a_t \;+\; \lambda E_{t-1}q_t \,] \quad \text{at date t}$$

(expectations conditional on
no default at t)

i.e., bonds are short-term & creditor is promised (a fraction θ of) expected project return next period + expected price of a new bond issued next period against residual flow of returns

collateral securing old bond

= expected project return

\+ expected sale price of new bond

from the price path $\{q_{t-1}, q_t, q_{t+1}, q_{t+2}, \dots\}$ we can compute the interbank interest rates:

effective risk-free interbank interest rate, r_{t-1},

between date t-1 and date t solves:

$$q_{t-1} = \frac{1 - \delta_t}{1 + r_{t-1}} \left[E_{t-1} a_t + \lambda E_{t-1} q_t \right]$$

where δ_t = probability of default at date t (endogenous)

NB in principle δ_t is bank-specific
– but see Corollary to Proposition below

A bank can issue "*household bonds*" (i.e. borrow from households) against its holding of interbank bonds. Household bonds mimic interbank bonds:

– a household bond issued at date t-1 promises to pay $[\, E_{t-1}a_t + \lambda E_{t-1}q_t \,]$ at date t

per interbank bond, bank can issue

$$\theta^* < 1 \quad \text{household bonds}$$

at price

$$q_{t-1}^* = \frac{1 - \delta_t}{1 + r^*} \left[\, E_{t-1}a_t + \lambda E_{t-1}q_t \,\right]$$

households lend at r*

These promised payments – on interbank and household bonds – are fixed at issue, date t-1, using that date's expectation (E_{t-1}) of future returns & bond prices:

bonds are unconditional (no state-dependence)

In the event of, say, a fall in returns, or
a fall in bond prices,

the debtor bank must honour its fixed payment obligations, or risk default & bankruptcy

Assume bankruptcy $\Rightarrow$ creditors receive nothing

typical bank's balance sheet at start of date t

assets	liabilities
	secured against
capital investment holdings (k_t)	interbank bonds issued ($\leq \theta k_t$)
	secured against
interbank bond holdings (b_t)	household bonds issued ($\leq \theta^* b_t$)
	own equity

lead bank's flow-of-funds (assuming no default)

rollover

$$i_t \quad \leq \quad a_t k_t \quad - \quad [\, E_{t-1}a_t \;+\; \lambda E_{t-1}q_t \,]\, \theta k_t$$

capital investment | returns | payments to other banks

$$+\; [\, E_{t-1}a_t \;+\; \lambda E_{t-1}q_t \,]\, b_t \;-\; [\, E_{t-1}a_t \;+\; \lambda E_{t-1}q_t \,]\, \theta^* b_t$$

payments from other banks | payments to households

rollover

$$+\quad q_t\theta\,(\lambda k_t \;+\; i_t\,)$$

sale of new interbank bonds

Hence, for a lead bank starting date t with (k_t, b_t),

$$b_{t+1} = 0$$

and $$k_{t+1} = \lambda k_t + i_t$$

where i_t is given by

$$\frac{(a_t - \theta E_{t-1}a_t)k_t + (1-\theta^*)\left[E_{t-1}a_t + \lambda E_{t-1}q_t \right]b_t + \theta(q_t - E_{t-1}q_t)\lambda k_t}{1 - \theta q_t}$$

non-lead bank's flow-of-funds

rollover

$$q_t b_{t+1} \leq a_t k_t - [E_{t-1}a_t + \lambda E_{t-1}q_t]\,\theta k_t$$

purchase of other banks' bonds — returns — payments to other banks

$$+ [E_{t-1}a_t + \lambda E_{t-1}q_t]\,b_t - [E_{t-1}a_t + \lambda E_{t-1}q_t]\,\theta^* b_t$$

payments from other banks — payments to households

rollover

$$+ \quad q_t \theta \lambda k_t \quad + \quad q_t^* \theta^* b_{t+1}$$

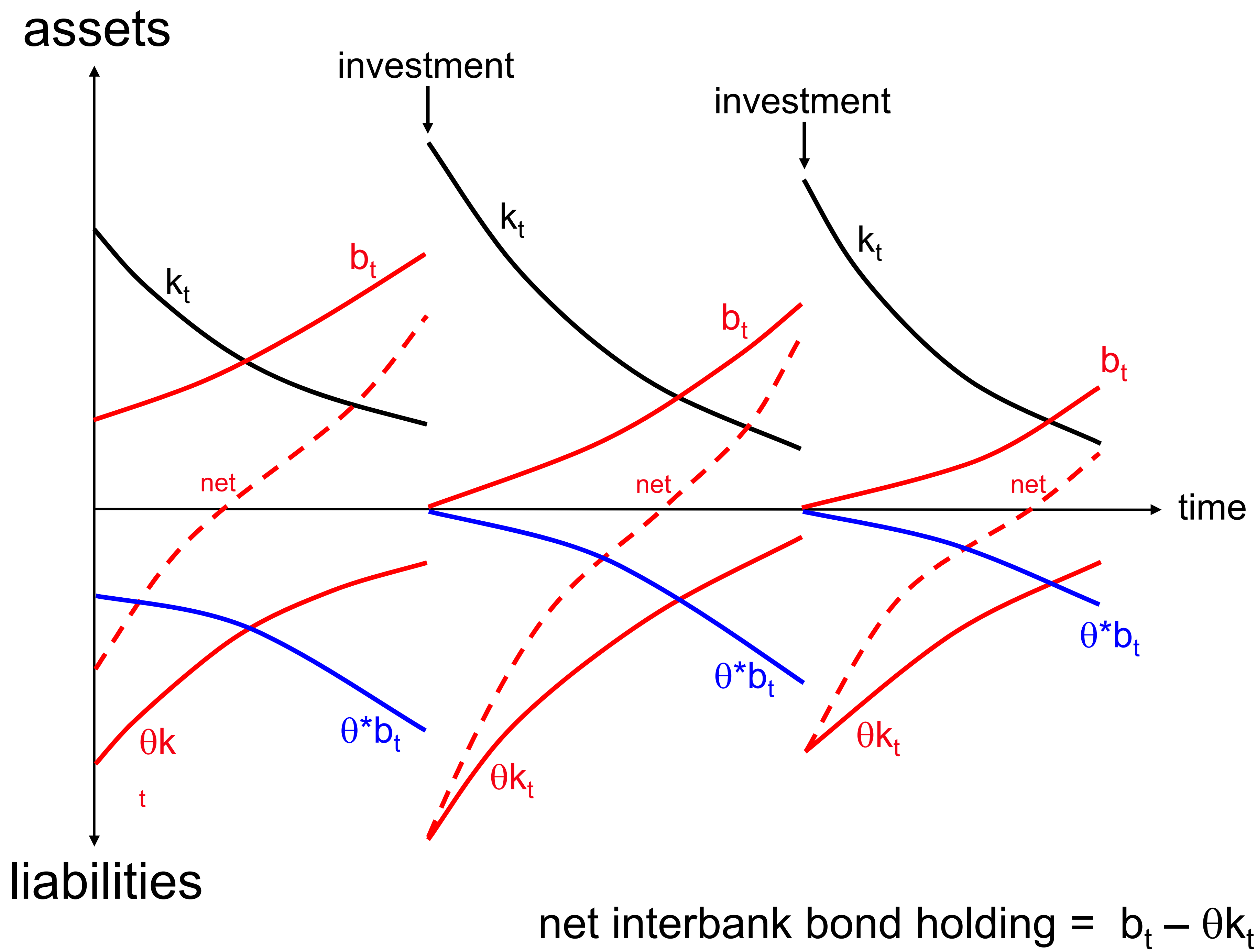
sale of new interbank bonds — sale of new household bonds

Hence, for a non-lead bank starting date t with (k_t, b_t),

$$k_{t+1} = \lambda k_t$$

and b_{t+1} is given by

$$b_{t+1} = \frac{(a_t - \theta E_{t-1} a_t) k_t + (1-\theta^*)\left[E_{t-1} a_t + \lambda E_{t-1} q_t \right] b_t + \theta (q_t - E_{t-1} q_t) \lambda k_t}{q_t - \theta^* q_t^*}$$


assets
investment
investment
k_t
b_t
k_t
b_t
k_t
b_t
net
net
net
time
θk
t
$\theta^* b_t$
θk_t
$\theta^* b_t$
θk_t
$\theta^* b_t$
liabilities
net interbank bond holding = $b_t - \theta k_t$

each bank has its personal history of, at each past date, being either a lead or a non-lead bank

$\Rightarrow$ in principle we should keep track of how the distribution of $\{k_t, b_t\}$'s evolves (hard)

however, the great virtue of our expressions for k_{t+1} and b_{t+1} is that they are linear in k_t and b_t

$\Rightarrow$ aggregation is easy

At the start of date t, let

K_t = banks' stock of capital investment

B_t = banks' stock of interbank bonds

$K_{t+1} = \lambda K_t + I_t$ where

I_t = banks' capital investment =

Investment is v sensitive to falls in the bond price

$$\frac{\pi \left\{ (a_t - \theta E_{t-1} a_t) K_t + (1 - \theta^*) [\, E_{t-1} a_t + \lambda E_{t-1} q_t \,] B_t + \theta (q_t - E_{t-1} q_t) \lambda K_t \right\}}{1 - \theta q_t}$$

and B_{t+1} is given by

$$\frac{(1-\pi)\left\{(a_t - \theta E_{t-1}a_t)K_t + (1-\theta^*)\left[E_{t-1}a_t + \lambda E_{t-1}q_t\right]B_t + \theta(q_t - E_{t-1}q_t)\lambda K_t\right\}}{q_t - \theta^* q_t^*}$$

Market clearing

Price q_t clears the market for interbank bonds at each date t:

$$\text{interbank banks' bond demand} = B_{t+1}$$

$$\text{interbank banks' bond supply} = \theta K_{t+1}$$

Posit additional demand from "outside banks":

$$\underbrace{D(\overset{\oplus}{r_t})}_{\uparrow} = q_t \left(\theta K_{t+1} - B_{t+1} \right)$$

outside banks' supply of loanable funds
is increasing in risk-free interest rate r_t

The following results hold near to steady-state

Throughout, assume that most interbank loans come from the other inside banks, not from outside banks:

$$q_t B_{t+1} \gg D(r_t)$$

As a preliminary, we need to confirm that non-lead banks will choose to lever their interbank lending with borrowing from households:

Lemma 1 $r_t > r^*$ iff

(A.1):

$$\theta > \pi\theta\theta^* + (1-\pi)(1-\lambda+\lambda\theta) + (1-\pi)(1-\theta\theta^*)r^*$$

Lemma 2a

A fall in a_t raises the current interest rate r_t

Intuition: $a_t \downarrow$ raises bond supply/demand ratio:

$$\frac{\text{inside banks' bond supply}}{\text{inside banks' bond demand}} = \frac{\theta\left(\lambda K_t + \frac{\pi}{1-\theta q_t} W_t\right)}{\frac{1-\pi}{q_t-\theta^* q_t^*} W_t}$$

which implies $r_t \uparrow$

where

$$W_t = \Big\{ (a_t - \theta E_{t-1} a_t) K_t + (1-\theta^*)\left[E_{t-1} a_t + \lambda E_{t-1} q_t \right] B_t + \theta (q_t - E_{t-1} q_t) \lambda K_t \Big\}$$

Lemma 2b

For $s \geq 0$, a rise in r_{t+s} raises r_{t+s+1}

Intuition: $r_{t+s} \uparrow \;\Rightarrow\; \underbrace{(1 + r_{t+s})D(r_{t+s})}_{\text{debt (inclusive of interest) owed by inside banks to outside banks at date t+s+1}} \uparrow$

$\Rightarrow\; W_{t+s+1} \downarrow$ (debt overhang)

$\Rightarrow\; r_{t+s+1} \uparrow$ (cf. Lemma 2a)

Lemma 2c

A rise in future interest rates raises the current interest rate if (A.2): $\theta^*\pi > (1 - \lambda + \lambda\pi)^2$

Intuition: a rise in any of $E_t r_{t+1}, E_t r_{t+2}, E_t r_{t+3}, \ldots$

$$\Rightarrow E_t q_{t+1} \downarrow \quad \Rightarrow \quad q_t^* = \frac{1-\delta_{t+1}}{1+r^*}\left\{E_t a_{t+1} + \lambda E_t q_{t+1}\right\} \downarrow$$

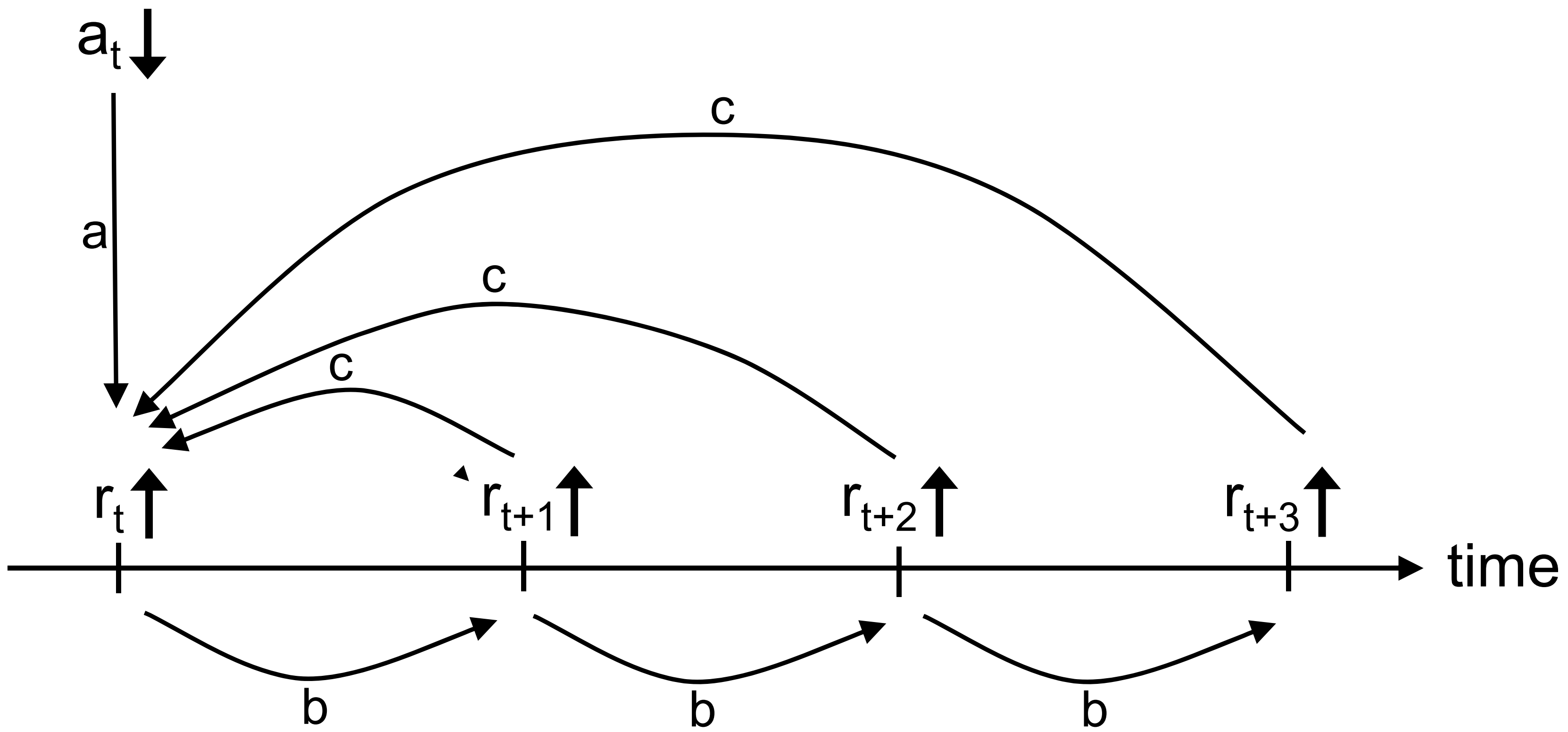
$\Rightarrow$ ratio of inside banks' bond supply/demand

$$= \frac{\theta\left(\lambda K_t + \frac{\pi}{1-\theta q_t} W_t\right)}{\frac{1-\pi}{q_t - \theta^* q_t^*} W_t} \uparrow \qquad \Rightarrow r_t \uparrow$$

$\theta^* q_t^* \downarrow$

under (A.2), this channel dominates (borrowing from households $\downarrow$)

amplification through interest rate cascades:



$\Rightarrow \; q_t \downarrow$

$\Rightarrow \; I_t \downarrow\downarrow$

collateral-value multiplier:

interbank bond prices ↓

↓

collateral values ↓

↓

borrowing from households ↓

↓

net interbank lending by non-lead banks ↓

↓

interbank interest rates ↑

broad intuition:

negative shock

$\Rightarrow$ interbank interest rates $\uparrow$ and bond prices $\downarrow$

$\Rightarrow$ banks' household borrowing limits tighten

$\Rightarrow$ funds are taken *from* banking system, just as they are most needed fall in interbank bond prices

$\Rightarrow$ banks may have difficulty rolling over their debt, and so be vulnerable to failure

"most vulnerable" banks:

banks that have just made maximal capital investment (because they hold no cushion of interbank bonds)

Failure of these banks can precipitate a failure of the entire banking system:

Proposition (systemic failure)

In addition to Assumption (A.1), assume

$$\text{(A.3):} \qquad \theta^* \;>\; (1-\pi)\,\lambda$$

If the aggregate shock is enough to cause the most vulnerable banks to fail, then *all* banks fail (in the order of the ratio of their capital stock to their holding of other banks' bonds).

NB In proving this Proposition, use is made of the steady-state (ergodic) distribution of the $\{k_t, b_t\}$'s across banks

Corollary

At each date t, the probability of default, δ_t, is the same for all inside banks

We implicitly assumed this earlier – in effect, we have been using a guess-and-verify approach

Banks make no attempt to self-insure – e.g. by lending to “less risky” banks (because there are none: all banks are equally risky)

Parameter consistency?

Assumptions (A.1), (A.2) and (A.3) are mutually consistent:

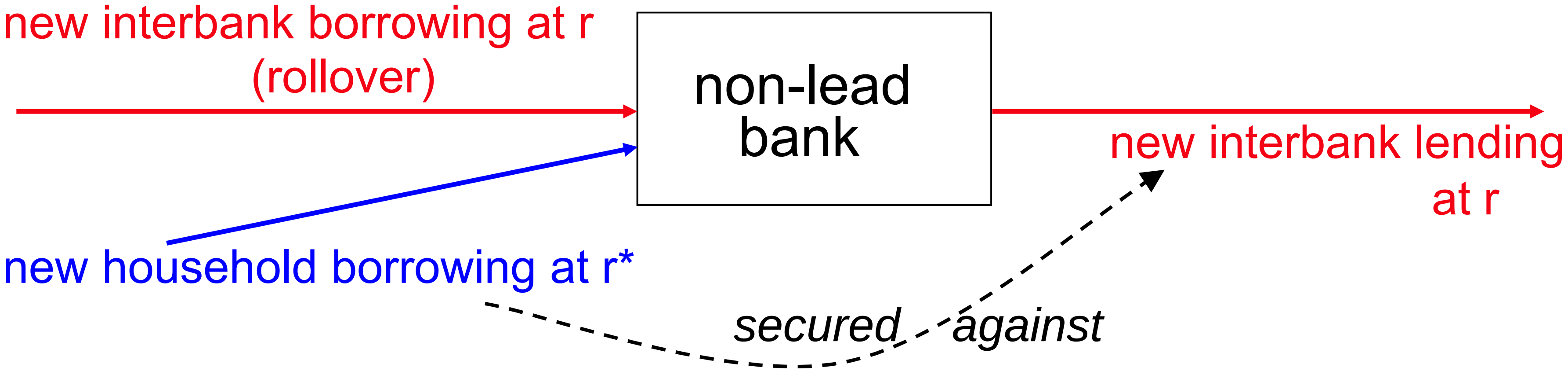
e.g. $\pi = 0.1$

$\lambda = 0.975$

$\theta = \theta^* = 0.9$

$r^* = 0.02$

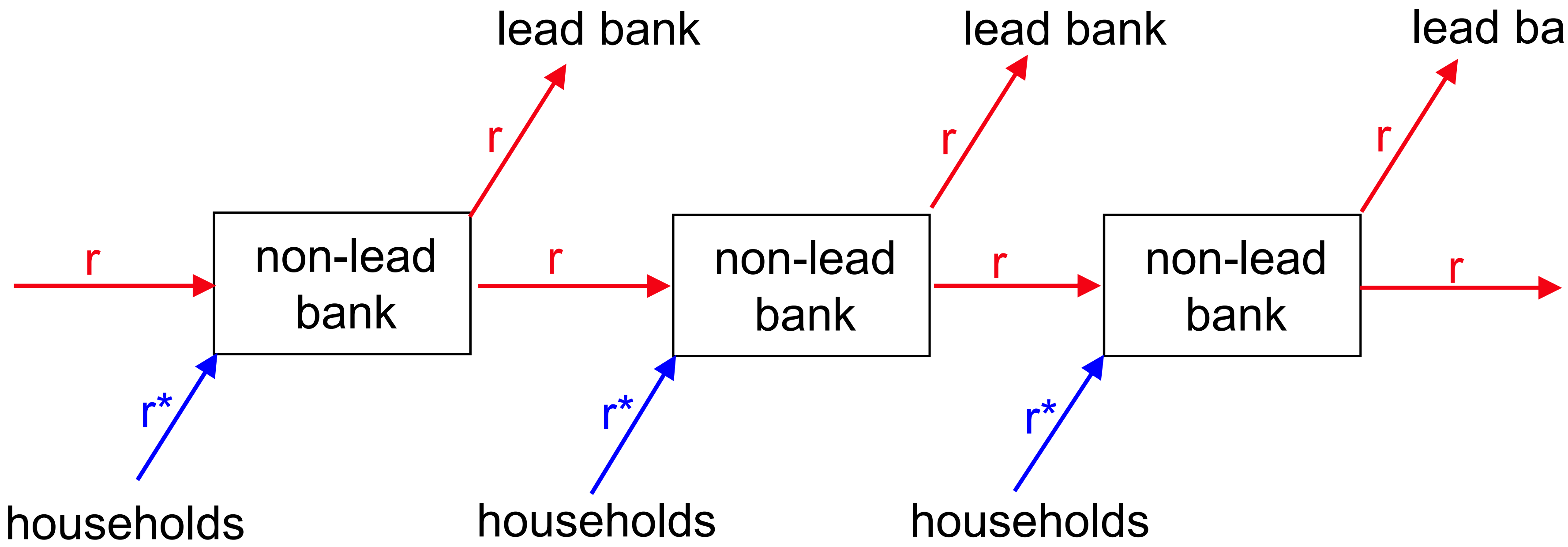
key point: non-lead banks are both borrowers and lenders in the interbank market



notice multiplier effect: if for some reason bank's value of new interbank borrowing $\downarrow$
(by x dollars, say)

$\Rightarrow$ bank's value of new interbank lending $\downarrow\downarrow$
(by >> x dollars, because of household leverage)

$\Rightarrow$ bank's *net* interbank lending $\downarrow$



if the “household-leverage multiplier”

exceeds the “leakage” to lead banks

then we get amplification along the chain