International Liquidity and Exchange Rate Dynamics

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Abstract

We provide a theory of the determination of exchange rates based on capital flows in imperfect financial markets. Capital flows drive exchange rates by altering the balance sheets of financiers that bear the risks resulting from international imbalances in the demand for financial assets. Such alterations to their balance sheets cause financiers to change their required compensation for holding currency risk, thus impacting both the level and volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets not only helps to rationalize the empirical disconnect between exchange rates and traditional macroeconomic fundamentals, but also has real consequences for output and risk sharing. Exchange rates are sensitive to imbalances in financial markets and seldom perform the shock absorption role that is central to traditional theoretical macroeconomic analysis. We derive conditions under which heterodox government financial policies, such as currency interventions and taxation of capital flows, can be welfare improving. Our framework is flexible; it accommodates a number of important modeling features within an imperfect financial market model, such as non-tradables, production, money, sticky prices or wages, various forms of international pricing-to-market, and unemployment.

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We provide a theory of exchange rate determination based on capital flows in imperfect financial markets. In our model, exchange rates are governed by financial forces because global shifts in the demand and supply of assets result in large scale capital flows that are intermediated by the global financial system. The demand and supply of assets in different currencies and the willingness of the financial system to absorb the resulting imbalances are first order determinants of exchange rates. A framework to characterize such forces and their implications for welfare and policy, while desirable, has proven elusive.

In our model, financiers absorb part of the currency risk originated by imbalanced global capital flows. Alterations to the size and composition of financiers' balance sheets induce them to differentially price currency risk, thus affecting both the level and the volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets differs from the traditional open macroeconomic model by introducing financial forces, such as portfolio flows, financiers' balance sheets, and financiers' risk bearing capacity as first order determinants of exchange rates.

We first present a basic theory of exchange rate financial determination in a two-period two-country model where capital flows are intermediated by global financiers. Each country borrows or lends in its own currency and financiers absorb all currency risk that is generated by the mismatch of global capital flows. Since financiers require compensation for holding currency risk in the form of expected currency appreciation, exchange rates are jointly determined by capital flows and by the financiers' risk bearing capacity. Our theory, therefore, is an elementary one whereby supply and demand determine a price, the exchange rate, that clears markets.

The exchange rate is disconnected from traditional macroeconomic fundamentals such as imports, exports, output, or inflation in as much as these same fundamentals correspond to different equilibrium exchange rates depending on financiers' balance sheets and risk bearing capacity. An extension to a multi-period model strengthens this intuition by solving the exchange rate as a present value relationship. The exchange rate discounts future current account balances, but the rate of discounting is determined in financial markets and therefore hinges on financiers' risk bearing capacity and balance sheets. Changes in such capacity affect both the level and volatility of the exchange rate. Financiers both act as shock absorbers, by using their risk bearing capacity to accommodate flows that result from fundamental shocks, and are themselves the source of financial shocks that distort exchange rates.

The financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing. To more fully analyze these consequences, we extend the basic model by introducing nominal exchange rates, monetary policy, and both flexible and sticky prices. In the presence of goods' prices that are sticky in the producers' currencies, a capital inflow or financial shock that produces an overly appreciated exchange rate causes a fall in demand

for the inflow-receiving country's exports and a corresponding fall in output.

Our theory yields novel predictions for policy analysis in the presence of flexible exchange rates. In a financial world, exchange rate movements are dominated by financial factors and seldom perform the benign expenditure switching role that is central to traditional macroeconomic analysis. In fact, the traditional macroeconomic rationale for prescribing pure floating exchange rates is that, in the presence of asymmetric real shocks, the exchange rate acts as a shock absorber by shifting global demand toward the country that has been most negatively affected by the shock via a depreciation of its currency.

By contrast, we show that in our framework the floating exchange rate is itself distorted by imbalances in financial markets and shocks to the financial system's risk bearing capacity; the exchange rate can often be the vehicle of transmission of financial shocks to the real economy. Our policy analysis suggests that novel trade-offs emerge when financial markets are disrupted or are less developed overall, and when output is far below its potential so that an exchange rate depreciation increases output via an increase in net exports. Heterodox policies, such as large scale currency interventions and capital controls, are shown to be beneficial in these specific circumstances.

We focus on providing a framework that is not only sufficiently rich to analyze the financial forces at the core of our theory in a full general equilibrium model, but also sufficiently tractable as to provide simple pencil-and-paper solutions that make the analysis as transparent as possible. While tractability requires some assumptions, we also verify that the core forces of our framework remain the leading forces of exchange rate determination even in more general setups, where a number of assumptions are relaxed and solutions (in some cases) have to be computed numerically.

Our model helps to make sense of a number of fundamental issues in open macroeconomics; these include the failure of the uncovered interest rate parity condition, external financial adjustment of creditors and debtor countries, the effectiveness of currency intervention, exchange rate disconnect from macroeconomic fundamentals, the failure of purchasing power parity, the failure of the Backus and Smith risk sharing condition, and the carry trade. While these issue have certainly been analyzed in other models, our work, while qualitative, provides a different, unified, and tractable treatment of these phenomena with a single main friction.

Our framework is flexible in accommodating a number of modeling features that are important in open economy analysis within an imperfect capital market model, such as non-tradables, production, money, sticky prices or wages, and various forms of international pricing-to-market. In each extension, we focused on simple and tractable modeling to map out in closed form both its basic channels and the interaction with the core forces of the paper. While the results are qualitative, we stress that the framework is versatile and can be easily employed in future research to

address a number of open questions in international macroeconomics.

We summarize our contribution as providing a tractable modern general-equilibrium framework for the determination of exchange rates in imperfect financial markets via capital flows and the risk-bearing capacity of financiers. Our general equilibrium framework combines financial forces such as risk taking and financial intermediation in imperfect capital markets with the traditional real economy analysis of production, import and export activities. A distinctive positive feature of our model is the direct relevance of flows, not just stocks, of assets for exchange rate determination. Our foundations allow us to study welfare and characterize optimal policy, such as foreign exchange rate intervention, in the presence of both financial and nominal frictions. We further show how the core force of the model, limited risk taking by the financiers, can help to rationalize a number of classic issues of international macroeconomics.

Related Literature Two important papers were published in 1976, the now classic exchange rate overshooting model (Dornbusch (1976)) and the portfolio balance model (Kouri (1976)). While we incorporate important aspects of the Keynesian tradition upon which Dornbusch builds, our model provides modern foundations to the spirit of Kouri's portfolio balance theory of exchange rates. Obstfeld and Rogoff (1995) brought the Keynesian approach into modern international economics by providing micro-foundations to the dynamic version of the Mundell-Fleming-Dornbusch model. Their foundations have been essential not only for the analysis of exchange rate determination, but also for that of optimal policy and welfare. However, financial forces play little role in this class of models. In the real version of these models, exchange rates are mostly determined by the demand and supply of domestic and foreign goods. Even in the nominal versions of the models, where the nominal exchange rate is often expressed as the present discounted sum of future monetary policy and other macroeconomic fundamentals, the impact of finance is limited because in most cases the uncovered interest parity holds, the demand for money is tightly linked to consumption expenditures, and/or the model is linearized.

This paper is related to three broad streams of literature: literature on portfolio balance in reduced form, micro-founded literature on portfolio demand in complete or incomplete markets, and micro-founded literature on frictions and asset demand.

As mentioned above, our paper is inspired by the early literature on portfolio balance modeled in reduced form. With respect to this literature our contribution consists of three main aspects: we provide a fully specified framework in general equilibrium with optimizing agents in the presence of frictions, we provide welfare and normative analysis, and we provide, via the foundations of the model, a theoretically distinct role for the balance sheet of financiers that is absent in the earlier literature. The closest paper in this early literature is Driskill and McCafferty (1980*a*)

who builds on earlier contributions by Kouri (1976).[1] Our main contribution of providing a full modern treatment of the forces sketched in these earlier papers is one that, in our view, was missing from the literature. For example, prominent economists have lamented that this earlier research effort "had its high watermark and to a large extent a terminus in Branson and Henderson (1985) handbook chapter" (see Obstfeld (2004)) and is "now largely and unjustly forgotten" (see Blanchard, Giavazzi and Sa (2005)). In our view, a major factor in the neglect of this sensible view of exchange rates is the lack of a modern micro-founded model that the field can build on in future work (on the contrary, the still popular Dornbusch model has received such fundamental uplifting in the celebrated article of Obstfeld and Rogoff (1995)).

The foundations provided in our model allow for the explicit analysis of policy and welfare. We provide a novel analysis of optimal foreign exchange intervention and show under which conditions it can be welfare improving. This could not be done without the foundations: for example, Backus and Kehoe (1989) provided a serious challenge to the portfolio balance literature claims on the effectiveness of intervention by showing that intervention is ineffective in a large set of modern micro-founded currency models with frictionless portfolios choice despite the currencies being imperfect substitutes due to risk premia.

A final distinction between our work and this earlier literature is that our foundations emphasize and provide a special role for the balance sheet of financiers and their risk bearing capacity while the previous work focused on the stock of external assets at the country level. Of course, the two need not be the same as showed in our work once the basic model is progressively extended. Understanding which flows and stocks are "stuck" on the financiers balance sheet provides both a new theoretical view and a novel avenue for future empirical work.

The literature that followed the earlier reduced-form modeling efforts has provided fully specified asset demand functions as well as general equilibrium effects. Most of the literature has focused on complete markets (Lucas, 1982, Backus, Kehoe and Kydland, 1992, Backus and Smith, 1993, Dumas, 1992, Verdelhan, 2010, Colacito and Croce, 2011, Hassan, 2013).[2] Pavlova and Rigobon (2007) analyze a real model with complete markets where countries' representative agents have logarithmic preferences affected by taste shocks similar to those considered in this paper.[3] A smaller literature has analyzed the importance of incomplete markets (for recent examples: Chari, Kehoe and McGrattan (2002), Corsetti, Dedola and Leduc (2008), Pavlova and Rigobon (2012)). With respect to both literatures we provide a different, and complementary,

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[1] An active early literature also includes: Allen and Kenen (1983), Henderson and Rogoff (1982), Dornbusch and Fischer (1980), Calvo and Rodriguez (1977), Branson, Halttunen and Masson (1979), Tobin and de Macedo (1979), Diebold and Pauly (1988), Driskill and McCafferty (1980b), de Macedo and Lempinen (2013). De Grauwe (1982) considers the role of the banking sector in generating portfolio demands.

[2] Among others see also: Farhi and Gabaix (2014), Martin (2011), and Stathopoulos (2012).

[3] Similar preferences are also used in Pavlova and Rigobon (2008, 2010).

approach by studying financial frictions. Our model has a set of distinct theoretical, and empirical, predictions, most prominently the importance of the balance sheet of financiers and of gross flows, not just stocks of assets in determining the exchange rate. Such gross flows matter in the presence of financial frictions, but would not matter in their absence both when markets are complete or incomplete.[4] This central prediction of our model has received confirmation in the data (Hau, Massa and Peress, 2010) as well as having been the focus of an important literature in other assets (De Long et al., 1990a,b), but had yet to be incorporated in a fully specified general equilibrium model of exchange rates.

The most closely related stream of the literature is the small set of papers that focused on exchange rate modeling in the presence of frictions. One set of papers are models of partial equilibrium. This set includes important contributions by Jeanne and Rose (2002), Evans and Lyons (2002), Hau and Rey (2006), Bruno and Shin (2014). Our contribution with respect to these papers is to provide a model of general equilibrium that merges the financial determination of exchange rates with the more traditional effects arising from the goods market via the trade balance. The general equilibrium set-up not only allows us to discuss positive predictions of the model about debtor/creditor countries and the exchange rate disconnect, but is also a requisite for the welfare analysis. One other important set of papers has a very different focus: informational frictions, infrequent portfolio rebalancing, or frictions in access to domestic money/funding market. Evans and Lyons (2012) focuses on how disaggregate order flows from customers might convey information about the economy fundamentals to exchange rate market makers who observe the consolidated flow. Bacchetta and Van Wincoop (2010) studies the implications of agents that infrequently rebalance their portfolio in an OLG setting. Alvarez, Atkeson and Kehoe (2002, 2009) and Maggiori (2014) are models of exchange rates where the frictions, a form of market segmentation, are only present in the domestic money market or funding market. Our model is both conceptually and empirically distinct from these papers because our frictions are only present on the international side and hence directly involve the exchange rate.

Recent economic events, such as the global financial and European crises, have rekindled an interest in the analysis of optimal policy and welfare in open economies. Aguiar, Amador and Gopinath (2009), Farhi, Gopinath and Itskhoki (2014), Farhi and Werning (2012a,b), Schmitt-Grohé and Uribe (2012), and Costinot, Lorenzoni and Werning (2014) provide innovative analyses of policies such as capital controls, fiscal transfers and fiscal devaluations in the context of the small-open-economy (new-Keynesian) model. We contribute to this literature by analyzing policies, and in particular foreign exchange rate intervention, in a two-country world where financial

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[4]This difference can easily be verified by considering the Basic Gamma model. The number of shocks ensures that the two bond economy when $\Gamma = 0$ is an incomplete market model with risk averse agents, yet f flows have no effect on the exchange rate as in Proposition 4.

flows are direct determinants of exchange rates and where the condition of financial markets is an important policy consideration. We find that public financial policies, which cannot be analyzed under the UIP-assumption, complete markets, or simple forms of market incompleteness, can be beneficial in specific circumstances. We characterize the policy instruments (FX interventions, currency-swaps, and capital controls) that can be used to implement these polices.

1 Basic Gamma Model

Let us start with a minimalistic model of financial determination of exchange rates in imperfect financial markets. This simple real model carries most of the economic intuition and core modeling that we will then extend to more general set-ups.

Time is discrete and there are two periods: $t = 0, 1$. There are two countries, the USA and Japan, each populated by a continuum of households. Households produce, trade (internationally) in a market for goods, and invest with financiers in risk-free bonds in their domestic currency.[5] Financiers intermediate the capital flows resulting from households' investment decisions. The basic structure of the model is displayed in Figure 1.

Figure 1: Basic Structure of the Model
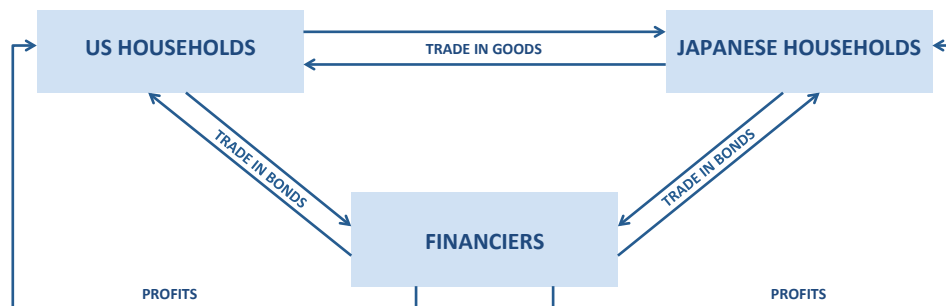
The players and structure of the flows in the goods and financial markets in the Basic Gamma Model.

Intermediation is not perfect because of the limited commitment of the financiers. The limited-commitment friction induces a downward sloping demand curve for risk taking by financiers. As a result, capital flows from households move financiers up and down their demand curve. Equilibrium is achieved by a relative price, in this case the exchange rate, adjusting so that international financial markets clear given the demand and supply of capital denominated in different currencies. In this sense, exchange rates are financially determined in an imperfect capital market.

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[5]In the absence of a nominal side to the model, in this section we intentionally abuse the word "currency" to mean a claim to the numéraire of the economy, and "exchange rate" to mean the real exchange rate. Similarly we abuse the words "Dollar or Yen denominated" to mean values expressed in units of non-tradable goods in each economy. As will shortly become clear, even this simple real model is set up so as to generalize immediately to a nominal model.

We now describe each of the model's actors, their optimization problems, and analyze the resulting equilibrium.

1.1 Households

Households in the US derive utility from the consumption of goods according to:

$$\theta_0 \ln C_0 + \beta \mathbb{E}\left[\theta_1 \ln C_1\right], \tag{1}$$

where C is a consumption basket defined as:

$$C_t \equiv \left[(C_{NT,t})^{\chi_t}(C_{H,t})^{a_t}(C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}, \tag{2}$$

where $C_{NT,t}$ is the US consumption of its non-tradable goods, $C_{H,t}$ is the US consumption of its domestic tradable goods, and $C_{F,t}$ is the US consumption of Japanese tradable goods. We use the notation $\{\chi_t, a_t, \iota_t\}$ for non-negative, potentially stochastic, preference parameters and we define $\theta_t \equiv \chi_t + a_t + \iota_t$.

The non-tradable good is the numéraire in each economy and, consequently, its price equals 1 in domestic currency ($p_{NT} = 1$). Non-tradable goods are produced by an endowment process that we assume for simplicity to follow $Y_{NT,t} = \chi_t$, unless otherwise stated.[6]

Households can trade both tradable goods in a frictionless goods market across countries, but can only trade non-tradable goods within their domestic country. Financial markets are incomplete and each country trades a risk-free domestic currency bond. The assumption that each country only trades in its own currency bonds is made here for simplicity and to emphasize the currency mismatch that the financiers have to absorb; we relax the assumption in later sections. *Risk-free* here refers to paying one unit of non-tradable goods in all states of the world and is therefore akin to "nominally risk free".

US households' optimization problem is:

$$\max_{\left(C_{NT,t},C_{H,t},C_{F,t}\right)_{t=0,1}} \theta_0 \ln C_0 + \beta \mathbb{E}\left[\theta_1 \ln C_1\right], \tag{3}$$

subject to (2),

$$\text{and} \quad \sum_{t=0}^{1} R^{-t}\left(Y_{NT,t} + p_{H,t}Y_{H,t}\right) = \sum_{t=0}^{1} R^{-t}\left(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t}\right). \tag{4}$$

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[6]The assumption, while stark, makes the analysis of the basic model most tractable. We stress that the assumption is one of convenience, and not necessary for the economics of the paper. The reader might find it useful to think of χ and Y_{NT} as constants and the equality between the two as a normalization that makes the closed form solutions of the paper most readable. Section 2 and the appendix provide more general results that do not impose this assumption.

US households maximize the utility by choosing their consumption and savings in dollar bonds subject to the state-by-state dynamic budget constraint. The households' optimization problem can be divided into two separate problems. The first is a static problem, whereby households decide, given their total consumption expenditure for the period, how to allocate resources to the consumption of various goods. The second is a dynamic problem, whereby households decide intertemporally how much to save and consume.

The static utility maximization problem takes the form:

$$\max_{C_{NT,t},C_{H,t},C_{F,t}} \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t} + \lambda_t \left(CE_t - C_{NT,t} - p_{H,t}C_{H,t} - p_{F,t}C_{F,t} \right), \quad (5)$$

where CE_t is aggregate consumption expenditure, which is taken as exogenous in this static optimization problem and later endogenized in the dynamic optimization problem, λ_t is the associated Lagrange multiplier, $p_{H,t}$ is the Dollar price in the US of US tradables, and $p_{F,t}$ is the Dollar price in the US of Japanese tradables. First-order conditions imply: $\frac{\chi_t}{C_{NT,t}} = \lambda_t$, and $\frac{\iota_t}{C_{F,t}} = \lambda_t p_{F,t}$. Our assumption that $Y_{NT,t} = \chi_t$, combined with the market clearing condition for non-tradables $Y_{NT,t} = C_{NT,t}$, implies that in equilibrium $\lambda_t = 1$. This yields:

$$p_{F,t}C_{F,t} = \iota_t,$$

i.e., the Dollar value of US imports is simply ι_t.

Japanese households derive utility from consumption according to: $\theta_0^* \ln C_0^* + \beta^* \mathbb{E}[\theta_1^* \ln C_1^*]$, where starred variables denote Japanese quantities and prices. By analogy with the US case, the Japanese consumption basket is: $C_t^* \equiv \left[(C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*} \right]^{\frac{1}{\theta_t^*}}$, where $\theta_t^* \equiv \chi_t^* + a_t^* + \xi_t$. The Japanese static utility maximization problem, reported for brevity in the appendix, together with the assumption $Y_{NT,t}^* = \chi_t^*$, leads to a Yen value of US exports to Japan, $p_{H,t}^* C_{H,t}^* = \xi_t$, that is entirely analogous to the import expression derived above.

The exchange rate e_t is defined as the quantity of dollars bought by 1 yen, i.e. the strength of the Yen. Consequently, an increase in e represents a Dollar depreciation.[7] The Dollar value of US exports is: $e_t \xi_t$. US net exports, expressed in dollars, are given by: $NX_t = e_t p_{H,t}^* C_{H,t}^* - p_{F,t}C_{F,t} = \xi_t e_t - \iota_t$.[8] We collect these results in the Lemma below.

Lemma 1 (Net Exports) *Expressed in dollars, US exports to Japan are $\xi_t e_t$; US imports from Japan are ι_t; so that US net exports are $NX_t = \xi_t e_t - \iota_t$.*

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[7]In this real model, the exchange rate is related to the relative price of non-tradable goods. Section 5.2 provides a full discussion of this exchange rate and its relationship to both the nominal exchange rate, formally introduced in Section 2.1 , and the CPI-based real exchange rate.

[8]Note that we chose the notation so that *i*mports are denoted by ι_t and *e*xports by ξ_t.

Note that this result is independent of the pricing procedure (e.g. price stickiness under either producer or local currency pricing). Under producer currency pricing (PCP) and in the absence of trade costs, the US Dollar price of Japanese tradables is p_H/e, while under local currency pricing (LCP) the price is simply p_H^*.

It follows that under financial autarky, i.e. if trade has to be balanced period by period, the equilibrium exchange rate is: $e_t = \frac{\iota_t}{\xi_t}$. In financial autarky, the Dollar depreciates ($\uparrow e$) whenever US demand for Japanese goods increases ($\uparrow \iota$) or whenever Japanese demand for US goods falls ($\downarrow \xi$). This has to occur because there is no mechanism, in this case, to absorb the excess demand/supply of dollars versus yen that a non-zero trade balance would generate.

The optimization problem (3) for the intertemporal consumption-saving decision leads to a standard optimality condition (Euler equation):

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\right] = \mathbb{E}\left[\beta R \frac{\chi_1/C_{NT,1}}{\chi_0/C_{NT,0}}\right] = \beta R, \tag{6}$$

where $U'_{t,C_{NT}}$ is the marginal utility at time t over the consumption of non-tradables. Given our simplifying assumption that $C_{NT,t} = \chi_t$, the above Euler equation implies that $R = 1/\beta$. An entirely similar derivation yields: $R^* = 1/\beta^*$.[9]

We stress that the aim of our simplifying assumptions is to create a real structure of the basic economy that captures the main forces (demand and supply of goods), while making the real side of the economy as simple as possible. This will allow us to analytically flesh out the crucial forces of the paper in the financial markets in the next sections without carrying around a burdensome real structure. Should the reader be curious as to the robustness of our model to relaxing some of the assumptions made so far, the quick answer is that it is quite robust. We will make such robustness explicit in Section 2, and in the appendix.

1.2   Financiers

Suppose that global financial markets are imbalanced, such that there is an excess supply of dollars versus yen resulting from, for example, trade or portfolio flows. Who will be willing to absorb such an imbalance by providing Japan those yen, and holding those dollars? We posit that the resulting imbalances are absorbed, at some premium, by global financiers.

We assume that there is a unit mass of global financial firms, each managed by a financier.

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[9]It might appear surprising that in a model with risk averse agents the equilibrium interest rate equals the rate of time preference. Of course, this occurs here because the marginal utility of non-tradable consumption, in which the bonds are denominated, is constant in equilibrium given the assumption $Y_{NT,t} = \chi_t$. This assumption is relaxed in later sections and the model still offers closed-form solutions.

Agents from the two countries are selected at random to run the financial firms for a single period.[10] Financiers start their jobs with no capital of their own and can trade bonds denominated in both currencies. Therefore, their balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, where q_0 is the Dollar value of Dollar-denominated bonds the financier is long of and $-\frac{q_0}{e_0}$ the corresponding value in Yen of Yen-denominated bonds. At the end of (each) period, financiers pay their profits and losses out to the households.

Our financiers are intended to capture a broad array of financial institutions that intermediate global financial markets. These institutions range from the proprietary desks of global investment banks such as Goldman Sachs and JP Morgan, to macro and currency hedge funds such as Soros Fund Management, to active investment managers and pension funds such as PIMCO and Black-Rock. While there are certainly significant differences across these intermediaries, we stress their common characteristic of being active investors that profit from medium-term imbalances in international financial markets, often by bearing the risks (*taking the other side*) resulting from imbalances in currency demand due both to trade and financial flows. They also share the characteristic of being subject to financial constraints that limit their ability to take positions, based on their risk bearing capacities and existing balance sheet risks.[11]

We assume that each financier maximizes the expected value of her firm:[12]

$$V_0 = \mathbb{E}\left[\beta\left(R - R^* \frac{e_1}{e_0}\right)\right] q_0 = \Omega_0 q_0. \tag{7}$$

In each period, after taking positions but before shocks are realized, the financier can divert a portion of the funds she intermediates. If the financier diverts the funds, her firm is unwound and the households that had lent to her recover a portion $1 - \Gamma\left|\frac{q_0}{e_0}\right|$ of their credit position $\left|\frac{q_0}{e_0}\right|$, where $\Gamma = \gamma\, var\,(e_1)^\alpha$, with $\gamma \geq 0, \alpha \geq 0$.[13] As will become clear below, our functional assumption regarding the diversion of funds is not only a convenient specification for tractability, but also

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[10]In this set-up, being a financier is an occupation for agents in the two countries rather than an entirely separate class of agents. The selection process is governed by a memoryless Poisson distribution. Of course, there are no selection issues in the one period basic economy considered here, but we proceed to describe a more general set-up that will also be used in the model extensions.

[11]An interesting literature also stresses the importance of global financial frictions for the international transmission of shocks, but does not study exchange rates: Kollmann, Enders and Müller (2011), Kollmann (2013), Dedola, Karadi and Lombardo (2013), Perri and Quadrini (2014).

[12]We derive this value function explicitly in the appendix. Here we only stress that this function does not require financiers to be risk neutral; in fact, it actually corresponds to the way in which US households would value currency trading, i.e their shadow Euler equation.

[13]Given that the balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, the Yen value of the financier's liabilities is always equal to $\left|\frac{q_0}{e_0}\right|$, irrespective of whether q_0 is positive or negative; hence the use of absolute value in the text above. More formally, the financier's creditors can recover a Yen value equal to: $\max\left(1 - \Gamma\left|\frac{q_0}{e_0}\right|, 0\right)\left|\frac{q_0}{e_0}\right|$. See the appendix for further details.

stresses the idea that financiers' outside options increase in the size and volatility, or complexity, of their balance sheet.[14] This constraint captures the relevant market practice in financial institutions whereby risk taking is limited not only by the overall size of the positions, position limits, but also by their expected riskiness, often measured by their variance. Since creditors, when lending to the financier, correctly anticipate the incentives of the financier to divert funds, the financier is subject to a credit constraint of the form:

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\underbrace{\frac{V_0}{e_0}}_{\substack{\text{Intermediary Value} \\ \text{in yen}}} \geq \underbrace{\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Total} \\ \text{Claims}}} \underbrace{\Gamma\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Diverted} \\ \text{Portion}}} = \underbrace{\Gamma\left(\frac{q_0}{e_0}\right)^2}_{\substack{\text{Total divertable} \\ \text{Funds}}}. \tag{8}
$$

Limited commitment constraints in a similar spirit have been popular in the literature; for earlier use as well as foundations see among others: Caballero and Krishnamurthy (2001), Kiyotaki and Moore (1997), Hart and Moore (1994), and Hart (1995). Here we follow most closely the formulation in Gertler and Kiyotaki (2010) and Maggiori (2014).[15]

For simplicity, we assume (for now and for much of this paper) that financiers rebate their profits and losses to the Japanese households, not the US ones. This asymmetry gives much tractability to the model, at fairly little cost to the economics.[16]

The constrained optimization problem of the financier is:

$$
\max_{q_0} V_0 = \mathbb{E}\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right]q_0, \qquad \text{subject to} \quad V_0 \geq \Gamma\frac{q_0^2}{e_0}. \tag{9}
$$

Since the value of the financier's firm is linear in the position q_0, while the right hand side of the constraint is convex in q_0, the constraint always binds.[17] Substituting the firm's value into

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[14]It is outside the scope of this paper to provide deeper foundations for this constraint. The reader can think of it as a convenient specification of a more complicated contracting problem. However, such foundations could potentially be achieved in models of financial complexity where bigger and riskier balance sheets lead to more complex positions. In turn, these more complex positions are more difficult and costly for creditors to unwind when recovering their funds in case of a financier's default.

[15]We generalize these constraints by studying cases where the outside option is directly increasing in the size of the balance sheet and its variance. Adrian and Shin (2013) provide foundations and empirical evidence for a value-at-risk constraint that shares some of the properties of our constraint above.

[16]For completeness, note that this assumption had already been implicitly made in deriving the US households' inter-temporal budget constraint in equation (4). This assumption is relaxed in the appendix, where we solve for general and symmetric payoff functions numerically.

[17]Intuitively, given any non-zero expected excess return in the currency market, the financier will want to either borrow or lend as much as possible in Dollar and Yen bonds. The constraint limits the maximum position and therefore binds. We make the very mild assumption that the model parameters always imply: $\Omega_0 \geq -1$. That is, we assume that the expected excess returns from currency speculation never exceed 100% in absolute value. This bound is several order of magnitudes greater than the expected returns in the data (of the order of 0-6%) and has no economic bearing on our model. See appendix for further details.

the constraint and re-arranging (using $R = 1/\beta$), we find: $q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. Integrating the above demand function over the unit mass of financiers yields the aggregate financiers' demand for assets: $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. We collect this result in the Lemma below.

Lemma 2 (Financiers' downward sloping demand for dollars) *The financiers' constrained optimization problem implies that the aggregate financial sector's optimal demand for Dollar bonds versus Yen bonds follows:*

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right].$$ (10)

where

$$\Gamma = \gamma(var(e_1))^\alpha.$$ (11)

The demand for dollars decreases in the strength of the dollar (i.e. increases in e_0), controlling for the future value of the Dollar (i.e. controlling for e_1). Notice that Γ governs the ability of financiers to bear risks; hence in the rest of the paper we refer to Γ as the financiers' risk bearing capacity. The higher Γ, the lower the financiers' risk bearing capacity, the steeper their demand curve, and the more segmented the asset market. To understand the behavior of this demand, let us consider two polar opposite cases. When $\Gamma = 0$, financiers are able to absorb any imbalances, i.e. they want to take infinite positions whenever there is a non-zero expected excess return in currency markets. So uncovered interest rate parity (UIP) holds: $\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right] = 0$. When $\Gamma \uparrow \infty$, then $Q_0 = 0$; financiers are unwilling to absorb any imbalances, i.e. they do not want to take any positions, no matter what the expected returns from risk-taking. In the intermediate cases $(0 < \Gamma < \infty)$ the model endogenously generates a deviation from UIP and relates it to financiers' risk taking. On the contrary, since the covered interest rate parity (CIP) condition is an arbitrage involving no risk it is always satisfied. Similarly the model smoothly converges to the frictionless benchmark $(\Gamma \downarrow 0)$ as the economy becomes deterministic $(var(e_1) \downarrow 0)$. Section 5.1 studies the carry trade and provides further analysis on UIP and CIP.

Since Γ, the financiers' risk bearing capacity, plays a crucial role in our theory, we refer hereafter to the setup described so far as the *basic Gamma model*. In many instances, like the one above, it is most intuitive to consider comparative statics on Γ rather than its subcomponents, and we do so for the remainder of the paper; in some instances it is interesting to consider the effect of each subcomponent γ and $var(e_1)$ separately.[18]

We stress that the above demand function captures the spirit of international financial intermediation by providing a simple and tractable specification for the constrained portfolio problem that generates the demand function that has been central to the limits of arbitrage theory pio-

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[18]The reader is encouraged either to intuitively consider the case $\alpha = 0$, or to follow the formal proofs that show the sign of the comparative statics to be invariant in Γ and γ.

neered by De Long et al. (1990*a*,*b*), Shleifer and Vishny (1997), and Gromb and Vayanos (2002). It yields a full characterization of the exchange rate and allows to characterize optimal policy and welfare. As we show in Section 4, welfare only has to consider the utility of households, who consume goods, with no direct weight attached to the well being of financiers.

While we emphasize the importance of a fully specified, general equilibrium analysis of exchange rates, we find that it is beyond the scope of this paper to provide the contract-theory foundations that determine which assets and contracts the financiers trade in equilibrium, and a detailed analysis of the origins of frictions. We follow the pragmatic tradition of macroeconomics and frictional finance, and we take as given the prevalence of frictions and short-term debt in different currencies, and proceed to analyze their equilibrium implications. This direct approach to modeling financial imperfections has a long standing tradition and has proved very fruitful with recent contributions by Kiyotaki and Moore (1997), Gromb and Vayanos (2002), Mendoza, Quadrini and Rıos-Rull (2009), Mendoza (2010), Gertler and Kiyotaki (2010), Garleanu and Pedersen (2011), Perri and Quadrini (2014).[19] Similarly the recent literature on externalities and macro prudential policies has employed a similar modeling approach often taking the constraints and frictions as given (Bianchi (2010), Farhi and Werning (2014)).

Before moving to the equilibrium, note that we are modeling the ability of financiers to bear substantial risks over a horizon that ranges from a quarter to a few years. Our model is silent on the high frequency market-making activities of currency desks in investment banks. To make this distinction intuitive, let us consider that the typical daily volume of foreign exchange transactions is estimated to be \$5.3 trillion.[20] This trading is highly concentrated among the market making desks of banks and is the subject of attention in the market microstructure literature pioneered by Evans and Lyons (2002). While these microstructure effects are interesting, we completely abstract away from these activities by assuming that there is instantaneous and perfect risk sharing across financiers, so that any trade that matches is executed frictionlessly and nets out. We are only concerned with the ultimate risk, most certainly a small fraction of the total trading volume, which financiers have to bear over quarters and years because households' demand is unbalanced.[21]

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[19]Even in the most recent macro-finance literature in closed economy, intense foundations of the contracting environment have either been excluded or relegated to separate companion pieces (Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013)).

[20]Source: Bank of International Settlements (2013).

[21]This is consistent with evidence that market-making desks in large investment banks, for example Goldman Sachs, might intermediate very large volumes on a daily basis but are almost always carrying no residual risk at the end of the business day. In contrast, proprietary trading desks (before recent changes in legislation) or investment management divisions of the same investment banks carry substantial amounts of risk over horizons ranging from a quarter to a few years. These investment activities are the focus of this paper. Similarly, our financiers capture the risk-taking activities of hedge funds and investment managers that have no market making interests and are therefore not the center of attention in the microstructure literature.

1.3 Equilibrium Exchange Rate

Recall that for simplicity we are for now only considering imbalances resulting from trade flows (imbalances from portfolio flows will soon follow). The key equations of the model are the financiers' demand:

$$Q_0 = \frac{1}{\Gamma} \mathbb{E} \left[e_0 - e_1 \frac{R^*}{R} \right], \tag{12}$$

and the equilibrium "flow" demand for dollars in the Dollar-Yen market at times $t = 0, 1$:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{13}$$

$$\xi_1 e_1 - \iota_1 - R Q_0 = 0. \tag{14}$$

Equation (13) is the market clearing equation for the Dollar against Yen market at time zero. It states that the net demand for Dollar against Yen has to be zero for the market to clear. The net demand has two components: $\xi_0 e_0 - \iota_0$, from US net exports, and Q_0, from financiers. Recall that we assume that US households do not hold any currency exposure: they convert their Japanese sales of ξ_0 yen into dollars, for a demand $\xi_0 e_0$ of dollars. Likewise, Japanese households have ι_0 dollars worth of exports to the US and sell them, as they only keep Yen balances.[22] At time one, equation (14) shows that the same net-export channel generates a demand for dollars of $\xi_1 e_1 - \iota_1$; while the financiers need to sell their dollar position $R Q_0$ that has accrued interest at rate R.[23] We now explore the equilibrium exchange rate in this simple setup.

Equilibrium exchange rate: a first pass To streamline the algebra and concentrate on the key economic content, we assume for now that $\beta = \beta^* = 1$, which implies $R = R^* = 1$, and that $\xi_t = 1$ for $t = 0, 1$. Adding equations (13) and (14) yields the US external intertemporal budget constraint:

$$e_1 + e_0 = \iota_0 + \iota_1. \tag{15}$$

Taking expectations on both sides: $\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0$. From the financiers' demand equation we have:

$$\mathbb{E}[e_1] = e_0 - \Gamma Q_0 = e_0 - \Gamma(\iota_0 - e_0) = (1 + \Gamma) e_0 - \Gamma \iota_0,$$

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[22]These assumptions are later relaxed in Sections 2.2 and in the appendix where households are allowed to have (limited) foreign currency positions.

[23]At the end of period 0, the financiers own Q_0 dollars and $-Q_0/e_0$ yen. Therefore, at the beginning of period one, they hold $R Q_0$ dollars and $-R^* Q_0/e_0$ yen. At time one, they unwind their positions and give the net profits to their principals, which we assume for simplicity to be the Japanese households. Hence they sell $R Q_0$ dollars in the Dollar-Yen market at time one.

where the second equality follows from equation (13). Equating the two expressions for the time-one expected exchange rate, we have:

$$\mathbb{E}\left[e_1\right] = \iota_0 + \mathbb{E}[\iota_1] - e_0 = (1+\Gamma)e_0 - \Gamma\iota_0.$$

Solving this linear equation for the exchange rate at time zero, we conclude:

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}.$$

We define $\{X\} \equiv X - \mathbb{E}[X]$ to be the innovation to a random variable X. Then, the exchange rate at time $t = 1$ is:

$$e_1 = \iota_0 + \iota_1 - e_0 = \iota_0 + \mathbb{E}[\iota_1] + \{\iota_1\} - e_0$$
$$= \{\iota_1\} + \iota_0 + \mathbb{E}[\iota_1] - \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1]}{2+\Gamma}.$$

This implies that $var(e_1) = var(\iota_1)$, so that, by (11), $\Gamma = \gamma\, var(\iota_1)^{\alpha}$.

We collect these results in the Proposition below.

Proposition 1 (Basic Gamma equilibrium exchange rate) *Assume that $\xi_t = 1$ for $t = 0, 1$, and that interest rates are zero in both countries. The exchange rate follows:*

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}, \tag{16}$$
$$e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1]}{2+\Gamma},$$

where $\{\iota_1\}$ is the time-one import shock. The expected Dollar appreciation is: $\mathbb{E}\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma(\iota_0 - \mathbb{E}[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}$. *Furthermore, $\Gamma = \gamma\, var(\iota_1)^{\alpha}$.*

Depending on Γ, the time-zero exchange rate varies between two polar opposites: the UIP-based and the financial-autarky exchange rates, respectively. Both extremes are important benchmarks of open economy analysis, and the choice of Γ allows us to modulate our model between these two useful benchmarks. $\Gamma \uparrow \infty$ results in $e_0 = \frac{\iota_0}{\xi_0}$, which we have shown in Section 1.1 to be the financial autarky value of the exchange rate. Intuitively, financiers have so little risk-bearing capacity that no financial flows can occur between countries and, therefore, trade has to be balanced period by period. When $\Gamma = 0$, UIP holds and we obtain $e_0 = \frac{\iota_0 + \mathbb{E}[\iota_1]}{2}$. Intuitively, financiers are so relaxed about risk taking that they are willing to take infinite positions in currencies whenever there is a positive expected excess return from doing so. UIP only imposes a constant exchange

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rate in expectation $\mathbb{E}[e_1] = e_0$; the level of the exchange rate is then obtained by additionally using the inter-temporal budget constraint in equation (15).

To further understand the effect of Γ, notice that at the end of period 0 (say, time 0^+), the US net foreign asset (NFA) position is $N_{0+} = \xi_0 e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1] - \iota_0}{2 + \Gamma}$. Therefore, the US has positive NFA at $t = 0^+$ *iff* $\iota_0 < \mathbb{E}[\iota_1]$. If the US has a positive NFA position, then financiers are long the Yen and short the Dollar. For financiers to bear this risk, they require a compensation: the Yen needs to appreciate in expectation. The required appreciation is generated by making the Yen weaker at time zero. The magnitude of the effect depends on the extent of the financiers' risk bearing capacity (Γ), as formally shown here by taking partial derivatives: $\frac{\partial e_0}{\partial \Gamma} = \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} = \frac{-N_{0+}}{2+\Gamma}$. We collect the result in the Proposition below.

Proposition 2 (Effect of financial disruptions on the exchange rate) *In the basic Gamma model, we have:* $\frac{\gamma}{\Gamma} \frac{de_0}{d\gamma} = \frac{\partial e_0}{\partial \Gamma} = \frac{-N_{0+}}{2+\Gamma}$, *where* $N_{0+} = \frac{\mathbb{E}[\iota_1] - \iota_0}{2+\Gamma}$ *is the US net foreign asset (NFA) position. When there is a financial disruption ($\uparrow \gamma, \uparrow \Gamma$), countries that are net external debtors ($N_{0+} < 0$) experience a currency depreciation ($\uparrow e$), while the opposite is true for net-creditor countries.*

Intuitively, net external-debtor countries have borrowed from the world financial system, thus generating a long exposure for financiers to their currencies. Should the financial system's risk bearing capacity be disrupted, these currencies would depreciate to compensate financiers for the increased (perceived) risk. This modeling formalizes a number of external crises where broadly defined global risk aversion shocks, embodied here in Γ, caused large depreciations of the currencies of countries that had recently experienced large capital inflows. Della Corte, Riddiough and Sarno (2014) confirm our theoretical prediction in the data. They show that net-debtor countries' currencies have higher returns than net-creditors' currencies, tend to be on the receiving end of carry trade related speculative flows, and depreciate when financial disruptions occur.

To illustrate how the results derived so far readily extend to more general cases, we report below expressions allowing for stochastic US export shocks ξ_t, as well as non-zero interest rates. Several more extensions can be found in Section 2.

Proposition 3 *With general trade shocks and interest rates (ι_t, ξ_t, R, R^*), the values of exchange rate at times $t = 0, 1$ are:*

$$e_0 = \frac{\mathbb{E}\left[\frac{\iota_0 + \frac{\iota_1}{R}}{\xi_1}\right] + \frac{\Gamma \iota_0}{R^*}}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma \xi_0}{R^*}}; \qquad e_1 = \mathbb{E}[e_1] + \{e_1\}, \qquad (17)$$

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where we again denote by $\{X\} \equiv X - \mathbb{E}[X]$ the innovation to a random variable X, and

$$\mathbb{E}[e_1] = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\xi_0 \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}, \tag{18}$$

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R \frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}\left\{\frac{1}{\xi_1}\right\}. \tag{19}$$

When ξ_1 is deterministic, $\Gamma = \gamma var(\frac{\iota_1}{\xi_1})^\alpha$. The proof of this Proposition reports the corresponding solution for Γ when ξ_1 is stochastic.

1.4 The Impact of Portfolio Flows

We now further illustrate how the *supply and demand of assets do matter for the financial determination of the exchange rate*. We stress the importance of portfolio flows in addition, and perhaps more importantly than, trade flows for our framework. The basic model so far has focused on current account, or net foreign asset, based flows; we introduce here pure portfolio flows that alter the countries' gross external positions. We focus here on the simplest form of portfolio flows from households, not so much for their complete realism, but because they allow for the sharpest analysis of the main forces of the model. The rest of the paper, as well as the appendix, extends this minimalistic section to more general flows.

1.4.1 Asset Flows Matter in the Gamma Model

Consider the case where Japanese households have, at time zero, an inelastic demand (e.g. some noise trading) f^* of Dollar bonds funded by an offsetting position $-f^*/e_0$ in Yen bonds. Both transactions face the financiers as counter parties.

While we take these flows as exogenous, they can be motivated as a liquidity shock, or perhaps as a decision resulting from bounded rationality or portfolio delegation. Technically, the maximization problem for the Japanese household is the one written before, where the portfolio flow is not a decision variable coming from a maximization, but is simply an exogenous action.[24]

The flow equations are now given by:

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0, \qquad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0. \tag{20}$$

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[24] The Japanese households' state-by-state budget constraint is $\sum_{t=0}^{1} \frac{Y_{NT,t}^* + p_{F,t}^* Y_{F,t} + \pi_t^*}{R^{*t}} = \sum_{t=0}^{1} \frac{C_{NT,t}^* + p_{H,t}^* C_{H,t} + p_{F,t}^* C_{F,t}^*}{R^{*t}}$, where π_t^* are FX trading profit to the Japanese, so $\pi_0^* = 0$, $\pi_1^* = (f^* + Q_0)(R - R^* \frac{e_1}{e_0})/e_1$ (recall that the financiers rebate their profits to the Japanese).

The financiers' demand is still $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right]$. The equilibrium exchange rate is derived in the Proposition below.

Proposition 4 (Gross capital flows and exchange rates) *Assume $\xi_t = R = R^* = 1$ for $t = 0, 1$. With an inelastic time-zero additional demand f^* for Dollar bonds by Japanese households who correspondingly sell $-f^*/e_0$ of Yen bonds, the exchange rates at times $t = 0, 1$ are:*

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma}; \qquad e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}.$$

Hence, additional demand f^* for dollars at time zero induces a Dollar appreciation at time zero, and subsequent depreciation at time one. However, the time-average value of the Dollar is unchanged: $e_0 + e_1 = \iota_0 + \iota_1$, independently of f^*. Furthermore, $\Gamma = \gamma\, var(\iota_1)^\alpha$.

   Proof. Define: $\tilde{\iota}_0 \equiv \iota_0 - f^*$, and $\tilde{\iota}_1 \equiv \iota_1 + f^*$. Given equations (20), our "tilde" economy is isomorphic to the basic economy considered in equations (13) and (14). For instance, import demands are now $\tilde{\iota}_t$ rather than ι_t. Hence, Proposition 1 applies to this "tilde" economy, thus implying that:

$$e_0 = \frac{(1+\Gamma)\,\tilde{\iota}_0 + \mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma},$$

$$e_1 = \{\tilde{\iota}_1\} + \frac{\tilde{\iota}_0 + (1+\Gamma)\mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}. \quad \square$$

An increase in Japanese demand for Dollar bonds needs to be absorbed by financiers, who correspondingly need to sell Dollar bonds and buy Yen bonds. To induce financiers to provide the desired bonds, the Dollar needs to appreciate on impact *as a result of the capital flow*, in order to then be expected to depreciate, thus generating an expected gain for the financiers' short Dollar positions. This example emphasizes that our model is an elementary one where a relative price, the exchange rate, has to move in order to equate the supply and demand of two assets, Yen and Dollar bonds. The capital flows considered in this section are gross flows that do not alter the net foreign asset position, thus introducing a first example of the distinct role for the financiers' balance sheet from the country net foreign asset position. In the data gross flows are much larger than net flows and we provide a reason why they play an important role in determining the exchange rate.[25]

This framework can analyze concrete situations, such as the recent large scale capital flows from developed countries into emerging market local-currency bond markets, say by US investors

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[25]One could extend the distinction between country level positions and financiers' balance sheet further by modeling situations where not all gross flows are stuck, either temporarily or permanently, on the balance sheet of the financiers.

into Brazilian Real bonds, that put upward pressure on the receiving countries' currencies. While such flows and their impact on currencies have been paramount in the logic of market participants and policy makers, they had thus far proven elusive in a formal theoretical analysis.

Hau, Massa and Peress (2010) provide direct evidence that plausibly exogenous capital flows impact the exchange rate in a manner consistent with the Gamma model. They show that, following a restating of the weights of the MSCI World Equity Index, countries that as a result experienced capital inflows (because their weight in the index increased) saw their currencies appreciate.

To stress the difference between our basic Gamma model of the financial determination of exchange rates in imperfect financial markets and the traditional macroeconomic framework, we next illustrate two polar cases that have been popular in the previous literature: the UIP-based exchange rate, and the complete market exchange rate.

Financial Flows in a UIP Model. Much of the now classic international macroeconomic analysis spurred by Dornbusch (1976) and Obstfeld and Rogoff (1995) either directly assumes that UIP holds or effectively imposes it by solving a first order linearization of the model.[26] The closest analog to this literature in the basic Gamma model is the case where $\Gamma = 0$, such that UIP holds by assumption. In this world, financiers are so relaxed, i.e. their risk bearing capacity is so ample, about supplying liquidity to satisfy shifts in the world demand for assets that such shifts have no impact on expected returns. Consider the example of US investors suddenly wanting to buy Brazilian Real bonds; in this case financiers would simply take the other side of the investors' portfolio demand with no effect on the exchange rate between the Dollar and the Real. In fact, equation (21) confirms that if $\Gamma = 0$, then portfolio flow f^* has no impact on the equilibrium exchange rate.[27]

Financial Flows in a Complete Market Model. Another strand of the literature has analyzed risk premia predominantly under complete markets. We now show that the exchange rate in a setup with complete markets (and no frictions) but otherwise identical to ours is *constant*, and therefore trivially not affected by the flows.

Lemma 3 (Complete Markets) *In an economy identical to the set-up of the basic Gamma model, other than the fact that financial markets are complete and frictionless, the equilibrium exchange rate is constant: $e_t = \nu$, where ν is the relative Negishi weight of Japan.*

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[26]Intuitively, a first order linearization imposes certainty equivalence on the model and therefore kills any risk premia such as those that could generate a deviation from UIP.

[27]These gross flows do not play a role in determining the exchange rate even in models, for example Schmitt-Grohé and Uribe (2003), that assume reduced-form deviations from UIP to be convex functions of the net foreign asset position.

Here, we only sketch the logic and the main equations; a full treatment is relegated to the appendix. Under complete markets, the marginal utility of US and Japanese agents must be equal when expressed in a common currency. Intuitively, the full risk sharing that occurs under complete markets calls for Japan and the US to have the same marginal benefit from consuming an extra unit of non-tradables. In our set-up, this risk sharing condition takes a simple form: $\frac{\chi_t/C_{NT,t}}{\chi_t^*/C_{NT,t}^*}e_t = v$, where v is a constant.[28] Simple substitution of the conditions $C_{NT,t} = \chi_t$ and $C_{NT,t}^* = \chi_t^*$ shows that $e_t = v$, i.e. the exchange rate is constant.[29]

1.4.2 Flows, not just Stocks, Matter in the Gamma model

In frictionless models only stocks matter, not flows per se. In the Gamma model, instead, flows per se matter. This is a distinctive feature of our model. To illustrate this, consider the case where the US has an exogenous Dollar-denominated debt toward Japan, equal to D_0 due at time zero, and D_1 due at time one.[30] For simplicity, assume $\beta = \beta^* = R = R^* = \xi_t = 1$ for $t = 0, 1$. Hence, total debt is $D_0 + D_1$. The flow equations now are:

$$e_0 - \iota_0 - D_0 + Q_0 = 0; \qquad e_1 - \iota_1 - D_1 + Q_1 = 0.$$

The exchange rate at time zero is:[31]

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma} + \frac{(1+\Gamma)D_0 + D_1}{2+\Gamma}.$$

Hence, when finance is imperfect ($\Gamma > 0$), both the timing of debt flows, as indicated by the term $(1+\Gamma)D_0 + D_1$, and the total stock of debt ($D_0 + D_1$) matter in determining exchange rates. The early flow, D_0, receives a higher weight $\left(\frac{1+\Gamma}{2+\Gamma}\right)$ than the late flow, D_1, $\left(\frac{1}{2+\Gamma}\right)$. In sum, flows, not just stocks, matter for exchange rate determination.

To highlight the contrast, let us parametrize the debt repayments as: $D_0 = F$ and $D_1 = -F + S$. The parameter F alters the flow of debt repayment at time zero, but leaves the total stock of debt ($D_0 + D_1 = S$) unchanged. The parameter S, instead, alters the total stock of debt, but does not affect the flow of repayment at time zero. We note that: $\frac{de_0}{dF} = \frac{\Gamma}{2+\Gamma}$, and $\frac{de_0}{dS} = \frac{1}{2+\Gamma}$. When $\Gamma \uparrow \infty$,

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[28]Formally, the constant is the relative Pareto weight assigned to Japan in the planner's problem that solves for complete-market allocations.

[29]The irrelevance of the f gross flows generalizes also to complete, and incomplete, market models where the exchange rate is not constant and the presence of a risk premium makes the two currencies imperfect substitutes. Intuitively in these models the state variables are ratios of stocks of assets, such as wealth, and since these gross flows do not alter the value of such stocks, they have no equilibrium effects because the agents can frictionlessly unwind them. In our model they have effects because these flows alter the balance sheet of constrained financiers.

[30]Hence, the new budget constraint is $\sum_{t=0}^1 R^{-t}(Y_{NT,t} + p_{H,t}Y_{H,t} - D_t) = \sum_{t=0}^1 R^{-t}(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t})$.

[31]The derivation follows from Proposition 6 by defining the pseudo imports as $\tilde{\iota}_t \equiv \iota_t + D_t$.

only flows affect the exchange rate at time zero; this is so even when flows leave the total stocks unchanged $\left(\frac{de_0}{dF} > 0 = \frac{de_0}{dS} \right)$. In contrast, when finance is frictionless ($\Gamma = 0$), flows have no impact on the exchange rate, and only stocks matter $\left(\frac{de_0}{dF} = 0 < \frac{de_0}{dS} \right)$. We collect the result in the Proposition below.

Proposition 5 (Stock Vs flow matters in the Gamma model) *Flows matter for the exchange rate when $\Gamma > 0$. In the limit when financiers have no risk bearing capacity ($\Gamma \uparrow \infty$), only flows matter. When risk bearing capacity is very ample ($\Gamma = 0$), only stocks matter.*

1.4.3 The Exchange Rate Disconnect

The Meese and Rogoff (1983) result on the inability of economic fundamentals such as output, inflation, exports and imports to predict, or even contemporaneously co-move with, exchange rates has had a chilling and long-lasting effect on theoretical research in the field (see Obstfeld and Rogoff (2001)).[32] The Gamma model helps to reconcile the disconnect by introducing financial forces, both the risk bearing capacity Γ and the balance sheet Q, as determinants of exchange rates. Intuitively a disconnect occurs because economies with identical fundamentals feature different equilibrium exchange rates depending on the incentives of the financiers to hold the resulting (gross) global imbalances.

Recently new evidence has been building in favor of these new financial channels. In addition to the instrumental variable approach in Hau, Massa and Peress (2010) discussed earlier, Froot and Ramadorai (2005), Adrian, Etula and Groen (2011), Hong and Yogo (2012), Kim, Liao and Tornell (2014), and Adrian, Etula and Shin (2014) find that flows, financial conditions, and financiers' positions provide information about expected currency returns. Froot and Ramadorai (2005) show that medium-term variation in expected currency returns is mostly associated with capital flows, while long-term variation is more strongly associated with macroeconomic fundamentals. Hong and Yogo (2012) show that speculators' positions in the futures currency market contain information that is useful, beyond the interest rate differential, to forecast future currency returns. Adrian, Etula and Groen (2011), Adrian, Etula and Shin (2014) show that empirical proxies for financial conditions and the tightness of financiers' constraints help forecast both currency returns and exchange rates. Kim, Liao and Tornell (2014) show that information extracted from the speculators' positions in the futures currency market helps to predict exchange rate changes at horizons between 6 and 12 months.

The model can also help to rationalize the co-movement across bilateral exchange rates and between exchange rates and other asset classes. Intuitively, this occurs because all these assets

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[32]Some forecastability of exchange rates using traditional fundamentals appears to occur at very-long horizons (e.g. 10 years) in Mark (1995) or for specific currencies, such as the US Dollar, using transformations of the balance of payments data (Gourinchas and Rey (2007*b*), Gourinchas, Govillot and Rey (2010)).

are traded by financiers and are therefore affected to some degree by the same financial forces. We formalize this intuition in an extension of the base model to multiple countries and assets in Appendix A.3.2. Verdelhan (2013) shows that there is substantial co-movement between bilateral exchange rates both in developed and emerging economies, while Dumas and Solnik (1995), Hau and Rey (2006), Farhi and Gabaix (2014), Verdelhan (2013), Lettau, Maggiori and Weber (2014) link movements in exchange rates to movements in equity markets.

1.5 Endowment Economy

Very little has been said so far about output; we now close the general equilibrium by describing the output market. To build up the intuition for our framework, we consider here a full endowment economy, and consider production economies under both flexible and sticky prices in Section 3.

Let all output stochastic processes $\{Y_{NT,t}, Y_{H,t}, Y_{NT,t}^*, Y_{F,t}\}_{t=0}^1$ be exogenous strictly-positive endowments. Assuming that all prices are flexible and that the law of one price (LOP) holds, one has: $p_{H,t} = p_{H,t}^* e_t$, and $p_{F,t} = p_{F,t}^* e_t$.

Summing US and Japanese demand for US tradable goods ($C_{H,t} = \frac{a_t}{p_{H,t}}$ and $C_{H,t}^* = \frac{\xi_t e_t}{p_{H,t}}$, respectively, which are derived as in Section 1.1), we obtain the world demand for US tradables: $D_{H,t} \equiv C_{H,t} + C_{H,t}^* = \frac{a_t + \xi_t e_t}{p_{H,t}}$. Clearing the goods market, $Y_{H,t} = D_{H,t}$, yields the equilibrium price in dollars of US tradables: $p_{H,t} = \frac{a_t + \xi_t e_t}{Y_{H,t}}$. An entirely similar argument yields: $p_{F,t}^* = \frac{a_t^* + \frac{l_t}{e_t}}{Y_{F,t}}$.

2 Nominal Exchange Rate, Interest Rates, and Capital Flows

We now extend the basic Gamma model from the previous section to account for the nominal side of the economy, direct (but limited) trading of foreign currency bonds by the households, and for a preexisting stock of external debt. Each of the extensions is not only of interest on its own, but also explores the flexibility of our framework by incorporating a number of features that are important in open-economy analysis within an imperfect-market general-equilibrium model.

We introduce each extension separately starting from the basic Gamma model and derive the extended version of the flow equations in the bond market (extensions of equations (13-14)). In all cases, except in the nominal extension of the model, the financiers' demand equation (equation (10)) is unchanged from the basic Gamma model. Finally, we solve in closed form for the equilibrium exchange rate resulting jointly from all extensions.

2.1 Nominal Exchange Rate

We have thus far considered a real model; we now investigate a nominal version of the Gamma model where the nominal exchange rate is determined, similarly to our baseline model, in an imperfect financial market.[33]

We assume that money is only used domestically by the households and that its demand is captured, in reduced form, in the utility function of households in each country.[34] Financiers do not use money, but they trade in nominal bonds denominated in the two currencies. The US consumption basket is now extended to include a real money balances component such that the consumption aggregator is: $C_t \equiv \left[\left(\frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t} \right]^{\frac{1}{\theta_t}}$, where M is the amount of money held by the households and P is the nominal price level so that $\frac{M}{P}$ is real money balances.[35] We maintain the normalization of preference shocks by setting $\theta_t \equiv \omega_t + \chi_t + a_t + \iota_t$. Correspondingly, the Japanese consumption basket is now: $C_t^* \equiv \left[\left(\frac{M_t^*}{P_t^*} \right)^{\omega_t^*} (C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*} \right]^{\frac{1}{\theta_t^*}}$.

Money is the numéraire in each economy, with local currency price equal to 1. The static utility maximization problem is entirely similar to the one in the basic Gamma model in Section 1.1, and standard optimization arguments lead to demand functions: $M_t = \frac{\omega_t}{\lambda_t}$; $p_{NT,t} C_{NT,t} = \frac{\chi_t}{\lambda_t}$; $p_{F,t} C_{F,t} = \frac{\iota_t}{\lambda_t}$, where, we recall from earlier sections, λ_t is the Lagrange multiplier on the households' static budget constraint.[36] Substituting for the value of the Lagrange multiplier, money demand is given by $M_t = \omega_t P_t C_t$ and is proportional to total nominal consumption expenditures; the coefficient of proportionality, ω_t, is potentially stochastic.[37]

Let us define $m_t \equiv \frac{M_t^s}{\omega_t}$ and $m_t^* \equiv \frac{M_t^{s*}}{\omega_t^*}$, where M_t^s and M_t^{s*} are the money supplies.[38] Notice

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[33]Notice that we have indeed set up the "real" model in the previous sections in such a way that non-tradables in each country play a role very similar to money and where, therefore, the exchange rate is rather similar to a nominal exchange rate (see Obstfeld and Rogoff (1996)[Ch. 8.3]). In this section we make such analogy more explicit. Section 5.2 provides a full discussion of the CPI-based real exchange rate in our model. Alvarez, Atkeson and Kehoe (2009) provide a model of nominal exchange rates with frictions in the domestic money markets, while our model has frictions in the international capacity to bear exchange-rate risk.

[34]A vast literature has focused on foundations of the demand for money; such foundations are beyond the scope of this paper and consequently we focus on the simplest approach that delivers a plausible demand for money and much tractability.

[35]See section 5.2 for details on the price index.

[36]The budget constraint of the households is now:

$$\sum_{t=0}^{1} R^{-t} (p_{NT,t} Y_{NT,t} + p_{H,t} Y_{H,t} + M_t^s) = \sum_{t=0}^{1} R^{-t} (p_{NT,t} C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} + M_t).$$

where M_t^s is the seignorage rebated lump-sum by the government, which is equal to M_t in equilibrium.

[37]The money demand equation is similar to that of a cash in advance constraint where money is only held by the consumers within the period, i.e. they need to have enough cash at the beginning of the period to carry out the planned period consumption. For constraints of this type see Helpman (1981), Helpman and Razin (1982).

[38]In the normative part, it will be convenient to consider the *cashless* limit of our economies by taking the limit case

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that since money (as in actual physical bank notes) is non-tradable across countries or with the financiers (but bonds that pay in units of money are tradable with the financiers as in the previous sections), the money market clearing implies that the central bank can pin down the level of nominal consumption expenditure ($m_t = \lambda_t^{-1}, m_t^* = \lambda_t^{*-1}$).[39] The nominal exchange rate e_t is the relative price of the two currencies. It is defined as the strength of the Yen, so that an increase in e_t is a Dollar depreciation.[40]

US nominal imports in dollars are: $p_{F,t}C_{F,t} = \frac{\iota_t}{\lambda_t} = \iota_t m_t$. Similarly, Japanese demand for US tradables is: $p_{H,t}^* C_{H,t}^* = \xi_t m_t^*$. Hence, US nominal exports in dollars are: $p_{H,t}^* C_{H,t}^* \, e_t = \xi_t e_t m_t^*$. We conclude that US nominal net exports in dollars are: $NX_t = \xi_t e_t m_t^* - \iota_t m_t$.

The key equations to solve for the equilibrium nominal exchange rate are the flow equations in the international bond market:

$$\xi_0 e_0 m_0^* - \iota_0 m_0 + Q_0 = 0; \qquad \xi_1 e_1 m_1^* - \iota_1 m_1 - R Q_0 = 0, \tag{21}$$

and the extended financiers' demand curve:[41]

$$Q_0 = \frac{m_0^*}{\Gamma} \mathbb{E}\left[e_0 - e_1 \frac{R^*}{R} \right]. \tag{22}$$

Finally, the nominal interest rates are given by the households' intertemporal optimality conditions (Euler Equations):

$$1 = \mathbb{E}\left[\beta R \frac{U_{1,C_{NT}}' / p_{NT,0}}{U_{0,C_{NT}}' \; p_{NT,1}} \right] = \mathbb{E}\left[\beta R \frac{\chi_1 / C_{NT,1}}{\chi_0 / C_{NT,0}} \frac{p_{NT,0}}{p_{NT,1}} \right] = \beta R \mathbb{E}\left[\frac{m_0}{m_1} \right],$$

so that $R^{-1} = \beta \mathbb{E}\left[\frac{m_0}{m_1} \right]$. Similarly, $R^{*-1} = \beta^* \mathbb{E}\left[\frac{m_0^*}{m_1^*} \right]$. These interest rate determination formulas extend those in equation (6) to the nominal setup.

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when $\{M_t^s, M_t^{s*}, \omega_t, \omega_t^*\} \downarrow 0$ such that $\{m_t, m_t^*\}$ are finite; however, this is not needed for the positive analysis.

[39]The central bank in each period choses money supply after the preference shocks are realized so that m and m^* are policy variables. We abstract here from issues connected with the zero lower bound (ZLB) on nominal interest rates. Notice the duality between money in the current setup and non-tradable goods in the basic Gamma model of Section 1. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the equations in Section 1, because the demand for money implies $\lambda_t = 1$, in which case the demand for non-tradables implies that $p_{NT,t} = 1$.

[40]We intentionally abuse the notation by denoting the nominal exchange rate by e_t the same symbol used for the exchange rate in the basic Gamma model. This allows the notation to be simpler and for the basic concepts of the paper to be more easily compared across a number of different extensions.

[41]Intuitively, scaling by m_0^* makes sure that the real demand is invariant to the level of the money supply. See appendix for further details.

2.2 Capital Flows

Section 1 focused, for simplicity, on capital flows originated by trade in the goods market. Section 1.4.1 provided a first extension to pure portfolio flows by allowing for a time-zero inelastic, or noise, demand by Japanese households for Dollar bonds in the amount of f^*.

In this section we allow households to directly trade foreign bonds, albeit in limited amounts.[42] These flows alter the composition of the countries' foreign assets and liabilities, thus extending results in Section 1 that focused on current account or net capital flows. We consider here demand functions for foreign bonds that depend on all fundamentals, but that do not directly depend on the exchange rate. These demand functions still allow the model to be solved in closed form. Rather than providing precise foundations for the many possible forms that these demands could take, we focus on a general theory of how they impact the equilibrium exchange rate.[43]

We allow the demand functions for foreign bonds from US and Japanese households, denoted by f and f^* respectively, to depend on all present and expected future fundamentals. We use the shorthand notation f and f^* to denote the generic functions: $f(R, R^*, \iota, \xi, ...)$ and $f^*(R, R^*, \iota, \xi, ...)$. For example, demand functions that load on a popular trading strategy, the carry trade, that invests in high interest rate currencies while funding the trade in low interest rate currencies can be expressed as $f = b + c(R - R^*)$ and $f^* = d + g(R - R^*)$, for some constants b, c, d, g.[44] The flow equations in the bond market are now given by:

$$e_0 \xi_0 - \iota_0 + Q_0 + f^* - fe_0 = 0; \qquad e_1 \xi_1 - \iota_1 - RQ_0 - Rf^* + R^* fe_1 = 0.$$

The above flow equations highlight that the demand for Dollar bonds at time zero is increasing in the Japanese households' demand for these bonds (f^*) and decreasing in the US households' demand for Yen bonds (f) that is funded by a corresponding short position in US bonds ($-fe_0$). The flows are reversed at time 1 after interest has accrued and at the new equilibrium exchange rate e_1.

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[42]While it is important that the households are not allowed to *optimally trade unlimited amounts* in foreign currency in order to avoid sidestepping the financiers that are at the core of this paper, limited direct trading or buy-and-hold positions can easily be accommodated in the model.

[43]The appendix extends the present results to demand functions that depend on the exchange rate directly by solving the model numerically.

[44]Possible microfoundations for these strategies range from the "boundedly rational" households who focus on the interest rate when investing without considering future exchange rate changes or covariance with marginal utility (as in Gabaix (2014)), to models of "reaching for yield" (Hanson and Stein, 2014) or rational models of portfolio delegation where the interest rate is an observable variable that is known, in equilibrium, to load on the sources of risk of the model (see Section 5.1). Interestingly, both gross capital flows and trade flows could be ultimately generated by financial frictions (see Antràs and Caballero (2009)). Dekle, Hyeok and Kiyotaki (2014) employ the reduced form approach and put holdings of foreign bonds directly in the utility function of domestic agents to generate flows.

2.3 External Debt, Currency Denomination, and Financial Adjustment

We consider here the impact of external debt and of its currency denomination on equilibrium exchange rates. We allow each country to start with a stock of foreign assets and liabilities. The US net foreign liabilities in dollars are D^{US} and Japan net foreign liabilities in yen are D^J. The flow equations, therefore, are now extended to be:

$$e_0 \xi_0 - \iota_0 + Q_0 - D^{US} + D^J e_0 = 0; \qquad e_1 \xi_1 - \iota_1 - RQ_0 = 0.$$

Notice that D^{US} and D^J enter the equations at time $t = 0$ because the stock of debt has to be intermediated and the different signs with which they enter correspond to their respective currency denomination.[45]

2.4 Equilibrium Exchange Rate in the Extended Setup

When we include all the extensions to the basic Gamma model considered in the previous Sections 2.1-2.3, the flow equations for the Dollar-Yen market become:

$$m_0^* \xi_0 e_0 - m_0 \iota_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0 = 0, \tag{23}$$

$$m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0 - Rf^* + R^* f e_1 = 0. \tag{24}$$

The above equations in conjunction with the financiers' extended demand curve, equation (22), can be used to solve for the equilibrium exchange rate. We show in the Proposition below that the solution method, even in this more general case, follows the simple derivation of the basic model by representing the current economy as a "pseudo" basic economy. We also note that these results do not impose that $Y_{NT,t} = \chi_t$ and $Y_{NT,t}^* = \chi_t^*$, thus generalizing the analysis in Section 1.

Proposition 6 *In the richer model above (with money, portfolio flows, external debt, and shocks to imports and exports) the values for the exchange rates e_0 and e_1 are those in Proposition 3, replacing imports (ι_t), exports (ξ_t), and the risk bearing capacity (Γ) by their "pseudo" counterparts $\{\widetilde{\iota}_t, \widetilde{\xi}_t, \widetilde{\Gamma}\}$, defined as: $\widetilde{\iota}_0 \equiv m_0 \iota_0 + D^{US} - f^*$; $\widetilde{\xi}_0 \equiv m_0^* \xi_0 + D^J - f$; $\widetilde{\iota}_1 \equiv m_1 \iota_1 + Rf^*$; $\widetilde{\xi}_1 \equiv m_1^* \xi_1 + R^* f$; $\widetilde{\gamma} \equiv \gamma/m_0^*$, $\widetilde{\Gamma} \equiv \Gamma/m_0^*$.*

Proof: Equations (23)-(24) reduce to the basic flow equations, equations (13)-(14), provided we replace ι_t and ξ_t by $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$. Similarly, equation (22) reduces to equation (10), provided we replace Γ by $\widetilde{\Gamma}$. Then the result follows from the proof of Proposition 3. □

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[45]We could have alternatively assumed that only a fraction α of the debt had to be intermediated in which case we would get a flow of αD at time zero and a flow $(1 - \alpha)RD$ at time 1.

Intuitively, the pseudo imports $(\widetilde{\iota})$ are composed of factors that lead consumers and firms to sell dollars and hence "force" financiers to be long the Dollar. An entirely symmetric intuition applies to the pseudo exports $\left(\widetilde{\xi}\right)$.

We collect here a number of qualitative results for the generalized economy. While some properties do not strictly depend on $\Gamma > 0$ and therefore can be derived even in UIP models, it is nonetheless convenient to provide a unified treatment in the present model. We assume that $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$ are positive at dates 0 and 1. Otherwise, various pathologies can happen, including the non-existence of an equilibrium (e.g. formally, a negative exchange rate).

Proposition 7 *The Dollar is weaker: 1)* (Imports-Exports) *when US import demand for Japanese goods (ι_t) is higher; when Japanese import demand for US goods (ξ_t) is lower; 2)* ("Myopia" from an imperfect financial system) *higher Γ increases the effects in point 1) by making current imports matter more than future imports;*[46] *3)* (Debts and their currency denomination) *when US net external liabilities in dollars (D^{US}) are higher; when Japanese net external liabilities in Yen (D^J) are lower; 4)* (Financiers' risk-bearing capacity) *when financial conditions are worse (Γ is higher), conditional on Japan being a net creditor at time 0^+ ($N_{0^+} < 0$); 5)* (Demand pressure) *when the noise demand for the Dollar (f^*) is lower, as long as $\Gamma > 0$; 6)* (Interest rates) *when the US real interest rate is lower; when the Japanese real interest rate is higher; 7)* (Money supply) *when the US current money supply (m_0) is higher; when the Japanese current money supply (m_0^*) is lower.*

Point 3 above highlights a valuation channel to the external adjustments of countries.[47] The exchange rate moves in a way that facilitates the re-equilibration of external imbalances. Interestingly, it is not just the net-external position of a country, its net foreign assets, that matters for external adjustment, but actually the (currency) composition of its gross external assets and liabilities (D^{US} and D^J). This basic result is consistent with the valuation channel to external adjustment highlighted in Gourinchas and Rey (2007*a,b*), Lane and Shambaugh (2010).

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[46]That is, $\partial e_0/\partial \iota_0$ and $\partial e_0/\partial \iota_1$ are positive and respectively increasing and decreasing in Γ.

[47]Various propositions take comparative statics with respect to (w.r.t) Γ. For instance, Proposition 2 studies $\frac{\partial e_0}{\partial \Gamma}$. As a matter of mathematics, $\frac{\partial e_0}{\partial \Gamma}$ is well-defined: it is simply the partial derivative w.r.t. Γ in the expression for e_0 in (13). The reader might wonder if taking the partial derivative w.r.t. to endogenous parameter is allowed. The answer is yes. For instance, economists routinely take the partial derivative of demand w.r.t. a price, even though it might be endogenous: this is well-defined mathematically. What happens if we take the derivative w.r.t the more primitive γ (recall that $\Gamma = \gamma var(e_1)^\alpha$). In almost all the expressions of quantities F in this paper, $\frac{dF}{d\gamma} = \frac{\Gamma}{\gamma} \frac{\partial F}{\partial \Gamma}$, so that the signs of comparative statics are preserved. This is because (in most of the paper, where ξ_1 is deterministic) $var(e_1) = var(\iota_1/\xi_1)$, so that that $\Gamma = \gamma var(\iota_1/\xi_1)^\alpha$. So, for instance, in Proposition 2 $\frac{de_0}{d\gamma} = \frac{\Gamma}{\gamma} \frac{\partial e_0}{\Gamma}$. The same logic is at work throughout the paper. In particular, in Propositions 2, 7 (provided $\widetilde{\xi}_1$ is deterministic), 12, 13, the signs of the comparative statics remain the same when they are taken w.r.t. γ rather than w.r.t. Γ.

3 Production and Price Rigidities

We extend here the endowment economy results from Section 1.5 to a production economy with and without price rigidities. Production, particularly in the presence of nominal rigidities, will allow us to illustrate the real effects of the financial determination of exchange rates. These effects will be at the core of the welfare and policy analysis in Section 4.

Production Without Price Rigidities. Let us introduce a minimal model of production that will allow us to formalize the effects of the exchange rate on output and employment. While we maintain the assumption that non-tradable goods in each country are given by endowment processes, we now assume that tradable goods in each country are produced with a technology linear in labor with unit productivity. In each country, labor L is supplied inelastically and is internationally immobile.

Simple profit maximization at the firm level yields a Dollar wage in the US of $w_{H,t} = p_{H,t}$. Under flexible prices, goods market clearing then implies full employment $Y_{H,t} = L$ and a US tradable price in dollars of: $p_{H,t}^{\circ} = \frac{a_t m_t + \xi_t m_t^* e_t}{L}$, where the circle in p° denotes a frictionless quantity. Likewise, for Japanese tradables the equilibrium features both full employment $Y_{F,t} = L$ and a Yen price of: $p_{F,t}^{*\circ} = \frac{a_t^* m_t^* + \iota_t m_t / e_t}{L}$.

Production With Price Rigidities. Let us now assume that wages are "downward rigid" in domestic currency at a preset level of $\{\bar{p}_H, \bar{p}_F^*\}$, where these prices are exogenous. Let us further assume that firms do not engage in pricing to market, so that prices are sticky in producer currency (PCP). Firm profit maximization then implies that: $p_{H,t} = \max\left(\bar{p}_H, p_{H,t}^{\circ}\right)$; or more explicitly: $p_{H,t} = \max\left(\bar{p}_H, \frac{a_t m_t + e_t \xi_t m_t^*}{L}\right)$. Hence:

$$Y_{H,t} = \min\left(\frac{a_t m_t + e_t \xi_t m_t^*}{\bar{p}_H}, L\right). \tag{25}$$

If demand is sufficiently low ($a_t m_t + \xi_t m_t^* e_t < \bar{p}_H L$), then output is demand-determined (i.e., it depends directly on: e_t, ξ_t, a_t, m_t, and m_t^*) and there is unemployment: $L - Y_{H,t} > 0$. Notice that in this case the exchange rate has an *expenditure-switching* effect: if the Dollar depreciates ($e_t \uparrow$), unemployment falls and output expands in the US. Intuitively, since US tradables' prices are sticky in dollars, these goods become cheap for Japanese consumers to buy when the Dollar depreciates. In a world that is demand constrained, this expansion in demand for US tradable is met by expanding production, thus raising US output and employment.[48]

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[48] Clearly, a similar expression and mechanism apply to Japanese tradables: $Y_{F,t} = \min\left(\frac{a_t^* m_t^* + \iota_t m_t / e_t}{\bar{p}_F^*}, L\right)$.

The expenditure switching role of exchange rates has been central to the Keynesian analysis of open macroeconomics of Dornbusch (1976), Obstfeld and Rogoff (1995). In the Gamma model, it is enriched by being the central channel for the transmission of financial forces affecting the exchange rate, such as the risk-bearing capacity and balance sheet of the financiers, into output and employment.

The financial determination of exchange rates has real consequences. Let us reconsider our earlier example of a sudden inflow of capital from US investors into Brazilian Real bonds. The exchange rate in this economy with production and sticky prices is still characterized by equation (21). As previously discussed, the capital inflow in Brazil causes the Real to appreciate $\left(\frac{\partial e_0}{\partial f} = -\frac{\Gamma}{2+\Gamma} < 0 \right)$, and, if the flow is sufficiently strong (f sufficiently high) or the financiers' risk bearing capacity sufficiently low (Γ sufficiently high), the appreciation (the increase in e_0) can be so strong as to make Brazilian goods uncompetitive on international markets; the corresponding fall in world demand for Brazilian output ($\downarrow C_H^* = \frac{\iota_0}{e_0 \bar{p}_F^*}$) causes an economic slump in Brazil with both falling output and increasing unemployment.[49]

Despite extensive debates on the possibility of this type of adverse effect on the real economy of fluctuations in exchange rates due to capital flows and imbalances in financial markets, little formal modeling has been carried out in the academic literature to provide the necessary theoretical foundations. We focused here on providing such foundations in a positive model; Section 4 provides the corresponding normative analysis.

The main focus of our model is to disconnect the exchange rate from fundamentals by altering the structure of financial markets. Of course, part of the disconnect in practice also comes from frictions in the goods makers. These frictions can be analyzed in our model; we illustrate this by considering prices that are sticky in the export destination currency (LCP). To make the point sharp, assume that prices for US tradable goods are exogenously set at $\{\bar{p}_H, \bar{p}_H^*\}$ in dollars in the US and in yen in Japan, respectively.

Lemma 4 (LCP vs PCP) *Under Local Currency Pricing the value of the exchange rate is the same as under Producer Currency Pricing, but US tradable output does not depend on the exchange rate:* $Y_{H,t} = \min \left(\frac{a_t m_t}{\bar{p}_H} + \frac{\xi_t m_t^*}{\bar{p}_H^*}, L \right).$

Proof Because of the log specification, the dollar value of US imports and exports is unchanged: they are still $\iota_t m_t$ and $e_t \xi_t m_t^*$. Consequently, net exports are unchanged, and the exchange rate is unchanged from the previous formulae. Total demand is derived as in Section 1.5. □.

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[49]The Brazilian Finance Minister Guido Mantega complained, as reported in Forbes Magazine (2011), that: "We have to face the currency war without allowing our productive sector to suffer. If we allow [foreign] liquidity to [freely] enter [the economy], it will bring the Dutch Disease to the economy."

LCP helps to further the disconnect between the exchange rate and fundamentals by preventing output in the tradable sector from responding to the exchange rate.[50]

4 Welfare and Heterodox Policies

The Gamma model of exchange rates considered so far in the positive analysis has made clear that exchange rates are affected by financial forces and that their behavior can be quite different from that implied by the traditional macroeconomic analysis. We have also shown how the financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing.

The Mundellian prescription of pure floating exchange rates rests on the idea that a country hit by a negative (asymmetric) real shock would be helped by a depreciating currency that in turn, under some form of sticky prices, would boost its exports and therefore alleviate the adverse impact of the shock on output and employment. In our model we focused on an alternative scenario whereby financial shocks and imbalances in financial markets might cause an appreciation of a country's exchange rate and depress its exports and therefore output.

The possibility of such perverse effects of floating exchange rates has been the subject of extensive debates (Rey (2013), Farhi and Werning (2013)), culminating in the threat of *currency wars*, but its theoretical analysis and the development of policies to improve welfare is still in its infancy. The renewed research effort on analyzing the welfare consequences of capital controls has mostly focused on fixed exchange rate regimes in the context of the small-open-economy new-Keynesian model. Farhi, Gopinath and Itskhoki (2014), Farhi and Werning (2012*a*,*b*), Magud, Reinhart and Rogoff (2011), and Schmitt-Grohé and Uribe (2012) provide innovative analyses of policies such as capital controls, fiscal transfers and fiscal devaluations in this context.[51]

We analyze welfare and policy in the Gamma model described in the previous sections. In particular, we focus on economic situations where the financial market imperfections that are at the core of our model, namely having $\Gamma > 0$ in the presence of capital flows, play an important role both in the welfare distortions and in the suggested policy reaction.

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[50]Devereux and Engel (2003) stressed the absence of exchange rate effects on output under LCP. The empirical evidence shows that, in practice, a combination of PCP, LCP and limited pass-through are present in the data (see Gopinath and Itskhoki (2010), Gopinath, Itskhoki and Rigobon (2010), Amiti, Itskhoki and Konings (2014), Burstein and Gopinath (2015)). For much of this paper, we focus on flexible prices or PCP as the basic cases. As shown in Proposition 4 above, our qualitative analysis can easily accommodate a somewhat more limited pass-through of exchange rate changes to local prices of internationally traded goods.

[51]See also the literature on macro-prudential regulation, amongst others: Mendoza (2010), Bianchi (2010), Korinek (2011).

4.1 Exchange Rate Manipulation

We first show under which conditions direct government interventions in the currency market can affect the equilibrium exchange rate. Then, we derive optimal currency management policy.

For notational simplicity, we set most parameters at 1: e.g. $\iota_0 = \xi_t = a_t = a_0^* = \beta = \beta^* = 1$. We allow ι_1 to be stochastic (keeping $\mathbb{E}[\iota_1] = 1$, and setting $a_1^* = \iota_1$ for symmetry) purely so that currency trading is risky. The reader is encouraged to proceed keeping in mind the intuition coming from the simpler case in which ι_1 is not stochastic and set equal to 1.[52]

Positive analysis At time 0, the US government intervenes in the currency market vis-à-vis the financiers: it buys q yen and sells qe_0 dollars. By Proposition 6 we immediately obtain the result below (as the government creates a flow $f = q$ in the currency market):

Lemma 5 *If the US government buys q yen and sells qe_0 dollars at time 0, the exchange rates satisfy (for small q): $e_0 = 1 + \frac{\Gamma}{2+\Gamma}q + O\left(q^2\right)$, and $\mathbb{E}[e_1] = 1 - \frac{\Gamma}{2+\Gamma}q + O\left(q^2\right)$.*

The intervention's impact on the *average* exchange rate is only second order: it induces a depreciation at time 0, and an appreciation at time 1. We call this effect the "*boomerang effect*". A currency intervention can change the level of the exchange rate in a given period, but not the average level of the exchange rate over multiple periods.

The imperfections in financial markets are at the core of the effects of FX intervention on exchange rates. Indeed, Backus and Kehoe (1989) show that in a general class of models in which currencies are imperfect substitutes due to risk premia, but in which there are no financial frictions, FX interventions have no effect on the exchange rate.[53] Lemma 5 correspondingly highlights the importance of the frictions: if $\Gamma = 0$, a frictionless set-up analogous to that in Backus and Kehoe (1989), there is no first order effect of the intervention on the exchange rate.

Normative analysis We assume that in the short run, i.e. period $t = 0$, US tradables' prices are sticky in domestic currency (PCP) as in Section 3; prices are flexible in the long run, i.e. period $t = 1$. We postulate that at time zero the price is downward rigid at a level $\overline{p}_H$ that is sufficiently high as to cause unemployment in the US tradable sector. Japanese tradable prices are assumed to be flexible. Currency intervention can be welfare improving in this economy. [54]

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[52]We make one more, largely technical, assumption. In this Section 4, we take the case $\alpha = 0$, so as to abstract from (potentially very small) feedback effect from exchange rate interventions to exchange rate volatility and then the exchange rate itself. We conjecture that the whole analysis remains the same, provided α is small enough.

[53]More specifically, sterilized foreign exchange rate interventions have no direct effect; if an effect is present, it is only due to possible distortionary fiscal measures adopted by the governments to redistribute profits and losses of the intervention. Similarly unsterilized intervention has no direct effect, and the only possible effects occur via the indirect change in the money supply.

[54] We make an ancillary assumption, that the proof of Proposition 8 makes precise. We also assume that $\overline{p}_H$ is above the market clearing price, but not too far above it.

Proposition 8 (FX intervention) *Assume that $\Gamma > 0$ and that at time zero US tradable goods are downward rigid at a price $\overline{p}_H$ that is sufficiently high to cause unemployment in the US tradable sector. A US government currency intervention, whereby the government buys $q \in [0, q^{opt}]$ worth of Yen bonds and sells qe_0 Dollar bonds at time zero, improves welfare both in the US and in Japan. The welfare improvement is monotonically increasing in the size of the intervention up to size q^{opt}, which is the smallest intervention that restores full employment in the US.*

Note that there are two preconditions for this intervention to be welfare improving. The first one is that prices are sticky (fixed) in the short run at a level that generates a fall in aggregate demand and induces an equilibrium output below the economy's potential. This condition, i.e. being in a *demand driven* state of the world, is central to the Keynesian analysis where a depreciation of the exchange rate leads to an increase in output via an increase in export demand. If this condition is satisfied a first order welfare loss would occur even in a world of perfect finance.[55]

The second precondition is that financial markets are imperfect, i.e. $\Gamma > 0$. Intuitively, Γ regulates the efficacy of an intervention. In fact, recall from Lemma 5 that the ability of the government to affect the time-zero exchange rate is inversely proportional to Γ. When markets are frictionless ($\Gamma = 0$) the government FX policy has no effect on the time-zero exchange rate, even if prices are sticky, because financiers would simply absorb the intervention without requiring a compensation for the resulting risk.[56]

Interestingly, the suggested policy is not of the *beggar thy neighbor* type: the US currency intervention, even with its aim to weaken the Dollar, actually increases welfare for both US and Japan. This occurs because the intervention induces first order welfare gains for both US and Japanese consumers by increasing US output, but only induces second order losses due to the ensuing inter-temporal distortions in consumption. We highlight that *currency wars* can only occur when both countries are in a slump and the post-intervention weaker dollar causes a first order output loss in Japan.

More generally, our results highlight that heterodox policies could be welfare improving when the currency appreciation is so strong as to actually cause a slump in domestic employment and when private markets are sufficiently disrupted for the government currency intervention to have a meaningful impact on the exchange rate.

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[55]Indeed, in this economy (before the government intervention) financiers optimally choose to not trade at all. The exchange rate is at 1, and is expected to remain at 1 on average, so that there are no gains from trading financial assets. Even if US households were to be allowed to directly trade Yen bonds, they would not change the value of the exchange rate.

[56]We note that after the government intervention the financiers are at their optimum and do not want to change their position. However, US households, that had no incentives to trade Yen bonds in the original equilibrium, would want to trade the Yen bonds after the government intervention. Of course, these unlimited trades are not possible due to the frictions in the intermediation process. Therefore the policy success relies on the presence of financial frictions rather than a direct failure of Ricardian equivalence.

Large-scale currency interventions, outspokenly justified by rationales very similar to the theory provided in this paper, were undertaken by the governments of Switzerland and Israel during the recent financial crisis.[57] Both governments aimed to relieve their currency appreciation in the face of turmoil in financial markets. By most accounts, the interventions successfully weakened the exchange rate and boosted the real economy.[58] Blanchard, de Carvalho Filho and Adler (2014) find empirical support for the efficacy of this policy.[59]

A classic criticism of portfolio balance models is that only extremely big interventions are effective because for an intervention to be effective it needs to alter very large stocks of assets: either the entire stock of assets outstanding or the country level gross external assets and liabilities. In our framework interventions are more effective because they need only alter Q, the balance sheet of financiers, which is potentially substantially smaller than the entire stock of assets.

4.2 Taxing International Finance

We now study a second policy instrument, taxation of the financiers, which is a form of capital controls. We consider a proportional (US) government tax on each financier's profits; the tax proceeds are rebated lump sum to financiers as a whole. Recall the imperfect intermediation problem in Section 1.2, we now assume that the after-tax value of the intermediary is $V_t(1 - \tau)$, where τ is the tax rate. The financiers' optimality condition, derived in a manner entirely analogous to the optimization problem in equation (9), is now: $Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right](1 - \tau)}{\Gamma}$. Notice that this is equivalent to changing Γ to an effective $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1 - \tau}$, so that the financiers' demand can be rewritten as $Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]}{\Gamma^{\text{eff}}}$. We collect the result in the Lemma below.[60]

Lemma 6 *A tax τ on finance is equivalent to lowering the financiers' risk bearing capacity by increasing Γ to $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1 - \tau}$. A higher tax increases the effective Γ^{eff}, thus reducing the financiers' risk bearing capacity.*

[57]The Czech Republic also intervened in the currency market in November 2013 with the aim of depreciating the Koruna to boost the domestic economy

[58]Israel central bank governor Stanley Fisher remarked: "I have no doubt that the massive purchases [of foreign exchange] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession." Levinson (2010)

[59]Empirical studies have to confront the thorny issue of endogeneity of the policy and future empirical work is necessary to provide a full empirical assessment. An earlier skeptical empirical literature, that mostly focused on interventions of considerable smaller size, is summarized by Sarno and Taylor (2001). Dominguez and Frankel (1993a,b) find empirical support for the effect of foreign exchange rate intervention via a portfolio balance channel.

[60]A tax might also plausibly impact the flows from carry traders. For instance, in the specification of Section 2.2, we could have $f^* = d + g(R - R^* - \tau)$. We do not explore here this possibility.

Positive analysis First we note that if the equilibrium before the government intervention features zero risk taking by the financiers ($Q_0 = 0$), as was the case in the economy studied in the previous subsection, then the tax τ is entirely ineffective. Intuitively, this occurs because there are zero expected profits to tax, and therefore the tax has no effect on ex-ante incentives.

More generally we recall from Proposition 2 that an increase in Γ, in this case an increase in Γ^{eff} due to an increase in τ, has the opposite effect on the exchange rate depending on whether the financiers are long or short the Dollar to start with, i.e. depending on the sign of Q_0 before the tax is imposed. For example, the tax would make the Dollar depreciate on impact if the financiers were long dollars to start with ($Q_0 > 0$), but the same tax would make the Dollar depreciate if the financiers had the opposite position to start with. In practice this means that policy makers who are considering imposing capital controls, or otherwise taxing international finance, should pay close attention to the balance sheets of financial institutions that have exposures to their currency. Basing the policy on reduced form approaches or purely on traditional macroeconomics fundamentals can not only be misguiding, but might actually generate the opposite outcome for the exchange rate from the desired one.

In order to study the impact of our tax policy, we start from an economy that features active risk taking by the financiers. In the interest of tractability, we focus on a special case of the basic Gamma model of Section 1. Namely, we assume that $\mathbb{E}[\iota_1] < \iota_0$ and set all other parameters of the model to 1. We maintain from the previous subsection the assumption that US tradable prices are downward rigid at time 0 and flexible at time 1. In this set-up, the Dollar is so strong compared to the Yen at time zero that US output is below potential and there is unemployment in the US. We study in the Lemma below the impact of the tax policy on the exchange rate.

Lemma 7 *If financiers are taxed at rate τ, the equilibrium exchange rate is:*

$$e_0(\tau) = \frac{\left(1 + \frac{\Gamma}{1-\tau}\right)\iota_0 + \mathbb{E}[\iota_1]}{2 + \frac{\Gamma}{1-\tau}}, \qquad e_1(\tau) = \frac{\iota_0 + \left(1 + \frac{\Gamma}{1-\tau}\right)\mathbb{E}[\iota_1]}{2 + \frac{\Gamma}{1-\tau}} + \{\iota_1\}. \tag{26}$$

Notice that while the tax affects the equilibrium exchange rate in each period, it has no effect on the average rate across the two periods $e_0 + e_1 = \iota_0 + \iota_1$. We, therefore, recover here the same "boomerang effect" that was noted in the previous section for FX interventions. As Γ^{eff} goes from Γ to infinity, i.e. as τ goes from zero to 1, the time zero exchange rate approaches ι_0 monotonically from below. Hence, given our assumption that $\mathbb{E}[\iota_1] < \iota_0$, a tax on capital flows devalues the time zero exchange rate.

Normative analysis To keep the analytics to a minimum, we make the countries symmetric by imposing that both have equal taste for foreign tradable goods, $a_t = \xi_t = 1$, $a_t^* = \iota_t$. We define

$\overline{e}_0 \equiv \overline{p}_H L - 1$ as the least-weak value of the Dollar versus the Yen that, in equilibrium, generates full employment in the US. [61]

Proposition 9 (Taxing international finance) *Assume that at time zero: the US Dollar is so strong as to induce a fall in US output below potential ($Y_{H,0} < L$), financial markets are imperfect ($\Gamma > 0$), and the US runs a trade deficit ($\iota_0 > \mathbb{E}[\iota_1]$). A US government tax on the financiers' profits at rate $\tau \in [0, \tau^{opt}]$, improves welfare in both the US and in Japan. The welfare improvement is monotonically increasing in the tax rate up to the tax rate τ^{opt}. This rate is the lowest tax rate that ensures full employment: $e_0(\tau^{opt}) = \overline{e}_0$.*

Note that there are three preconditions for this policy to be welfare improving. As for the case of an FX intervention described in the previous subsection, two preconditions are that output has to be demand driven and that financial markets have to be imperfect. The latter condition is necessary, but in this case no longer sufficient, for the policy to affect the exchange rate. As the reader will recall from our discussion of Lemma 6, a new condition is necessary, and sufficient jointly with $\Gamma > 0$, for the policy to have an effect: financiers need to have a non-zero exposure to currency before the policy is implemented.

4.3 Joint Optimal Monetary and Financial Policy

We have analyzed above optimal financial policy in the form of FX intervention and taxation of international finance in a Gamma world. We analyze here the optimal mix of monetary policy and financial policy.

In order to perform the analysis with minimal algebra, we study a particularly clean case where most model parameters, $(\beta, \beta^*, \iota_t, \xi_t, ...)$, are set to 1. We also initialize $m_t = m_t^* = 1$ for $t = 0, 1$. Before the disruption, the equilibrium exchange rates are $e_0 = e_1 = 1$. The US economy starts with rigid (in dollars) tradable prices $\overline{p}_H$ at a level that just clears the goods market at full employment (i.e., a slightly higher price would lead to unemployment). We assume that the US government's objective function is: $\mathbb{E}[U_0 + U_1] - g(m_0)$, where U_t is the household utility at time t as in equation (1), and $g(m_0)$ is a convex cost of monetary policy surprises, minimal when $m_0 = 1$.[62]

We consider two types of "shocks": an unexpected increase in the Japanese money supply at time zero, from $m_0^* = 1$ to $m_0^{*\prime} > 1$, while keeping m_1^* constant so that the interest rate R^* falls, and an increase in the pre-set price for US tradables ($\overline{p}_H$).[63] We first analyze the monetary shock.

Proposition 10 ("Benign neglect" of foreign monetary shocks) *If the Dollar appreciation is simply due to a monetary shock (increase in m_0^*), then no US policy response is needed ($m_0 = 1$, $q = 0$) to restore full employment. The shock has no real impact, and the economies maintain full employment.*

We next consider the optimal policy reaction to an increase in the pre-set price for US tradables.

Proposition 11 (Trade-off between FX intervention and monetary accommodation) *Suppose that at time zero $\overline{p}_H$ is downwards rigid at a level above the full-employment flexible price, and that at time 1 it is either: 1) flexible ("short-lasting rigidity") or 2) rigid ("long-lasting rigidity"). If the rigidity is short-lasting, then the optimal policy reaction combines both FX intervention and monetary policy, where at the optimum $q > 0$ and $m_0 > 1$. The policy relies more heavily on the FX intervention compared to monetary intervention (i.e. $|m_0 - 1|$ is lower) the higher Γ is. If the rigidity is long-lasting, then the optimal policy reaction employs only monetary policy. In this case, a currency intervention is welfare reducing.*

Intuitively, FX interventions are more desirable when financial markets are more constrained, because an higher Γ makes it is easier for the policy to affect the exchange rate, and when distortions are of shorter duration, because an FX intervention cannot move the exchange rate forever. The reader will recall the "boomerang effect" of a currency intervention: it depreciates the exchange rate at time 0, but then appreciates it at time 1. If the price rigidities are long lasting, in this case last for both periods, then the intervention only generates negative net welfare benefits because of the inter-temporal distortions that it causes.

Note that the government actions would not be taken by the households in the competitive equilibrium. After the shock (in m_0^* or $\overline{p}_H$), there is no incentive for US consumers to intervene and trade the Yen because the (expected) currency returns are zero ($e_0 = R^* \mathbb{E}[e_1]$). The US government chooses to intervene because it internalizes the wage externality (i.e. internalizes the impact on unemployment).

Hence, there are at least three policy tools, (i) monetary policy (ii) FX interventions, (iii) taxation of finance. In general, all might be used, and FX interventions are particularly potent when Γ is high – when financial market are disrupted. While (i) was already well-understood, our analysis allows us to analyze (ii) and (iii) in a way that was difficult in previous treatments that did not have a model of imperfect financial markets.

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lines of Propositions 8 and 9. Then the Propositions in this subsection hold, up to second order terms in the standard deviation of that fundamental uncertainty.

5 Revisiting Canonical Issues with the Gamma Model

We consider in this section a number of canonical issues of international macroeconomics via the lenses of the Gamma model. While these classic issues have also been the subject of previous literature, our analysis not only provides new insights, but also allows us to illustrate how the framework built in the previous sections provides a unified and tractable rationalization of empirical regularities that are at the center of open-economy analysis.

5.1 The Carry Trade in the Presence of Financial Shocks

In the Gamma model there is a profitable carry trade. Let us give the intuition in terms of the most basic model first and then extend it to a set-up with shocks to the financiers' risk bearing capacity (Γ shocks).

First, imagine a world in which countries are in financial autarky because the financiers have zero risk bearing capacity ($\Gamma = \infty$), suppose that Japan has a 1% interest rate while the US has a 5% interest rate, and that all periods ($t = 0, ..., T$) are ex-ante identical with $\xi_t = 1$ and ι_t a martingale. Thus, we have $e_t = \iota_t$, and the exchange rate is a random walk $e_0 = \mathbb{E}[e_1] = ... = \mathbb{E}[e_T]$. A small financier with some available risk bearing capacity, e.g. a small hedge fund, could take advantage of this trading opportunity and pocket the 4% interest rate differential. In this case, there is a very profitable carry trade. As the financial sector risk bearing capacity expands (Γ becomes smaller, but still positive), this carry trade becomes less profitable, but does not disappear entirely unless $\Gamma = 0$, in which case the UIP condition holds. Intuitively, the carry trade in the basic Gamma model reflects the risk compensation necessary to induce the financiers to intermediate global financial flows.

In the most basic model, the different interest rates arise from different rates of time preferences, such that $R = \beta^{-1}$ and $R^* = \beta^{*-1}$. Without loss of generality, assume $R < R^*$ so that the Dollar is the "funding" currency, and the Yen the "investment" currency. The return of the carry trade is: $R^c \equiv \frac{R^*}{R} \frac{e_1}{e_0} - 1$. For notational convenience we define the carry trade expected return as $\overline{R}^c \equiv \mathbb{E}[R^c]$. The calculations in Proposition 3 allow us to immediately derive the equilibrium carry trade.

Proposition 12 *Assume $\xi_t = 1$. The expected return to the carry trade in the basic Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}[\iota_1] - \iota_0}{(R^* + \Gamma)\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1]}, \qquad where \quad \Gamma = \gamma \, var(\iota_1)^{\alpha}. \tag{27}$$

Hence the carry trade return is bigger (i) when the return differential R^*/R is larger (ii) when the funding country is a net foreign creditor (iii) when finance is more imperfect (higher Γ).

To gain further intuition on the above result, consider first the case where $\iota_0 = \mathbb{E}[\iota_1]$. The first order approximation to $\overline{R}^c$ in the case of a small interest rate differential $R^* - R$ is: $\overline{R}^c = \frac{\Gamma}{2+\Gamma}(R^* - R)$. Notice that we have both $\frac{\partial \overline{R}^c}{\partial \Gamma} > 0$ and $\frac{\partial \overline{R}^c}{\partial (R^*-R)} > 0$, so that the profitability of the carry trade increases the more limited the risk bearing capacity of the financiers and the larger the interest rate differential.[64]

The effects of broadly defined "global risk aversion", here proxied by Γ, on the profitability of the carry trade have been central to the empirical analysis of for example Brunnermeier, Nagel and Pedersen (2009), Lustig, Roussanov and Verdelhan (2011), and Lettau, Maggiori and Weber (2014). Here we have shown that the carry trade is more profitable the lower the risk bearing capacity of the financiers; we next formally account for shocks to such capacity in the form of a stochastic Γ.

In addition to a pure carry force due to the interest rate differential, our model features global imbalances as a separate risk factor in currency risk premia. The reader should recall Proposition 2 that showed how net-external-debtor countries' currencies have a positive excess return and depreciate whenever risk bearing capacity decreases ($\uparrow \Gamma$). This effect occurs even if both countries have the same interest rate, thus being theoretically separate from the pure carry trade. Della Corte, Riddiough and Sarno (2014) test these theoretical predictions and find evidence of a global imbalance risk factor in currency excess returns. Notice that we built the model so that financial forces have no effect on the interest rates and the exchange rate makes all the adjustment; while this sharpens the model, we could extend the framework to allow for effects of imbalances on both the exchange rate and interest rates.

The exposure of the carry trade to financial disruptions   We now expand on the risks of the carry trade by studying a three period ($t = 0, 1, 2$) model with stochastic shocks to the financiers' risk bearing capacity in the middle period. To keep the analysis streamlined, we take period 2 to be the "long run". Intuitively, period 2 will be a long-run steady state where countries have zero net foreign assets and run a zero trade balance. This allows us to quickly focus on the short-to-medium-run exchange rate dominated by financial forces and the long-run exchange rate completely anchored by fundamentals. We jump into the analysis, and provide many of the background details of this model in the appendix.[65]

We assume that time-1 financial conditions, Γ_1, are stochastic. In the 3-period economy with

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[64]The first effect occurs because, given an interest rate differential, expected returns to the carry trade have to increase whenever the risk bearing capacity of the financiers goes down to induce them to intermediate financial flows. The second effect occurs because, given a level of risk bearing capacity for the financiers, an increase in the interest rate differential will not be offset one to one by the expected exchange rate change due to the risk premium.

[65]The flow demand equations in the Yen / Dollar market are: $e_t - \iota_t + Q_t = 0$ for $t = 0, 1$, and in the long-run period $e_2 - \iota_2 = 0$, with the financiers' demand for dollars: $Q_t = \frac{e_t - \mathbb{E}[e_{t+1}]\frac{R^*}{R}}{\Gamma_t}$.

a long-run last period, the equilibrium exchange rates are:

$$e_0 = \frac{\Gamma_0 \iota_0 + \frac{R^*}{R} \mathbb{E}_0 \left[\frac{\Gamma_1 \iota_1 + \iota_2 R^*/R}{\Gamma_1 + 1} \right]}{\Gamma_0 + 1}; \qquad e_1 = \frac{\Gamma_1 \iota_1 + \frac{R^*}{R} \mathbb{E}_1 \left[\iota_2 \right]}{\Gamma_1 + 1}; \qquad e_2 = \iota_2. \qquad (28)$$

Recall that the carry-trade return between period 0 and 1 is: $R^c \equiv \frac{R^*}{R} \frac{e_1}{e_0} - 1$. Interestingly, in this case the carry trade also has "exposure to financial conditions". Notice that $\frac{\partial e_1}{\partial \Gamma_1} < 0$ in the equations above, so that the Dollar (the funding currency) appreciates whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$, $\downarrow e_1$). Since in our chosen parametrization the carry trade is short Dollar and long Yen, we correspondingly have: $\frac{\partial R^c}{\partial \Gamma_1} < 0$, the carry trade does badly whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$). This is consistent with the intuition and the empirical findings in Brunnermeier, Nagel and Pedersen (2009); we obtain this effect here in the context of an equilibrium model. We formalize and prove the results obtained so far in the proposition below.

Proposition 13 (Determinants of expected carry trade returns) *Assume that $R^* > R$, $1 = \iota_0 = \mathbb{E}_0 [\iota_1]$ and $\iota_1 = \mathbb{E}_1 [\iota_2]$. Define the "certainty equivalent" $\overline{\Gamma}_1$ by $\frac{\overline{\Gamma}_1 + R^*/R}{\overline{\Gamma}_1 + 1} \equiv \mathbb{E}_0 \left[\frac{\Gamma_1 + R^*/R}{\Gamma_1 + 1} \right]$. Consider the returns to the carry trade, R^c. The corresponding expected return $\overline{R}^c \equiv \mathbb{E}_0 [R^c]$ is*

$$\overline{R}^c = (\mathscr{R}^* - 1) \Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathscr{R}^*}{\overline{\Gamma}_1 (\Gamma_0 + \mathscr{R}^*) + \Gamma_0 + (\mathscr{R}^*)^2}.$$

with $\mathscr{R}^* \equiv \frac{R^*}{R}$. We have:

1. *An adverse shock to financiers affects the returns to carry trade negatively : $\frac{\partial R^c}{\partial \overline{\Gamma}_1} < 0$.*

2. *The carry trade has positive expected returns: $\overline{R}^c > 0$.*

3. *The expected return to the carry trade is higher the worse the financial conditions are at time 0 $\left(\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0 \right)$, the better the financial conditions are expected to be at time 1 $\left(\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0 \right)$, and the higher the interest rate differential ($\frac{\partial \overline{R}^c}{\partial R^*} > 0$, $\frac{\partial \overline{R}^c}{\partial R} < 0$).*

The Fama Regression The classic UIP regression of Fama (1984) is in levels:[66]

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta_{\text{UIP}} (R - R^*) + \varepsilon_1.$$

Under UIP, we would find $\beta_{\text{UIP}} = 1$. However, a long empirical literature finds $\beta_{\text{UIP}} < 1$, and sometimes even $\beta_{\text{UIP}} < 0$. The proposition below rationalizes these findings in the context of an

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[66]The regression is most commonly performed in its logarithmic approximation version, but the levels prove more convenient for our theoretical treatment without loss of economic content.

equilibrium model.

Proposition 14 (Fama regression and market conditions) *The coefficient of the Fama regression is* $\beta_{UIP} = \frac{1+\overline{\Gamma}_1-\Gamma_0}{(1+\Gamma_0)(1+\overline{\Gamma}_1)}$. *Therefore one has* $\beta_{UIP} < 1$ *whenever* $\Gamma_0 > 0$. *In addition, one has* $\beta_{UIP} < 0$ *if and only if* $\overline{\Gamma}_1 + 1 < \Gamma_0$, *i.e. if risk bearing capacity is very low in period 0 compared to period 1.*

Intuitively financial market imperfections always lead to $\beta_{UIP} < 1$ and very bad current market imperfections compared to future ones lead to $\beta_{UIP} < 0$. This occurs because any positive Γ leads to a positive risk premium on currencies that the financiers are long of and hence to a deviation from UIP ($\beta_{UIP} < 1$). If, in addition, financial conditions are particularly worse today compared to tomorrow the risk premium is so big as to induce currencies that have temporarily high interest rates to appreciate on average ($\beta_{UIP} < 0$).

The intuition for $\beta_{UIP} < 1$ is as follows. In the language of Fama (1984), when Japan has high interest rates, the risk premium on the Yen is high. The reason is that the risk premium is not entirely eliminated by financiers, who have limited risk-bearing capacity. In the limit where finance is eliminated ($\Gamma = \infty$), an interest rate of 1% translate one-for-one into a risk premium of 1% ($\beta_{UIP} = 0$).

If riskiness (assuming $\alpha > 0$) or financial frictions go to 0, then β_{UIP} goes to 1.[67] In all cases, however, Covered Interest Rate Parity (CIP) holds in the model. This is because we allow financiers to eliminate all riskless arbitrages. The machinery to justify this is spelled out in the online appendix (section A.3.3). There, we formulate a version of our basic demand (equation (10)), that applies to an arbitrary number of assets, and is arbitrage-free. One upshot is that CIP is respected.

Exchange Rate Excess Volatility In the data exchange rates are more volatile than fundamentals, a fact often referred to as exchange rate excess volatility. The Gamma model helps to rationalize this volatility not only by directly introducing new sources of variation, for example shocks to the risk bearing capacity of the financiers (γ_t) and gross flows (f_t), but also indirectly by endogenously amplifying fundamental volatility via the financial constraints. The intuition is that higher fundamental volatility tightens financial constraints, tighter constraints lead to higher volatility, thus generating a self-reinforcing feedback loop. We formalize this more subtle effect in the Lemma below and sharpen it by not only maintaining the assumption that $\xi_t = 1$ at all dates, but also by considering the case of deterministic (γ_t), so that the only source of volatility is fundamental, and no information revelation about future shocks $E_1[\iota_2] = E_0[\iota_2]$ and $Var_1[\iota_2] = Var_0[\iota_2]$.

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[67]As riskiness ($var(e_1), var(e_2)$) goes to 0, Γ_0 and $\overline{\Gamma}_1$ go to 0, so β_{UIP} goes to 1.

Lemma 8 (Endogenous Amplification of Volatility) *The volatility of the exchange rate at time one is:* $var(e_1) = \left(\frac{\Gamma_1}{1+\Gamma_1}\right)^2 var(\iota_1)$, *where* $\Gamma_1 = \gamma_1 var(\iota_2)^\alpha$. *If* $\alpha > 0$ *and* $\gamma_1 > 0$, *then fundamental volatility is endogenously amplified by the financial constraint:* $\frac{\partial var(e_1)}{\partial var(\iota_2)} > 0$. *Notice that if* $\gamma_1 = 0$, *then* $\frac{\partial var(e_1)}{\partial var(\iota_2)} = 0$.

5.2   Nominal and Real Exchange Rates

We explore here the relationship between the nominal and the real CPI-based exchange rate in our framework. The real exchange rate can be defined as the ratio of two broad price levels, one in each country, expressed in the same numéraire. It is most common to use consumer price indices (CPI) adjusted by the nominal exchange rate, in which case one has: $\mathscr{E} \equiv \frac{P^*e}{P}$. Notice that a fall in $\mathscr{E}$ is a US Dollar real appreciation.

Consider the nominal version of the basic Gamma model in Section 2.1. Standard calculations reported in the appendix imply that the real CPI-based exchange rate is:

$$\mathscr{E} = \tilde{\theta} \frac{(p_H^*)^{\xi'}(p_F^*)^{a^{*'}}(p_{NT}^*)^{\chi^{*'}}}{(p_H)^{a'}(p_F)^{\iota'}(p_{NT})^{\chi'}} \, e_t, \tag{29}$$

where $\tilde{\theta}$ is a function of exogenous shocks also reported in the appendix, and primed variables are normalized by θ. The above equation is the most general formulation of the relationship between the CPI-RER and the nominal exchange rate in the Gamma model. If we impose further assumptions, we can drive a useful special case.

The Basic Gamma Model   Assume that $\omega = \omega^* = 0$ and $p_{NT} = p_{NT}^* = 1$ so that there is no money and the numéraire in each economy is the non-tradable good. Recall that in the basic Gamma model of Section 1 the law of one price holds for tradables, so we have $p_H = p_H^*e$ and $p_F = p_F^*e$. Equation (29) then reduces to: $\mathscr{E} = \tilde{\theta} \, (p_H)^{\xi'-a'}(p_F)^{a^{*'}-\iota'}e^{\chi^{*'}}$. This equation describes the relationship between the RER as defined in the basic Gamma model and the CPI-based RER. Notice that the two are close proxies of each other whenever the baskets' shares of tradables are symmetric across countries (i.e. $\xi' \approx a'$ and $a^{*'} \approx \iota'$) and the non-tradable goods are a large fraction of the Japanese overall basket (i.e. $\chi^{*'} \approx 1$).

6   Conclusion

We presented a theory of exchange rate determination in imperfect capital markets where financiers bear the risks resulting from global imbalances in the demand and supply of international assets. Exchange rates are determined by the balance sheet risks and risk bearing capacity of

these financiers. Exchange rates in our model are disconnected from traditional macroeconomic fundamentals, such as output, inflation and the trade balance and are instead more connected to financial forces such as the demand for assets denominated in different currencies. We have shown how seemingly heterodox policies, such as government interventions in currency markets, can be welfare improving in this context. Our model is tractable, with simple to derive closed form solutions, and can be generalized to address a number of both classic and new issues in international macroeconomic analysis.

References

Adrian, Tobias, and Hyun Song Shin. 2013. "Procyclical leverage and value-at-risk." *Review of Financial Studies*.

Adrian, Tobias, Erkko Etula, and Hyun Song Shin. 2014. "Risk appetite and exchange rates." *Federal Reserve Bank of New York Staff Report*.

Adrian, Tobias, Erkko Etula, and Jan JJ Groen. 2011. "Financial amplification of foreign exchange risk premia." *European Economic Review*, 55(3): 354–370.

Aguiar, Mark, Manuel Amador, and Gita Gopinath. 2009. "Investment cycles and sovereign debt overhang." *Review of Economic Studies*, 76(1): 1–31.

Allen, Polly R., and Peter B. Kenen. 1983. *Asset Markets and Exchange Rates: Modeling an Open Economy.* Cambridge University Press, Cambridge, UK.

Alvarez, Fernando, Andrew Atkeson, and Patrick J Kehoe. 2002. "Money, interest rates, and exchange rates with endogenously segmented markets." *Journal of political economy*, 110(1): 73–112.

Alvarez, Fernando, Andrew Atkeson, and Patrick J Kehoe. 2009. "Time-varying risk, interest rates, and exchange rates in general equilibrium." *Review of Economic Studies*, 76(3): 851–878.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2014. "Importers, Exporters, and Exchange Rate Disconnect." *American Economic Review*, 104(7): 1942–78.

Antràs, Pol, and Ricardo J Caballero. 2009. "Trade and Capital Flows: A Financial Frictions Perspective." *Journal of Political Economy*, 117(4): 701–744.

Bacchetta, Philippe, and Eric Van Wincoop. 2010. "Infrequent portfolio decisions: A solution to the forward discount puzzle." *American Economic Review*, 100(3): 870–904.

Backus, David K., and Gregor W. Smith. 1993. "Consumption and real exchange rates in dynamic economies with non-traded goods." *Journal of International Economics*, 35(3-4): 297–316.

Backus, David K., and Patrick J. Kehoe. 1989. "On the denomination of government debt : A critique of the portfolio balance approach." *Journal of Monetary Economics*, 23(3): 359–376.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy*, 100(4): 745–75.

Bank of International Settlements. 2013. "Triennial Central Bank Survey." *Preliminary Report*.

Bianchi, Javier. 2010. "Credit externalities: Macroeconomic effects and policy implications." *American Economic Review*, 100(2): 398–402.

Blanchard, Olivier, Francesco Giavazzi, and Filipa Sa. 2005. "International Investors, the U.S. Current Account, and the Dollar." *Brookings Papers on Economic Activity*, 36(1): 1–66.

Blanchard, Olivier, Irineu de Carvalho Filho, and Gustavo Adler. 2014. "Can Sterilized Foreign Exchange Intervention Stem Exchange Rate Pressures from the Global Financial Cycle?" *Unpublished Manuscript*.

Branson, William H., and Dale W. Henderson. 1985. "The specification and influence of asset markets." In *Handbook of International Economics*. Vol. 2 of *Handbook of International Economics*, , ed. R. W. Jones and P. B. Kenen, Chapter 15, 749–805. Elsevier.

Branson, William H., Hannu Halttunen, and Paul Masson. 1979. "Exchange rates in the short run : Some further results." *European Economic Review*, 12(4): 395–402.

Brunnermeier, Markus, and Yuliy Sannikov. 2014. "A macroeconomic model with a financial sector." *American Economic Review*, 104(2): 379–421.

Brunnermeier, Markus K., Stefan Nagel, and Lasse H. Pedersen. 2009. "Carry Trades and Currency Crashes." In *NBER Macroeconomics Annual 2008, Volume 23*. *NBER Chapters*, 313–347. NBER, Inc.

Bruno, Valentina, and Hyun Song Shin. 2014. "Cross-Border Banking and Global Liquidity." *Forthcoming Review of Economic Studies*.

Burstein, Ariel, and Gita Gopinath. 2015. "Chapter 7 - International Prices and Exchange Rates." In *Handbook of International Economics*. Vol. 4 of *Handbook of International Economics*, , ed. Kenneth Rogoff Elhanan Helpman and Gita Gopinath, 391 – 451. Elsevier.

Caballero, Ricardo J., and Arvind Krishnamurthy. 2001. "International and domestic collateral constraints in a model of emerging market crises." *Journal of Monetary Economics*, 48(3): 513–548.

Calvo, Guillermo A, and Carlos Alfredo Rodriguez. 1977. "A model of exchange rate determination under currency substitution and rational expectations." *The Journal of Political Economy*, 617–625.

Carvalho, Carlos, and Fernanda Nechio. 2011. "Aggregation and the PPP Puzzle in a Sticky-Price Model." *American Economic Review*, 101(6): 2391–2424.

Chari, Varadarajan V, Patrick J Kehoe, and Ellen R McGrattan. 2002. "Can sticky price models generate volatile and persistent real exchange rates?" *The Review of Economic Studies*, 69(3): 533–563.

Colacito, Riccardo, and Mariano M. Croce. 2011. "Risks for the Long Run and the Real Exchange Rate." *Journal of Political Economy*, 119(1): 153–181.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2008. "International risk sharing and the transmission of productivity shocks." *The Review of Economic Studies*, 75(2): 443–473.

Costinot, Arnaud, Guido Lorenzoni, and Ivan Werning. 2014. "A theory of capital controls as dynamic terms-of-trade manipulation." *Journal of Political Economy*, 122(1): 77–128.

Dedola, Luca, Peter Karadi, and Giovanni Lombardo. 2013. "Global implications of national unconventional policies." *Journal of Monetary Economics*, 60(1): 66 – 85. Carnegie-NYU-Rochester Conference.

De Grauwe, Paul. 1982. "The exchange rate in a portfolio balance model with a banking sector." *Journal of International Money and Finance*, 1(1): 225–239.

Dekle, Robert, Jeong Hyeok, and Nobuhiro Kiyotaki. 2014. "Dynamics of Firms and Trade in General Equilibrium." *Unpublished Manuscript Princeton University*.

Della Corte, Pasquale, S. Riddiough, and Lucio Sarno. 2014. "Currency Premia and Global Imbalances." *Unpublished manuscript, Imperial College and City University*.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. 1990*a*. "Noise Trader Risk in Financial Markets." *Journal of Political Economy*, 703–738.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. 1990*b*. "Positive feedback investment strategies and destabilizing rational speculation." *Journal of Finance*, 45(2): 379–395.

de Macedo, Jorge Braga, and Urho Lempinen. 2013. "Exchange rate dynamics revisited." National Bureau of Economic Research, Inc NBER Working Papers 19718.

Devereux, Michael B., and Charles Engel. 2003. "Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility." *Review of Economic Studies*, 70(4): 765–783.

Diebold, Francis X., and Peter Pauly. 1988. "Endogenous risk in a portfolio-balance rational-expectations model of the Deutschemark-Dollar rate." *European Economic Review*, 32(1): 27–53.

Dominguez, Kathryn M, and Jeffrey A Frankel. 1993*a*. "Does Foreign-Exchange Intervention Matter? The Portfolio Effect." *American Economic Review*, 83(5): 1356–69.

Dominguez, K.M., and J.A. Frankel. 1993*b*. *Does Foreign Exchange Intervention Work?* Institute for International Economics.

Dornbusch, Rudiger. 1976. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy*, 84(6): 1161.

Dornbusch, Rudiger, and Stanley Fischer. 1980. "Exchange Rates and the Current Account." *American Economic Review*, 70(5): 960–71.

Driskill, Robert, and Stephen McCafferty. 1980*a*. "Exchange-rate variability, real and monetary shocks, and the degree of capital mobility under rational expectations." *Quarterly Journal of Economics*, 95(3): 577–586.

Driskill, Robert, and Stephen McCafferty. 1980*b*. "Speculation, rational expectations, and stability of the foreign exchange market." *Journal of International Economics*, 10(1): 91–102.

Dumas, Bernard. 1992. "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World." *Review of Financial Studies*, 5(2): 153–80.

Dumas, Bernard, and Bruno Solnik. 1995. "The World Price of Foreign Exchange Risk." *Journal of Finance*, 50(2): 445–79.

Evans, Martin, and Richard K. Lyons. 2002. "Order Flow and Exchange Rate Dynamics." *Journal of Political Economy*, 110(1): 170–180.

Evans, Martin D. D., and Richard K. Lyons. 2012. "Exchange Rate Fundamentals and Order Flow." *Quarterly Journal of Finance*, 2(4): 1–63.

Fama, Eugene F. 1984. "Forward and spot exchange rates." *Journal of Monetary Economics*, 14(3): 319–338.

Farhi, Emmanuel, and Ivan Werning. 2012*a*. "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates." *Unpublished manuscript, Harvard University and MIT*.

Farhi, Emmanuel, and Ivan Werning. 2012*b*. "Fiscal Unions." *Unpublished manuscript, Harvard University and MIT*.

Farhi, Emmanuel, and Ivan Werning. 2013. "Dilemma not trilemma? Capital controls and exchange rates with volatile capital flows." *IMF working paper*. Harvard University and MIT.

Farhi, Emmanuel, and Ivan Werning. 2014. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Unpublished manuscript, Harvard University and MIT*.

Farhi, Emmanuel, and Xavier Gabaix. 2014. "Rare Disasters and Exchange Rates." *Unpublished manuscript, Harvard University and NYU*.

Farhi, Emmanuel, Gita Gopinath, and Oleg Itskhoki. 2014. "Fiscal Devaluations." *Forthcoming Review of Economic Studies*.

Feenstra, Robert C. 1986. "Functional equivalence between liquidity costs and the utility of money." *Journal of Monetary Economics*, 17(2): 271–291.

Forbes Magazine. 2011. *http://www.forbes.com/sites/afontevecchia/2011/04/18/mantega-inflation-under-control-in-brazil-as-qe2-pressures-real/*.

Froot, Kenneth A, and Jeremy C. Stein. 1991. "Exchange rates and foreign direct investment: an imperfect capital markets approach." *Quarterly Journal of Economics*, 106(4): 1191–1217.

Froot, Kenneth A., and Tarun Ramadorai. 2005. "Currency Returns, Intrinsic Value, and Institutional-Investor Flows." *Journal of Finance*, 60(3): 1535–1566.

Froot, Kenneth A., Paul G. J. O'Connell, and Mark S. Seasholes. 2001. "The portfolio flows of international investors." *Journal of Financial Economics*, 59(2): 151–193.

Gabaix, Xavier. 2014. "A Sparsity-Based Model of Bounded Rationality." *Quarterly Journal of Economics*.

Garleanu, Nicolae, and Lasse Heje Pedersen. 2011. "Margin-based asset pricing and deviations from the law of one price." *Review of Financial Studies*, 24(6): 1980–2022.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*. Vol. 3 of *Handbook of Monetary Economics*, Chapter 11, 547–599. Elsevier.

Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of monetary Economics*, 58(1): 17–34.

Gopinath, Gita, and Oleg Itskhoki. 2010. "Frequency of Price Adjustment and Pass-Through." *The Quarterly Journal of Economics*, 125(2): 675–727.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon. 2010. "Currency Choice and Exchange Rate Pass-Through." *American Economic Review*, 100(1): 304–36.

Gourinchas, Pierre-Olivier, and Helene Rey. 2007*a*. "From world banker to world venture capitalist: US external adjustment and the exorbitant privilege." In *G7 Current Account Imbalances: Sustainability and Adjustment*.

Gourinchas, Pierre-Olivier, and Hélène Rey. 2007*b*. "International financial adjustment." *Journal of Political Economy*, 115(4): 665–703.

Gourinchas, Pierre-Olivier, and Helene Rey. 2013. "External Adjustment, Global Imbalances and Valuation Effects." *Forthcoming Handbook of International Economics*.

Gourinchas, Pierre-Olivier, Nicolas Govillot, and Hélène Rey. 2010. "Exorbitant Privilege and Exorbitant Duty." *Unpublished manuscript, UC Berkeley and LBS*.

Gromb, Denis, and Dimitri Vayanos. 2002. "Equilibrium and welfare in markets with financially constrained arbitrageurs." *Journal of Financial Economics*, 66(2): 361–407.

Hanson, Samuel G., and Jeremy C. Stein. 2014. "Monetary policy and long-term real rates." *Journal of Financial Economics*.

Hart, Oliver. 1995. *Firms, contracts, and financial structure.* Oxford University Press.

Hart, Oliver, and John Moore. 1994. "A theory of debt based on the inalienability of human capital." *Quarterly Journal of Economics*, 109(4): 841–879.

Hassan, Tarek A. 2013. "Country Size, Currency Unions, and International Asset Returns." *Journal of Finance*, 68(6): 2269–2308.

Hau, Harald, and Hélène Rey. 2006. "Exchange Rates, Equity Prices, and Capital Flows." *Review of Financial Studies*, 19(1): 273–317.

Hau, Harald, Massimo Massa, and Joel Peress. 2010. "Do Demand Curves for Currencies Slope Down? Evidence from the MSCI Global Index Change." *Review of Financial Studies*, 23(4): 1681–1717.

Helpman, Elhanan. 1981. "An exploration in the theory of exchange-rate regimes." *The Journal of Political Economy*, 865–890.

Helpman, Elhanan, and Assaf Razin. 1982. "Dynamics of a Floating Exchange Rate Regime." *Journal of Political Economy*, 90(4): pp. 728–754.

Henderson, Dale W., and Kenneth Rogoff. 1982. "Negative net foreign asset positions and stability in a world portfolio balance model." *Journal of International Economics*, 13(1-2): 85–104.

He, Zhiguo, and Arvind Krishnamurthy. 2013. "Intermediary Asset Pricing." *American Economic Review*, 103(2): 732–70.

Hong, Harrison, and Motohiro Yogo. 2012. "What does futures market interest tell us about the macroeconomy and asset prices?" *Journal of Financial Economics*, 105(3): 473–490.

Ivashina, Victoria, David S. Scharfstein, and Jeremy C. Stein. 2012. "Dollar Funding and the Lending Behavior of Global Banks." NBER Working Paper 18528.

Jeanne, Olivier, and Andrew K. Rose. 2002. "Noise trading and exchange rate regimes." *Quarterly Journal of Economics*, 117(2): 537–569.

Kim, Young Ju, Zhipeng Liao, and Aaron Tornell. 2014. "Speculators Positions and Exchange Rate Forecasts: Beating Random Walk Models." *Unpublished Manuscript UCLA*.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211–248.

Kollmann, Robert. 2013. "Global Banks, Financial Shocks, and International Business Cycles: Evidence from an Estimated Model." *Journal of Money, Credit and Banking*, 45(s2): 159–195.

Kollmann, Robert, Zeno Enders, and Gernot J Müller. 2011. "Global banking and international business cycles." *European Economic Review*, 55(3): 407–426.

Korinek, Anton. 2011. "The new economics of prudential capital controls: A research agenda." *IMF Economic Review*, 59(3): 523–561.

Kouri, Pentti J. K. 1976. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach." *Scandinavian Journal of Economics*, 78(2): 280–304.

Lane, Philip R, and Jay C Shambaugh. 2010. "Financial exchange rates and international currency exposures." *American Economic Review*, 100(1): 518–540.

Lettau, Martin, Matteo Maggiori, and Michael Weber. 2014. "Conditional risk premia in currency markets and other asset classes." *Journal of Financial Economics*, 114(2): 197 – 225.

Levinson, Charles. 2010. "Interview Transcript: Stanley Fischer." *Wall Street Journal.* `http://online.wsj.com/news/articles/SB10001424052702304510704575562410676436730` *(accessed January 15, 2014)*.

Lucas, Robert E. 1982. "Interest rates and currency prices in a two-country world." *Journal of Monetary Economics*, 10(3): 335–359.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. 2011. "Common risk factors in currency markets." *Review of Financial Studies*, 24(11): 3731–3777.

Maggiori, Matteo. 2014. "Financial Intermediation, International Risk Sharing, and Reserve Currencies." *Unpub-*

lished manuscript, Harvard University.

Magud, Nicolas E., Carmen M. Reinhart, and Kenneth S. Rogoff. 2011. "Capital Controls: Myth and Reality - A Portfolio Balance Approach." National Bureau of Economic Research, Inc NBER Working Papers 16805.

Mark, Nelson C. 1995. "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability." *American Economic Review*, 85(1): 201–18.

Martin, Ian. 2011. "The Forward Premium Puzzle in a Two-Country World." NBER Working Paper 17564.

Meese, Richard A., and Kenneth Rogoff. 1983. "Empirical exchange rate models of the seventies : Do they fit out of sample?" *Journal of International Economics*, 14(1-2): 3–24.

Mendoza, Enrique G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review*, 100(5): 1941–1966.

Mendoza, Enrique G, Vincenzo Quadrini, and Jose-Vıctor Rıos-Rull. 2009. "Financial Integration, Financial Development, and Global Imbalances." *Journal of Political Economy*, 117(3).

Obstfeld, Maurice. 2004. "External adjustment." *Review of World Economics*, 140(4): 541–568.

Obstfeld, Maurice, and Kenneth Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy*, 103(3): 624–660.

Obstfeld, Maurice, and Kenneth Rogoff. 2001. "The six major puzzles in international macroeconomics: is there a common cause?" In *NBER Macroeconomics Annual 2000, Volume 15.* , ed. Ben S. Bernanke and Kenneth Rogoff, 339–412. The MIT Press.

Obstfeld, Maurice, and Kenneth S. Rogoff. 1996. *Foundations of International Macroeconomics.* MIT Press.

Pavlova, Anna, and Roberto Rigobon. 2007. "Asset Prices and Exchange Rates." *Review of Financial Studies*, 20: 1139–1180.

Pavlova, Anna, and Roberto Rigobon. 2008. "The Role of Portfolio Constraints in the International Propagation of Shocks." *Review of Economic Studies*, 75(4): 1215–1256.

Pavlova, Anna, and Roberto Rigobon. 2010. "An asset-pricing view of external adjustment." *Journal of International Economics*, 80(1): 144–156.

Pavlova, Anna, and Roberto Rigobon. 2012. "Equilibrium Portfolios and External Adjustment Under Incomplete Markets." *Unpublished Manuscript LBS*.

Perri, Fabrizio, and Vincenzo Quadrini. 2014. "International recessions."

Rey, Hélène. 2013. "Dilemma not Trilemma: The global financial cycle and monetary policy independence." Jackson Hole Economic Symposium.

Sarno, Lucio, and Mark P. Taylor. 2001. "Official Intervention in the Foreign Exchange Market: Is It Effective and, If so, How Does It Work?" *Journal of Economic Literature*, 39(3): pp. 839–868.

Schmitt-Grohé, Stephanie, and Martın Uribe. 2003. "Closing small open economy models." *Journal of international Economics*, 61(1): 163–185.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2012. "Pegs and Pain." *Working paper Columbia University*.

Shleifer, Andrei, and Robert W. Vishny. 1997. "The limits of arbitrage." *Journal of Finance*, 52(1): 35–55.

Stathopoulos, Andreas. 2012. "Asset prices and risk sharing in open economies." *Unpublished manuscript, University of Southern California.*

Tobin, James, and Jorge B. de Macedo. 1979. "The Short-Run Macroeconomics of Floating Exchange Rates: An Exposition." Cowles Foundation for Research in Economics, Yale University Cowles Foundation Discussion Papers 522.

Verdelhan, Adrien. 2010. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *Journal of Finance*, 65(1): 123–146.

Verdelhan, Adrien. 2013. "The share of systematic variation in bilateral exchange rates." *Unpublished Manuscript MIT*.

Appendix to "International Liquidity and Exchange Rate Dynamics"

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A.1 Analytical Generalization of the Model

A.1.1 The Exchange Rate with Infinite Horizon

We provide here the infinite-horizon extension of the model. In this model the flow equation is: $\xi_t e_t - \iota_t - RQ_{t-1} + Q_t = 0$.

Proposition A.1 (Exchange rate with infinite horizon) *Assume a non-negative stochastic process for imports ι_t and that $\xi_t = 1$ for all t. Denote by RQ_{t-1} the accrued net foreign liabilities of the US (denominated in dollars) to be absorbed by the financiers at t. The equilibrium exchange rate is:*

$$e_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \Lambda^{s-t} (1 - \Lambda) \iota_s \right] + (1 - \Lambda) RQ_{t-1}, \tag{A.1}$$

where $\Lambda \equiv \frac{(1+R+\Gamma) - \sqrt{(1+R+\Gamma)^2 - 4R}}{2R} \in (0, \frac{1}{R}]$. If $\Gamma = 0$, then $\Lambda = \frac{1}{R}$; otherwise, Λ is decreasing in Γ.

This Proposition shows that Γ is akin to inducing myopia about future flows. If $\Gamma = 0$ so that UIP holds and $\Lambda = 1/R$, then future flows are discounted at the US interest rate. When $\Gamma > 0$, future flows are discounted at a rate higher than this interest rate.

It is interesting to analyze the model in the presence of portfolio flows, f_t^*, along the lines of Sections 1.4.1 and 2.2, so that the flow equation is, for all t, $\xi_t e_t - \iota_t + f_t^* + Q_t - RQ_{t-1} = 0$. Recall that for analytical tractability we assume that these flows can depend on most fundamental variables, e.g. the interest rate, but cannot depend directly on the exchange rate.[68] We perform a Taylor expansion in the interest rate differential, linearizing around $R_t^* = R_t \approx R$, assuming that ι_t is close to its steady state value ι, and assuming that RQ_{t-1} is close to 0, so that we can expand the exchange rate around $\bar{e} \equiv \iota$. Then

$$e_t = \mathbb{E}_t \sum_{s=t}^{\infty} \Lambda^{s-t} \left[(1 - \Lambda)(\iota_s - f_s^*) + \Lambda \bar{e} \frac{R_s^* - R_s}{R} \right] + (1 - \Lambda) R_t Q_{t-1}. \tag{A.2}$$

The above expression is exact in its treatment of the term in $(\iota_s - f_s^*)$, but contains a Taylor expansion in the $R_s^* - R_s$ term.

This generalizes to an infinite horizon the effects of portfolio flows on the exchange rate, which were the focus of Proposition 7 in the two period model. The Yen is stronger: if Japan is a creditor ($Q_{t-1} > 0$); if there is high import demand for Japanese goods ($\iota_s > 0$); if interest rates are higher in Japan than in the US ($R_s^* - R_s > 0$); and if there is selling pressure on the Dollar ($f_s^* < 0$). Compared to the UIP case, the first effect is amplified, the import-demand and interest-rate effects are simply discounted at a higher rate (we have $\Lambda < \frac{1}{R}$ in this Gamma model, rather than $\Lambda = \frac{1}{R}$ in the UIP case). The last effect (the f_s^* term) is entirely specific to the Gamma model.[69]

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[68] Section A.4 provides numerical solutions for the case of portfolio flows that depend directly on the exchange rate.
[69] The derivation of equation (A.2) in the Proof section of this appendix provides details for this latter effect.

A.1.1.1 Slow Digestion of Imbalances

To further understand the dynamics of exchange rates, consider a very simple example: there is an initial debt Q_0^- at time 0, ι_t is i.i.d. with mean 1 with $R_t = R_t^* = R$ for all t.

Proposition A.2 (Slow digestion of imbalances when $\Gamma > 0$) *Given an initial debt Q_0^-, the dynamics of the exchange rate are: $\mathbb{E}_0 e_t = 1 + (1-\Lambda)Q_0^- \lambda^t$. US net foreign liabilities evolve according to: $R\mathbb{E}_0 Q_t = Q_0^- \lambda^t$, where $\lambda \equiv R\Lambda$. If $\Gamma = 0$, then $\lambda = 1$, and otherwise $0 < \lambda < 1$.*

In the UIP case ($\Gamma = 0$) the debt and the exchange rate are constant in expectation. The Dollar is permanently weak. For the general case, we reason in expectation. When $\Gamma > 0$, and therefore $\lambda < 1$, the debt slowly shrinks to 0, and the Dollar slowly mean-reverts to its fundamental value with no debt ($e_\infty = 1$). Intuitively, this "slow digestion" occurs because the US has a debt at time zero and the financiers need to be convinced to hold it. Given that interest rates are the same in both countries, the Dollar needs to be expected to appreciate to give a capital gain to the financiers. Hence, the Dollar is weak initially, and it will appreciate as long as there is debt remaining. Indeed, it appreciates until $Q_t = 0$, which happens in the limit as $t \to \infty$. Viewed in a different way, the Dollar is depreciated, so US exports are higher, which allows the US to repay its debt slowly.

Hence, the financial imperfections modeled in this Gamma framework lead to qualitatively different dynamics compared to the traditional UIP case.

A.1.2 Japanese Households and the Carry Trade

In most of the main body of the paper, consumers do not do the carry trade themselves. In this subsection, we analyze the case where Japanese consumers buy a quantity f^* of dollar bonds, financing the purchase by shorting an equivalent amount of Yen bonds. We let this demand take the form:

$$f^* = b(R - R^*).$$

If $b \geq 0$ the Japanese household demand is a form of carry trade. The flow equations now are:

$$NX_0 + Q + f^* = 0; \qquad NX_1 - R(Q + f^*) = 0.$$

We summarize the implications for the equilibrium carry trade in the Proposition below.

Proposition A.3 *When Japanese consumers do the carry trade, the expected return to the carry trade in the basic Gamma model is:*
$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}[\iota_1] - \iota_0 + f^*(1+R^*)}{(R^* + \Gamma)\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1] - \Gamma f^*}.$$

Hence the carry trade return is bigger: (i) when R^*/R is higher, (ii) when the funding country is a net foreign creditor, or (iii) when consumers do the carry trade less (f^* increases).

If consumers do the carry trade on too large a scale (f^* too negative), then the carry trade becomes unprofitable, $\overline{R}^c < 0$.

A.2 Further Details for the Main Body of the Paper

A.2.1 A more abstract version of the market structure

It may be useful to have a more abstract presentation of the basic model. We focus on the US side, as the Japanese side is entirely symmetric.

For generality, we present the monetary model, and then show how the real model can be viewed as a special case of it. We call

$$c = (C_H, C_F, C_{NT}, M, 0, 0),$$

the vector of consumptions of C_H US tradables, C_F Japanese tradables, C_{NT} US non-tradables, and a quantity M of US money, respectively. The last 2 slots in vector c (set at 0) are the consumption of Japanese non-tradables, and Japanese money: they are zero for the US consumer. Likewise, the Japanese consumer has consumption:

$$c^* = (C_H^*, C_F^*, 0, 0, C_{NT}^*, M^*).$$

The Japanese household consumes C_H^* US tradables, C_F^* Japanese tradables, 0 US non-tradables, 0 US money, C_{NT}^* Japanese non-tradables, and M^* Japanese money.

The US production vector is

$$y = (y_H, 0, y_{NT}, M^s, 0, 0).$$

This shows that the US produces y_H US tradables, 0 Japanese tradables, y_{NT} US non-tradables, and 0 Japanese non-tradables and money . Here M^s is the money supply given by the government to the household. Japanese production is similarly

$$y^* = (0, y_F^*, 0, 0, y_{NT}^*, M^{s*}).$$

The vector of prices in the US is

$$p = (p_H, p_F, p_{NT}, 1, 0, 0).$$

Utility is $u(c_t, \phi_t)$, where ϕ_t is a taste shock. In the paper, $\phi_t = (a_t, \iota_t, \chi_t, \omega_t, 0, 0)$, so that in the utility function is

$$u(c_t, \phi_t) = \sum_{i=1}^{6} \phi_{it} \ln c_{it}, \text{ for } t = 0, \dots, T.$$

Consumptions are non-negative, $c_{it} \geq 0$ for all i, t.

The fourth and sixth components of the above vectors correspond to money. In the real model they are set to 0. Then, in this real money, the numéraire is the non-tradable good, so that $p_{NT} = 1$.

We call $\Theta_t = (\Theta_t^{US}, \Theta_t^J)$ the holding by the US of US bonds and Japanese bonds, $P_t = (1, e_t)$ the price of bonds in dollars.

The US consumers' problem is:

$$\max_{(c_t, \Theta_t^{US})_{t \leq T}} \mathbb{E} \sum_{t=0}^{T} \beta^t u(c_t, \phi_t), \tag{A.3}$$

s.t.

$$p_t \cdot (y_t - c_t) + P_t \cdot D_t \Theta_{t-1} + \pi_t^F = P_t \cdot \Theta_t, \text{ for } t = 0, \dots, T, \tag{A.4}$$

and

$$\Theta_T = 0. \tag{A.5}$$

Here $D_t = diag(R, R^*)$ is the diagonal matrix expressing the gross rate of return of bonds in each currency and π_t^F is a profit rebated by financiers. The left-hand side of (A.4) is the households' financial wealth (in dollars) after period t. US firms are fully owned by US households. Because the economy is fully competitive, they make no profit. The entire production comes as labor income, whose value is $p_t \cdot y_t$. The budget constraint is the terminal asset holdings should be 0, which is expressed by (A.5). Finally, as is usual, c_t and Θ_t^{US} are adapted process, i.e. they depend only on information available at date t.

In the above maximization problems, US consumers choose optimally their consumption vector c_t and

their dollar bond holdings Θ_t^{US}. However, they do not choose their holding of Japanese bonds Θ_t^J optimally. In the basic model we preclude such holdings and set $\Theta_t^J = 0$. In the extended model, we allow for such holdings and study simple and intuitive cases: for instance, at time 0 the holdings of Japanese bonds can be an endowment $\Theta_{-1}^J = D^J$ (or Japanese debt denoted in Yen). Alternatively, they could be a liquidity (noise trader) shock $\Theta_0^J = -f$, or we could have f be a function of observables, but not the exchange rate directly, e.g. $f = b + c(R - R^*)$ for a carry-trader (see Section 2.2). We do not focus on the foundations for each type of demand, but actually take the demands as exogenously specified. Possible microfoundations for these demands range from rational models of portfolio delegation where the interest rate is an observable variable that is known, in equilibrium, to load on the sources of risk of the model (see Section 5.1), to models of "reaching for yield" (Hanson and Stein, 2014), or to the "boundedly rational" households who focus on the interest rate when investing without considering future exchange rate changes or covariance with marginal utility (as in Gabaix (2014)).

To summarize, while all goods are frictionlessly traded within a period (with the non-tradable goods being traded only within a country), asset markets are restricted: only US and Japanese bonds are traded (rather than a full set of Arrow-Debreu securities).

The *goods market clearing condition* is:

$$y_t + y_t^* = c_t + c_t^* \text{ at all dates } t \leq T. \tag{A.6}$$

Firms produce and repatriate their sales at every period. They have net asset flows,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* (e_t, -1),$$

$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right).$$

The first equation expresses the asset flows of US exporters : in Japan, they have sales of $p_{Ht}^* c_{Ht}^*$ Yen in Japan market; they repatriate those yens (hence a flow of $-p_{Ht}^* c_{Ht}^*$ in Yen), to buy dollars (hence a flow of $p_{Ht}^* c_{Ht}^* e_t$ dollars).

For instance, in the model with the log specification,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* (e_t, -1) = m_t^* \xi_t (e_t, -1),$$

$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right) = m_t \iota_t \left(-1, \frac{1}{e_t}\right),$$

so that $\Theta_t^{firms} + \Theta_t^{firms,*} = (m_t^* \xi_t e_t - m_t \iota_t) \left(1, -\frac{1}{e_t}\right)$. The real model is similar, replacing m_t and m_t^* by 1.

The gross demand by financiers is $Q_t (1, -1/e_t)$. Each period the financiers sell the previous period position, so that their net demand is:

$$Q_t (1, -1/e_t) - D_t Q_{t-1} (1, -1/e_{t-1}) = (1 - D_t \mathscr{L}) Q_t (1, -1/e_t), \tag{A.7}$$

where $\mathscr{L}$ is the lag operator, $\mathscr{L} X_t = X_{t-1}$.

Financiers choose Q_t optimally, given the frictions, as in the main body of the paper and we do not restate their problem here for brevity. In the last period, holdings are 0, i.e. $Q_T = 0$.

The *asset market clearing condition* is that the net demand for bonds is 0

$$\Theta_t^{firms} + \Theta_t^{firms,*} + (1 - D_t \mathscr{L})(\Theta_t + \Theta_t^* + Q_t (1, -1/e_t)) = 0. \tag{A.8}$$

For instance, for consumers, $(1 - D_t \mathscr{L})\Theta_t$ is the increased asset demand by the agent. To gain some intu-

ition, the first coordinate of equation (A.8), evaluated at time $t = 0$, in the case where $\Theta_t = \Theta_t^* = 0$, gives equation (21) of the paper:

$$\xi_0 e_0 m_0^* - \iota_0 m_0 + Q_0 = 0;$$

and in the real case (corresponding to $m_0 = m_0^* = 1$), we obtain the basic equation (13) of the paper:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0.$$

We now state formally the definition of equilibrium in the case of flexible prices. Recall that we assume the law of one price in goods market to hold such that:

$$p_{Ht}^* = p_{Ht}/e_t, \, p_{Ft}^* = p_{Ft}/e_t. \tag{A.9}$$

Definition 1 *A competitive equilibrium consists of allocations* $\left(c_t, c_t^*, \Theta_t, \Theta_t^*, \Theta_t^{firms}, \Theta_t^{firms,*}, Q_t\right)$, *prices* p_t, p_t^*, *exchange rate* e_t, *for* $t = 0, \ldots, T$ *such that the US consumers optimize their utility function (A.3) under the above constraints (A.4-A.5), Japanese consumers optimize similarly, goods markets clear (A.6), and asset markets clear (A.8), and the low of one price (A.9) holds.*

As explained in the paper (Lemma 4), if we use local currency pricing (i.e. change (A.9), and replace the value of p_{Ht}^* and p_{Ft} by other, potentially arbitrary, values), the equilibrium value of the exchange rate does not change (though consumptions do change).

The timing was already stated in the paper, but for completeness we restate it here. At time 0, producers produce, consumers demand and consume, exporters repatriate their sales, financiers take their FX positions, and asset and goods market clear (simultaneously, like in Arrow-Debreu). The potential diversion by the financiers happens at time 0^+, right after time 0 (of course, no diversion happens on the equilibrium path). Then, at time 1 and potentially future periods, the same structure is repeated (with no financiers' position in the last period).

A.2.2 Maximization Problem of the Japanese Household

We include there many details excluded from Section 1 for brevity. The dynamic budget constraint of Japanese households (which holds state by state) is:

$$\sum_{t=0}^{1} \frac{Y_{NT,t}^* + p_{F,t}^* Y_{F,t}^* + \pi_t^*}{R^{*t}} = \sum_{t=0}^{1} \frac{C_{NT,t}^* + p_{H,t}^* C_{H,t}^* + p_{F,t}^* C_{F,t}^*}{R^{*t}},$$

where π_t^* are the financiers' profits remittances to the Japanese, $\pi_0^* = 0$, $\pi_1^* = Q_0(R - R^* e_1/e_0)/e_1$.

The static utility maximization problem of the Japanese household:

$$\max_{C_{NT,t}^*, C_{H,t}^*, C_{F,t}^*} \chi_t^* \ln C_{NT,t}^* + \xi_t \ln C_{H,t}^* + a_t^* \ln C_{F,t}^* + \lambda_t^* \left(CE_t^* - C_{NT,t}^* - p_{H,t}^* C_{H,t}^* - p_{F,t}^* C_{F,t}^*\right),$$

where CE_t^* is aggregate consumption expenditure of the Japanese household, λ_t^* is the associated Lagrange multiplier, p_H^* is the Yen price in Japan of US tradables, and p_F^* is the Yen price in Japan of Japanese tradables. Standard optimality conditions imply:

$$C_{NT,t}^* = \frac{\chi_t^*}{\lambda_t^*}; \qquad p_{H,t}^* C_{H,t}^* = \frac{\xi_t}{\lambda_t^*}; \qquad p_{F,t}^* C_{F,t}^* = \frac{a_t^*}{\lambda_t^*}.$$

Our assumption that $Y_{NT,t}^* = \chi_t^*$, combined with the market clearing condition for Japanese non-tradables $Y_{NT,t}^* = C_{NT,t}^*$, implies that in equilibrium $\lambda_t^* = 1$. We obtain:

$$p_{H,t}^* C_{H,t}^* = \xi_t; \qquad p_{F,t}^* C_{F,t}^* = a_t^*.$$

A.2.3 The Euler Equation when there are Several Goods

We state the general Euler equation when there are several goods.

With utility $u^t(\mathsf{C}_t) + \beta u^{t+1}(\mathsf{C}_{t+1})$, where C_t is the vector of goods consumed (for instance, $\mathsf{C}_t = (C_{NT,t}, C_{H,t}, C_{F,t})$ in our setup), if the consumer is at his optimum, we have:

Lemma A.1 *When there are several goods, the Euler equation is:*

$$1 = \mathbb{E}_t \left[\beta R \frac{u_{c_{j,t+1}}^{t+1} / p_{j,t+1}}{u_{c_{i,t}}^t / p_{i,t}} \right] \text{ for all } i, j. \tag{A.10}$$

This should be understood in "nominal" terms, i.e. the return R is in units of the (potentially arbitrary) numéraire.

Proof. It is a variant on the usual one: the consumer can consume $d\varepsilon$ fewer dollars' worth (assuming that the "dollar" is the local unit of account) of good i at time t (hence, consume $dc_{i,t} = -\frac{d\varepsilon}{p_{i,t}}$), invest them at rate R, and consume the proceeds, i.e. $Rd\varepsilon$ more dollars of good j at time $t+1$ (hence, consume $dc_{j,t+1} = \frac{Rd\varepsilon}{p_{j,t+1}}$). The total utility change is:

$$dU = u_{c_{i,t}}^t dc_{i,t} + \beta \mathbb{E}_t u_{c_{j,t+1}}^{t+1} dc_{j,t+1} = \mathbb{E}_t \left(-u_{c_{i,t}}^t / p_{i,t} + \beta R u_{c_{j,t+1}}^{t+1} / p_{j,t+1} \right) d\varepsilon.$$

At the margin, the consumer should be indifferent, so $dU = 0$, hence (A.10). $\square$

Applying this to our setup, with $i = j = NT$, with $p_{NT,t} = 1$ and $u_{c_{NT,t}}^t = \frac{\chi_t}{C_{NT,t}} = 1$ for $t = 0, 1$, we obtain: $1 = \mathbb{E} \left[\beta R \frac{1/1}{1/1} \right]$, hence $R = 1/\beta$.

A.2.4 The Financiers' Demand Function

The financiers' optimization problem. We clarify here the role of the mild assumption, made in footnote 17, that $1 \geq \Omega_0 \geq -1$. Formally, the financiers' optimization problem is:

$$\max_{q_0} V_0 = \Omega_0 q_0, \quad \text{subject to } V_0 \geq \min \left(1, \Gamma \frac{|q_0|}{e_0} \right) |q_0|, \quad \text{where } \Omega_0 \equiv \mathbb{E} \left[1 - \frac{R^*}{R} \frac{e_1}{e_0} \right].$$

Notice that Ω_0 is unaffected by the individual financier's decisions and can be thought of as exogenous here.

Consider the case in which $\Omega_0 > 0$, then the optimal choice of investment has $q_0 \in (0, \infty)$. Notice that $\Omega_0 \leq 1$ trivially. Then one has $V_0 \leq q_0$. In this case, the constraint can be rewritten as: $V_0 \geq \Gamma \frac{q_0^2}{e_0}$, because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma \frac{|q_0|}{e_0}$ reaches 1. This yields the simpler formulation of the constraint adopted in the main text.

Now consider the case in which $\Omega_0 < 0$, then the optimal choice of investment has $q_0 \in (-\infty, 0)$. It is a property of currency excess returns that Ω_0 has no lower bound. In this paper, we assume that the parameters of the model are as such that $\Omega_0 > -1$, i.e. we assume that the worst possible (discounted) expected returns from being long a Dollar bond and being short a Yen bond is -100%. Economically this is an entirely innocuous assumption given that the range of expected excess returns in the data is approximately [-6%,+6%]. With this assumption in hand we have $V_0 \leq |q_0|$, and hence we can once again adopt the simpler

formulation of the constraint because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma\frac{|q_0|}{e_0}$ reaches 1.

The financiers' value function and households' valuation of currency trades. We now analyze, in the context of the basic Gamma model of Section 1, the connection between the financiers' value function, equation (7), and the households' optimal demand for foreign currency in the absence of frictions. If US households were allowed to trade Yen bonds as well as Dollar bonds we would recover the standard Euler equation:

$$0 = \mathbb{E}\left[\beta\frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[\beta\frac{\chi_1/C_{NT,1}}{\chi_0/C_{NT,0}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right],$$

where the last equality follows from the assumption that $C_{NT,t} = \chi_t$ and the result that $\beta R = 1$ derived in the main text (see equation (6)). US households optimally value the currency trade according to its expected (discounted at R) excess returns. Notice that this mean-return criterion holds despite the households being risk averse. The simplification occurs because variations in marginal utility are exactly offset by variations in the relative price of non-tradable goods, so that marginal utility in terms of the numéraire (the NT good) is constant across states of the world.

The reader can verify from our discussion above that, in the absence of frictions, the first order condition for the investment q_t in the basic Gamma model would be: $0 = \mathbb{E}\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right]$, where we have again made use of $\beta R = 1$. We conclude, therefore, that our assumption about the financiers' value function reflects the risk adjusted value of the currency trade to the US households. In the absence of frictions the financiers are a "veil" and choose exactly the same currency trade as the US household would choose. It is the friction $\Gamma > 0$ that makes the financiers' problem interesting in our set-up. As pointed out in the numerical generalization section of this appendix (Section A.4), more general (and non-linear) value functions would apply depending on who the financiers' repatriate their profit and losses to. In the main draft, we maintain the assumption that the financiers' use the US household valuation criterion; this makes the model most tractable while very little economic content is lost. The numerical generalizations in this appendix provide robustness checks by solving the non-linear cases.

The financiers' demand in the extended model. In the main body of the paper, when we consider setups that are more general than the basic Gamma model of Section 1, we maintain the simpler formulation of the financiers' demand function. We do not directly derive the households' valuation of currency trades in these more general setups. Our demand functions are very tractable and carry most of the economic content of more general treatments; we leave it for the extension Section A.4 to characterize numerically financier value functions more complex than those analyzed, in closed form, in the main parts of the paper. We provide here a few details regarding the monetary model. We assume that the financiers solve:

$$\max_{q_0} V_0 = \Omega_0 q_0, \quad \text{subject to } V_0 \geq \min\left(1, \Gamma\frac{|q_0|}{m_0^* e_0}\right)|q_0|, \quad \text{where } \Omega_0 \equiv \mathbb{E}_0\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right].$$

Notice that m_0^* is now scaling the portion of nominal assets that the financiers' can divert to ensure that such fraction is scale invariant to the level of the Japanese money supply and hence the nominal value in Yen of the assets. [70]

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[70]The constraint $\Gamma = \gamma var(e_1)^\alpha$ can become $\Gamma = \gamma var(e_1\frac{m_1^*}{m_1})^\alpha$, to make the model invariant to predictable changes to money supply.

A.2.5 A "Short-Run" Vs "Long-Run" Analysis

As in undergraduate textbooks, it is handy to have a notion of the "long run". We develop here a way to introduce it in our model. We have periods of unequal length: we say that period 0 is short, but period "1" lasts for a length T. The equilibrium flow equations in the dollar-yen market become:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0,$$
$$T\left(\xi_1 e_1 - \iota_1\right) - R Q_0 = 0. \tag{A.11}$$

The reason for the "T" is that the imports and exports will occur over T periods. We assume a zero interest rate "within period 1". This already gives a good notion of the "long run".[71]

Some extra simplicity is obtained by taking the limit $T \to \infty$. The interpretation is that period 1 is "very long" and period 0 is "very short". The flow equation (A.11) can be written: $\xi_1 e_1 - \iota_1 - \frac{R Q_0}{T} = 0$. So in the large T limit we obtain: $\xi_1 e_1 - \iota_1 = 0$. Economically, it means the trades absorbed by the financiers are very small compared to the trades in the goods markets in the long run. We summarize the environment and its solution in the following proposition.[72]

Proposition A.4 *Consider a model with a "long-run" last period. Then, the flow equations become* $\xi_0 e_0 - \iota_0 + Q_0 = 0$ *and* $e_1 - \iota_1 = 0$, *while we still have* $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$. *The exchange rates become:*

$$e_0 = \frac{\frac{R^*}{R}\mathbb{E}\left[\frac{\iota_1}{\xi_1}\right] + \Gamma \iota_0}{1 + \Gamma \xi_0}; \qquad e_1 = \frac{\iota_1}{\xi_1}.$$

In this view, the "long run" is determined by fundamentals $e_1 = \frac{\iota_1}{\xi_1}$, while the "short run" is determined both by fundamentals and financial imperfections (Γ) with short-run considerations (ι_0, ξ_0). In the simple case $R = R^* = \xi_t = 1$, we obtain: $e_0 = \frac{\Gamma \iota_0 + \mathbb{E}[\iota_1]}{\Gamma + 1}$ and $e_1 = \iota_1$.

Application to the carry trade with three periods. In the 3-period carry trade model of Section 5.1, we take period 2 to be the "long run". We assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. That allows us to analyze more clearly the dynamic environment. Without the "long-run" period 2, the expressions are less intelligible, but the economics is the same.

A.2.6 Price Indices, Nominal and Real Exchange Rates

We report here a few details omitted for brevity in Section 5.2.

Let us first derive the price indices $\{P, P^*\}$. The US price index P is defined as the minimum cost, in units of the numéraire (money), of obtaining one unit of the consumption basket:

$$C_t \equiv \left[\left(\frac{M_t}{P_t}\right)^{\omega_t}(C_{NT,t})^{\chi_t}(C_{H,t})^{a_t}(C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}.$$

Let us define a "primed" variable as being normalized by the sum of the preference coefficients θ_t; so that, for example, $\chi_t' \equiv \frac{\chi_t}{\theta_t}$. Substituting the optimal demand for goods (see the first order conditions at the

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[71]The solution is simply obtained by Proposition 3, setting $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$.

[72]One derivation is as follows. Take Proposition 3, set $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$, and take the limit $T \to \infty$.

beginning of Section 5.2) in the consumption basket formula we have:

$$1 = \left(\omega' P \right)^{\omega'} \left(a' \frac{P}{p_H} \right)^{a'} \left(\iota' \frac{P}{p_F} \right)^{\iota'} \left(\chi' \frac{P}{p_{NT}} \right)^{\chi'}.$$

Hence:

$$P = (p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'} \left[(\omega_t')^{-\omega_t'} (\iota_t')^{-\iota_t'} (a_t')^{-a_t'} (\chi_t')^{-\chi_t'} \right].$$

The part in square brackets is a residual and not so interesting. Similarly for Japan, we have:

$$P^* = (p_H^*)^{\xi'} (p_F^*)^{a^{*'}} (p_{NT}^*)^{\chi^{*'}} \left[(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t')^{-\xi_t'} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}} \right].$$

The CPI-RER in equation (29) is then obtained by substituting the price indices above in the definition of the real exchange rate $\mathscr{E} \equiv \frac{P^* e}{P}$. For completeness, we report below the full expression for the function $\tilde{\theta}$ that enters in equation (29):

$$\tilde{\theta}_t = \frac{(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t')^{-\xi_t'} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}}}{(\omega_t')^{-\omega_t'} (\iota_t')^{-\iota_t'} (a_t')^{-a_t'} (\chi_t')^{-\chi_t'}}.$$

The Basic Complete Market Model   The main text of the paper illustrated the relationship between the CPI-RER and the definition of the real exchange rate in the basic Gamma model. For completeness we include here a similar analysis for the case of complete and frictionless markets. We maintain all the assumptions from the paragraph on the Basic Gamma model in Section 5.2, except that we now assume markets to be complete and frictionless. Recall from Lemma 3 that we then obtain $e_t = v$. Hence, the CPI-RER now follows: $\mathscr{E} = \tilde{\theta} \, (p_H)^{\xi' - a'} (p_F)^{a^{*'} - \iota'} v^{\chi^{*'}}$. Notice that while the real exchange rate (e) is constant in complete markets in the basic Gamma model, the CPI-RER will in general not be constant as long as the CPI baskets are not symmetric and relative prices of goods move.

A.2.7   The Backus and Smith Condition

In the spirit of re-deriving some classic results of international macroeconomics with the Gamma model, let us analyze the Backus and Smith condition (Backus and Smith (1993)). Let us first consider the basic Gamma set-up but with the additional assumption of complete markets as in Lemma 3. Then by equating margin utility growth in the two countries and converting, via the exchange rate, in the same units, we have: $\frac{P_0 C_0 / \theta_0}{P_1 C_1 / \theta_1} = \frac{P_0^* C_0^* / \theta_0^*}{P_1^* C_1^* / \theta_1^*} \frac{e_0}{e_1}$. Re-arranging we conclude:

$$\frac{C_0 / \theta_0}{C_1 / \theta_1} = \frac{C_0^* / \theta_0^*}{C_1^* / \theta_1^*} \frac{\mathscr{E}_0}{\mathscr{E}_1}, \tag{A.12}$$

where the reader should recall the definition $\mathscr{E} = \frac{P^* e}{P}$. This is the Backus and Smith condition in our set-up under complete markets: the perfect risk sharing benchmark equation.

Of course, this condition fails in the basic Gamma model because agents not only cannot trade all Arrow-Debreu claims, but also have to trade with financiers in the presence of limited commitment problems. In our framework (Section 1), however, an extended version of this condition holds:

$$\frac{C_0 / \theta_0}{C_1 / \theta_1} = \frac{C_0^* / \theta_0^*}{C_1^* / \theta_1^*} \frac{\mathscr{E}_0}{\mathscr{E}_1} \frac{e_1}{e_0}. \tag{A.13}$$

The simple derivation of this result is reported below. The above equation is the extended Backus-Smith

condition that holds in our Gamma model. Notice that our condition in equation (A.13) differs from the standard Backus-Smith condition in equation (A.12) by the growth rate of the "nominal" exchange rate $\frac{e_1}{e_0}$. Since exchange rates are much more volatile in the data than consumption, this omitted term creates an ample wedge between the complete market and the Gamma version of the Backus-Smith condition.

The condition in equation (A.12) can be verified as follows:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*} \frac{\mathscr{E}_0}{\mathscr{E}_1} \frac{e_1}{e_0} \iff \frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*} \iff \frac{1}{1} = \frac{1}{1},$$

where the first equivalence simply makes use of the definition $\mathscr{E} \equiv \frac{P^* e}{P}$, and the second equivalence follows from $P_t C_t = \theta_t$ and $P_t^* C_t^* = \theta_t^*$ for $t = 0, 1$. These latter equalities (we focus here on the US case) can be recovered by substituting the households' demand functions for goods in the static household budget constraint: $P_t C_t = C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} = \chi_t + a_t + \iota_t = \theta_t$.

A.3 Model Extensions: Multi-Country, Multi-Asset Model, and Additional Material on the Variance in the Constraint

We provide below generalizations of the model. In particular, we develop a multi-asset, multi-country model.

A.3.1 Verification of the tractability of the model when the variance is in the constraint

In the paper, we propose a formulation of $\Gamma = \gamma var(e_1)^\alpha$. We verify that it leads to a tractable model in the core parts of the paper. In this subsection of the appendix, we check that we also keep a tractable model in other important cases.

Infinite horizon model We verify that the infinite horizon model in A.1 is also tractable with our endogenous Γ given in (1). Suppose that ι_t follows an AR(1): $\iota_t - 1 = \rho(\iota_{t-1} - 1) + \varepsilon_{\iota t}$. An innovation $\varepsilon_{\iota t}$ to import creates an innovation to ι_{t+s} equal to $\rho^s \varepsilon_{\iota t}$, hence an innovation to the exchange rate equal to

$$\{e_t\} = \sum_{i \geq 0} \Lambda^i (1-\Lambda) \rho^i \varepsilon_{\iota t} = \frac{1-\Lambda}{1-\rho\Lambda} \varepsilon_{\iota t},$$

implying that $var_t(e_t) = \left(\frac{1-\Lambda}{1-\rho\Lambda}\right)^2 \sigma_\iota^2$. Hence,

$$\Gamma = \gamma \left(\frac{1-\Lambda}{1-\rho\Lambda}\right)^{2\alpha} \sigma_\iota^{2\alpha}. \tag{A.14}$$

Next, we need to solve for the endogenous Λ. Recall that $\Lambda^2 - (1+R+\Gamma)\Lambda + R = 0$. This means:

$$(1-\Lambda)(R-\Lambda) = \Lambda\Gamma = \gamma\Lambda \left(\frac{1-\Lambda}{1-\rho\Lambda}\right)^{2\alpha} \sigma_\iota^{2\alpha}.$$

Hence,

$$R - \Lambda = \gamma \Lambda \frac{(1-\Lambda)^{2\alpha-1}}{(1-\rho\Lambda)^{2\alpha}} \sigma_\iota^{2\alpha}. \tag{A.15}$$

When $\alpha = 1$, we obtain a cubic polynomial in the equation for Λ.

T **period model**   When ξ_t is deterministic, the formulation remains tractable. We obtain each Γ_t in closed form.[73] Let us work out explicitly a 3-period example. We take $\xi_t = R = R^* = 1$ for simplicity. The equations are:

$$e_0 - \iota_0 + Q_0 = 0,$$
$$e_1 - \iota_1 - Q_0 + Q_1 = 0,$$
$$e_2 - \iota_2 - Q_1 = 0,$$
$$Q_t = \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right]}{\Gamma_t} \text{for } t = 0,1,$$
$$\Gamma_t = \gamma \, var_t \left(e_{t+1}\right)^\alpha.$$

Notice that the model at $t = 1, 2$: is like the basic model with 2 periods, except for the pseudo-import term $\widetilde{\iota}_1 = \iota_1 - Q_1$. Hence, we have $\{e_2\} = \{\iota_2\}$, and

$$\Gamma_1 = \gamma \sigma_{\iota_2}^{2\alpha}. \tag{A.16}$$

This also implies that (by Proposition 3 applied to (e_1, e_2) rather than (e_0, e_1)): $\{e_1\} = \frac{1+\Gamma_1}{2+\Gamma_1}\{\iota_1\}$, which gives:

$$\Gamma_0 = \gamma \left(\frac{1+\Gamma_1}{2+\Gamma_1}\sigma_{\iota_1}\right)^{2\alpha}, \tag{A.17}$$

so we endogenize Γ_0. Note that the σ_{ι_1} is, in general, the variance of pseudo-imports, hence it would include the volatility due to financial flows. Notice also that fundamental variance is endogenously amplified by the imperfect financial market: $var(e_1)$ depends positively on Γ_1, that itself depends positively on fundamental variance.

The same idea and procedure applies to an arbitrary number of periods, and indeed to the infinite period model. We could also have correlated innovations in ι_t.

A.3.2   A tractable multi-country model

A.3.2.1   General formulation

We call e_i^t the exchange rate of country i at date t, with a high e_i^t being an appreciation of country i's currency versus the USD. There is a central country 0, for which we normalize $e_0^t = 1$ at all dates t. As a short hand, we call this country "the US". For $i \neq j$, call $\xi_{ij} < 0$ exports of country i to country j (*minus* the Cobb-Douglas weight), and $x_i = -\xi_{i0} > 0$ exports of country i to country 0. Define the import weight as:

$$\xi_{ii} \equiv - \sum_{j=0,\dots,n,j\neq i} \xi_{ji} > 0,$$

---

[73]However, when ξ_t is stochastic, the formulation is more complex. We obtain a fixed point problem not just in Γ_0 (like in the 2-period model), but in $(\Gamma_0,\dots,\Gamma_{T-1})$.

so that ξ_{ii} equals total imports of country i. Call θ_i the holdings of country i's bonds by financiers, expressed in number of bonds: so, the dollar value of those bond holdings is $q_i \equiv \theta_i e_i^0$.

Hence, the net demand for currency i in the currency i / USD spot market, expressed in dollars, is:

$$-\sum_{j \neq 0} \xi_{ij}^0 e_j^0 + x_i^0 + \theta_i e_i^0, = 0, \tag{A.18}$$

and has to be 0 in market equilibrium. Indeed, at time 0 the country imports a dollar value $\xi_{ii}^0 e_i^0$, creating a negative demand $-\xi_{ii}^0 e_i^0$ for the currency. It also exports a dollar value $-\sum_{j \neq 0} \xi_{ij}^0 e_j^0 + x_i^0$ (recall that $\xi_{ij} < 0$ for $i \neq j$); as those exports are repatriated, they lead to a demand for the currency. Finally, financiers demand a dollar value $\theta_i e_i^0$ of the country's bonds. Using $q_i \equiv \theta_i e_i^0$, equation (A.18) can be rewritten in vector form:

$$-\xi^0 e^0 + x^0 - q = 0. \tag{A.19}$$

The flow equation at time $t = 1$ is (again, net demand for currency i in the dollar-currency i market, expressed in dollars):

$$-\sum_{j \neq 0} \xi_{ij}^1 e_j^1 + x_i^1 - \theta_i e_i^1 + \Pi_i = 0 \tag{A.20}$$

where Π_i is the time-1 rebate of financiers profits to country i. In the first equation, imports enter as $-\xi_{ii} e_i^0 < 0$, creating a net negative demand for currency i, and exports to other countries enter as $-\sum_{j \neq 0, i} \xi_{ij} e_j^0 > 0$.

Total financiers' profit is: $\Pi \equiv \sum_i \Pi_i = \sum_i \theta_i \left(e_i^1 - e_i^0 \right)$. We posit the following rule for the rebate Π_i to country i: $\Pi_i = \theta_i \left(e_i^1 - e_i^0 \right)$. Then, (A.20) becomes: $-\sum_{j \neq 0} \xi_{ij}^1 e_j^1 + x_i^1 - \theta_i e_i^1 = 0$, i.e., in vector form:

$$-\xi^1 e^1 + x^1 - q = 0. \tag{A.21}$$

Finally, we will have the generalized demand for assets:

$$q = \Gamma^{-1} \mathbb{E} \left[e^1 - e^0 \right], \tag{A.22}$$

where q, e^t are vectors, and Γ is a matrix. We provide foundations for this demand in section A.3.3. The financiers buy a dollar value q_i of country i's bonds at time 0, and $-\sum_{i=1}^n q_i$ dollar bonds, so that the net time-0 value of their initial position is 0. The correspondence with the basic Gamma model (with only 2 countries) is $q = -Q$, $x_{it} = \iota_{it}$.

We summarize the set-up below.

Lemma A.2 *In the extended n-country model, the basic equations describing the vectors of exchange rates e^t are:*

$$\xi^0 e^0 - x^0 - q = 0, \tag{A.23}$$

$$\xi^1 e^1 - x^1 + q = 0, \tag{A.24}$$

$$\mathbb{E} \left[e^1 - e^0 \right] = \Gamma q. \tag{A.25}$$

Those are exactly the equations of the $2-$country model (with $Q^{\text{Gamma}} = -q^{\text{here}}$), and $\iota^{\text{Gamma}} = x^{\text{here}}$, but with n countries (so $e^t \in \mathbb{R}^{n-1}$). Hence the solution is the same (using matrices). We assume that ξ^1 is deterministic.

Proposition A.5 *The exchange rates in the n−country model are given by the following vectors:*

$$e^0 = \left(\xi^0 + \xi^1 + \xi^1 \Gamma \xi^0\right)^{-1} \left((1 + \xi^1 \Gamma) x^0 + \mathbb{E}\left[x^1\right]\right), \tag{A.26}$$

$$e^1 = \left(\xi^0 + \xi^1 + \xi^0 \Gamma \xi^1\right)^{-1} \left(x^0 + (1 + \xi^0 \Gamma) \mathbb{E}\left[x^1\right]\right) + \left(\xi^1\right)^{-1} \left\{x^1\right\}. \tag{A.27}$$

Hence, the above model has networks of trade in goods, and multi-country asset demand.

A.3.2.2 Case of a star network

The model becomes particularly transparent (at little cost for the economics) in the case

$$\xi^t = I, \quad \text{where } I \text{ is the identity matrix.}$$

i.e. country i always imports 1 (in country i's domestic currency units) from country 0, nothing from the other countries, and exports x_i^t to country 0 (in country 0's currency units). This is usually called a "star" network. Transposition of our earlier results give the following.

Lemma A.3 *Assume that $R_i = 1$ for all i. The exchange rate vectors e^t are characterized by the following equations:*

$$e^0 - x^0 - q = 0,$$
$$e^1 - x^1 + q = 0,$$
$$\mathbb{E}\left[e^1 - e^0\right] = \Gamma q.$$

Proposition A.6 *(Basic exchange rates with n countries) In the n-country model with a start network (with $R_i = 1$ for all i), the (vectors of) exchange rates are:*

$$e^0 = (2 + \Gamma)^{-1} \left((1 + \Gamma) x^0 + \mathbb{E}\left[x^1\right]\right), \tag{A.28}$$

$$e^1 = (2 + \Gamma)^{-1} \left(x^0 + (1 + \Gamma) \mathbb{E}\left[x^1\right]\right) + \left\{x^1\right\}, \tag{A.29}$$

or, defining the matrix A as

$$A = (2 + \Gamma)^{-1}, \tag{A.30}$$

we have:

$$e^0 = (1 - A) x^0 + A \mathbb{E}\left[x^1\right], \tag{A.31}$$

$$e^1 = A x^0 + (1 - A) \mathbb{E}\left[x^1\right] + \left\{x^1\right\}. \tag{A.32}$$

Note that, here, the exchange rates are vectors. We lighten up the notation by using 1 rather than I, the identity matrix of the correct size, $(n-1) \times (n-1)$. Using the I notation, we can write the basic equations as

$$e^0 = (2I + \Gamma)^{-1} \left((I + \Gamma) x^0 + \mathbb{E}\left[x^1\right]\right),$$

and $e^0 = (I - A) x^0 + A \mathbb{E}\left[x^1\right]$.

Explicit calculations. Suppose that

$$\Gamma = \begin{pmatrix} \gamma & \rho \\ \rho & \gamma \end{pmatrix}.$$

where $|\rho| \leq \gamma$. This captures the fact that the exchange rate innovations are correlated. The case $\rho = 0$ corresponds to entirely decoupled countries. That implies, for $A = (2 + \Gamma)^{-1}$,

$$A = \begin{pmatrix} \alpha & -\beta \\ -\beta & \alpha \end{pmatrix}, \tag{A.33}$$

where

$$\alpha = \frac{2 + \gamma}{(2 + \gamma)^2 - \rho^2}; \qquad \beta = \frac{\rho}{(2 + \gamma)^2 - \rho^2}, \tag{A.34}$$

$$0 \leq \alpha \leq \frac{1}{2}; \qquad |\beta| \leq \alpha; \qquad \alpha + \beta \leq \frac{1}{2}; \qquad sign(\beta) = sign(\rho). \tag{A.35}$$

It is more convenient to do calculations in terms of α, β rather than Γ. When $\gamma = 0$, $(\alpha, \beta) = \left(\frac{1}{2}, 0\right)$. When $\gamma \to \infty$, with $\rho = \overline{\rho}\gamma$ for a constant $\overline{\rho}$, $(\alpha, \beta) \sim \frac{1}{\gamma(1-\overline{\rho})^2}(1, \overline{\rho})$. α and $|\beta|$ are decreasing in γ and increasing in $|\rho|$.

Hence, using (A.31), we have that

Proposition A.7 *The exchange rates of country i are*

$$e_i^0 = (1 - \alpha)x_i^0 + \alpha \mathbb{E}x_i^1 + \beta \left(x_{-i}^0 - \mathbb{E}x_{-i}^1\right), \tag{A.36}$$
$$e_i^1 = \alpha x_i^0 + (1 - \alpha)\mathbb{E}x_i^1 - \beta \left(x_{-i}^0 - \mathbb{E}x_{-i}^1\right) + \left\{x_i^1\right\}. \tag{A.37}$$

where x_i^t are exports of country i at time t, and x_{-i}^t are exports of the other country at time t.

Contagion in the $n-$country model We obtain the intuition of the basic model, but now with "contagion".

Proposition A.8 *(Contagion via finance) Suppose that $\rho > 0$, i.e. the financiers' demands for the two countries are correlated. Suppose that all $x_i^t = 1$, except $x_1^0 < 1$: country 1 has a negative export shock at time 0. That naturally devalues country 1's currency. Because of imperfect finance, that also devalues country 2's currency: $e_1^0 < e_2^0 < 1$. However, country 1's currency is also devalued at time 1, while country 2's currency bounces back: $e_1^1 < 1 < e_2^1$.*

Proof: Using (A.36)-(A.37),

$$e_1^0 = (1 - \alpha)x_1^0 + \alpha,$$
$$\mathbb{E}e_1^1 = \alpha x_1^0 + 1 - \alpha,$$
$$e_2^0 = 1 + \beta \left(x_1^0 - 1\right),$$
$$\mathbb{E}e_2^1 = 1 - \beta \left(x_1^0 - 1\right),$$

hence, calling $x_1^0 = 1 - y$ for $y > 0$,

$$e_1^0 = 1 - (1 - \alpha)y,$$
$$\mathbb{E}e_1^1 = 1 - \alpha y,$$
$$e_2^0 = 1 - \beta y,$$
$$\mathbb{E}e_2^1 = 1 + \beta y,$$

A.14

$$\mathbb{E}\left[\Delta e_1\right] = (1 - 2\alpha)y,$$
$$\mathbb{E}\left[\Delta e_2\right] = 2\beta y,$$

hence we have that (using $0 < \beta < \alpha < \frac{1}{2}$),

$$e_1^0 < \mathbb{E}e_1^1 < 1,$$

that is, country 1's currency is permanently depreciated, more at time 0 and less at time 1. We also have that

$$e_2^0 < 1 < \mathbb{E}e_2^1,$$

country 2's currency is depreciated at time 0, and appreciated at time 1. Furthermore,

$$e_1^0 < e_2^0,$$

the depreciation of country 1's currency is larger than that of country 2's currency. Finally,

$$0 < \mathbb{E}\left[\Delta e_2\right] < \mathbb{E}\left[\Delta e_1\right].$$

both countries' currencies appreciate in the second period, with country 1's currency appreciating more. $\square$

Here we see how real (or financial) shocks can create contagion.

A.3.3   Foundation of the multi-asset, multi-country demand

We derive the financiers' demand function in a multi-asset case. We start with a general asset case, and then specialize our results to exchange rates.

A.3.3.1   General asset pricing case

Basic case   We use notations that are valid in general asset pricing, as this makes the exposition clearer and more general. We suppose that there are assets $a = 1, \ldots, A$, with initial price p^0, and period 1 payoffs p^1 (all in $\mathbb{R}^A$). Suppose that the financiers hold a quantity position $\theta \in \mathbb{R}^A$ of those assets, so that the terminal value is $\theta \cdot p^1$. We want to compute the equilibrium price at time 0.

Let

$$\pi = \mathbb{E}\left[p^1\right] - p^0,$$

denote the expected gain (a vector), and

$$V = var\left(p^1\right),$$

denote the variance-covariance matrix of period 1 payoffs.

Given a matrix G, our demand will generate the relation

$$\pi = G\theta^*, \tag{A.38}$$

This is a generalization to an arbitrary number of assets of the basic demand of Lemma 2, $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$. The traditional mean-variance case is $G = \gamma V$. The present machinery yields more general terms: for example, we could have $G = VH'$, for a "twist" matrix H. The mean-variance case is $H = \gamma I_n$, for a risk-aversion scalar γ. The H can, however, represent deviations from that benchmark, e.g. source-dependent risk aversion (if $H = diag\left(\gamma_1, \ldots, \gamma_A\right)$, we have a "risk aversion" scalar γ_a for source a), or tractability-inducing twists (our main application here). Hence, the machinery we develop here will allow to go beyond the traditional mean-variance setup.

The financiers' profits (in dollars) are: $\theta \cdot \left(p^1 - p^0\right)$, and their expected value is $\theta'\pi$, where $\pi :=$ $\mathbb{E}\left[p^1\right] - p^0$. We posit that financiers solve:

$$\max_{\theta \in R^A} \theta'\pi \text{ s.t. } \theta'\pi \geq \theta'S\theta,$$

where S is a symmetric, positive semi-definite matrix. This a limited commitment constraint: the financiers' outside option is $\theta'S\theta$. Hence, the incentive-compatibility condition is $\theta'\pi \geq \theta'S\theta$. Again, this is a generalization (to an arbitrary number of assets) of the constraint in the paper in Equation (8).

The problem implies:

$$\pi = S\theta^*,$$

where θ^* is the equilibrium θ.[74]

Hence, we would deliver (A.38) if we could posit $S = G$. However, this is not exactly possible, because S must be symmetric, and G is not necessarily symmetric.

We posit that the outside option $\theta'S\theta$ equals:[75]

$$\theta'S\theta \equiv \sum_{i,j} \theta_i^2 \frac{1_{\theta_i^* \neq 0}}{\theta_i^*} G_{ij}\theta_j^*, \tag{A.39}$$

where θ is chosen by the financier under consideration, and θ^* is the equilibrium demand of *other* financiers (in equilibrium, $\theta = \theta^*$). This functional form captures the fact that as the portfolio or balance sheet expands (θ_i high), it is "more complex" and the outside option of the financiers increases. In addition, (if say $G = \gamma V$), it captures that high variance assets tighten the constraint more (perhaps again because they are more "complex" to monitor). The non-diagonal terms indicate that "similar" assets (as measured by covariance) matter. Finally, the position of other financiers matter. Mostly, this assumption is made for convenience. However, it captures the idea (related to Basak and Pavlova (2013)) that the positions of other traders influence the portfolio choice of a given trader. The influence here is mild: when G is diagonal, there is no influence at all.

We will make the assumption that

$$\forall i, \; sign\left(\pi_i\right) = sign\left(\theta_i^*\right), \text{ where } \pi^* \equiv G\theta^*. \tag{A.40}$$

This implies that S is a positive semi-definite matrix: for instance, when $\theta_i^* \neq 0$, $\sum_j \frac{1}{\theta_i^*}G_{ij}\theta_j^* \geq 0$. Equation (A.40) means that the sign of the position θ_i^* is equal to the sign of the expected return π_i. This is a mild assumption, that rules out situations where hedging terms are very large.

We summarize the previous results. Recall that we assume (A.40).

Proposition A.9 *(General asset pricing case: foundation for the financiers' demand) With the above microfoundation, the financiers' equilibrium holding θ^* satisfies:*

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta^*, \tag{A.41}$$

with G a matrix. When G is invertible, we obtain the demand $\theta^* = G^{-1}\mathbb{E}\left[p^1 - p^0\right]$.

Proof: First, take the case $\theta_i^* \neq 0$. Deriving (A.39) w.r.t. θ_i: $2\left(S\theta\right)_i = \sum_j \frac{2\theta_i}{\theta_i^*}G_{ij}\theta_j^*$, so that $\left(S\theta^*\right)_i = \sum_j G_{ij}\theta_j^* = \left(G\theta^*\right)_i$. When $\theta_i^* = 0$, assumption (A.40) implies again $\left(S\theta^*\right)_i = \sum_j S_{ij}\theta_j^* = 0 = \pi_i^* = \left(G\theta^*\right)_i$.

---

[74]The proof is as follows. Set up the Lagrangian $\mathscr{L} = \theta'\pi + \lambda\left(\theta'\pi - \theta'S\theta\right)$. The first-order condition reads $0 = \mathscr{L}_{\theta'} = (1+\lambda)\pi - 2\lambda S\theta$. So, $\pi = \frac{2\lambda}{1+\lambda}S\theta$. Left-multiplying by θ' yields $\theta'\pi = \frac{2\lambda}{1+\lambda}\theta'S\theta$. Since $\theta'\pi \geq \theta'S\theta$, we need $\lambda \geq 1$. Hence, $\pi = S\theta$.

[75]This is, $S_{ij} = 1_{i=j}\frac{1}{\theta_i^*}G_{ij}\theta_j^*$ if $\theta_i^* \neq 0$, $S_{ij} = 0$ if $\theta_i^* = 0$.

Thus, $S\theta^* = G\theta^*$. Hence, the foundation induces $\pi = S\theta^* = G\theta^*$. $\square$

Proposition A.10 *Suppose that we can write $G = VH'$, for some matrix H. Then, a riskless portfolio simply offers the riskless US return, and in that sense the model is arbitrage-free.*

Proof: Suppose that you have a riskless, 0-investment portfolio κ: $\kappa'V = 0$. Given $\pi = VH'\theta$, we have $\kappa'\pi = \kappa'G\theta = \kappa'VH'\theta^* = 0$, i.e. the portfolio has 0 expected return, hence, as it is riskless, the portfolio has 0 return. $\square$

Proposition A.12 offers a stronger statement that the model is arbitrage-free.

A.3.3.2 Extension with derivatives and other redundant assets

The reader may wish to initially skip the following extension. When there are redundant assets (like derivatives), some care needs to be taken when handling indeterminacies (as many portfolios are functionally equivalent). Call Θ the full portfolio, including redundant assets, and P^t the full price vector. We say that assets $a \le B$ are a basis, and we reduce the portfolio Θ into its "basis-equivalent" portfolio in the basis, $\theta \in \mathbb{R}^B$, with price p^t, defined by:

$$\Theta \cdot P^1 = \theta \cdot p^1, \quad \text{for all states of the world.}$$

For instance, if asset c is redundant and equal to asset a minus asset b ($p_c^1 = p_a^1 - p_b^1$), then $(\theta_a, \theta_b) = (\Theta_a + \Theta_c, \Theta_b - \Theta_c)$.

More generally, partition the full portfolio into basis assets Θ_B and derivative assets Θ_D, $\Theta = (\Theta_B, \Theta_D)$, and similarly partition prices in $P = (p, p_D)$. As those assets are redundant, there is a matrix Z such that

$$p_D^1 = Zp^1.$$

Then the basis-equivalent portfolio $\theta = \Theta_B + Z'\Theta_D$.[76]

Then, we proceed as above, with the "basis-equivalent portfolio". This gives the equilibrium pricing of the basis assets, p_B^0. Then, derivatives are priced by arbitrage:

$$p_D^0 = Zp^0,$$

A.3.3.3 Formulation with a Stochastic Discount Factor

The following section is more advanced, and may be skipped by the reader.

It is often useful to represent pricing via a Stochastic Discount Factor (SDF). Let us see how to do that here. Call $w = P_1'\Theta = p_1'\theta$ the time-1 wealth of the financiers. Recall that we have $\pi = G\theta$, with $G = VH'$.

If we had traditional mean-variance preferences, with $\pi = \gamma V\theta$, we could use a SDF: $M = 1 - \gamma\{w\}$, for a scalar γ. We want to generalize that idea.

As before, we define

$$\{X\} \equiv X - \mathbb{E}[X],$$

to be the innovation to a random variable X.

Recall that we are given B basis assets $a = 1, \dots, B$ (i.e., $\left(\{p_a^1\}\right)_{a=1,\dots,B}$ are linearly independent), while assets $a = B+1, \dots, A$ are derivatives (e.g. forward contracts), and so their payoffs are spanned by the vector $\left(p_a^1\right)_{a \le B}$.

---

[76]Proof: the payoffs are $\Theta'P^1 = \Theta_B'p^1 + \Theta_D'p_D^1 = \Theta_B'p^1 + \Theta_D'Zp_B^1 = \theta'p_B^1$ with $\theta' = \Theta_B' + \Theta_D'Z$.

Next, we choose a twist operator Ψ for the basis assets, that maps random variables into random variables.[77] It is characterized by:

$$\Psi\left\{p_a^1\right\} = \sum_b H_{ab}\left\{p_b^1\right\} \text{ for } a = 1,\ldots,B,$$

or, more compactly:

$$\Psi\left\{p^1\right\} = H\left\{p^1\right\}.$$

This is possible because the $\{p_1^a\}$ are linearly independent. The operator extends to the whole space S of traded assets (including redundant assets).

Proposition A.11 *The pricing is given by the SDF:*

$$M = 1 - \Psi\{w\}, \tag{A.42}$$

where $w = P_1'\Theta = p_1'\theta$ is the time-1 wealth of the financiers.

Proposition A.12 *If the shocks $\{p^1\}$ are bounded and the norm of matrix H, $\|H\|$, is small enough, then $M > 0$ and the model is arbitrage-free.*

In addition, it shows that the SDF depends linearly on the agents' total terminal wealth w, including their proceeds from positions in derivatives.

Proof. We need to check that this SDF generates: $p^0 = \mathbb{E}\left[Mp^1\right]$. Letting $M = 1 - m$ with $m = \Psi\{w\}$, we need to check that $p^0 = \mathbb{E}\left[p^1 - mp^1\right] = \mathbb{E}\left[p^1\right] - \mathbb{E}\left[mp^1\right]$, i.e. $\pi := \mathbb{E}\left[p^1 - p^0\right] = \mathbb{E}\left[mp^1\right]$. Recall that we have $\pi = G\theta = VH'\theta$.

Hence, we compute:

$$\begin{aligned}
\mathbb{E}\left[mp_a^1\right] &= \mathbb{E}\left[p_a^1(\Psi\{w\})\right] \\
&= \mathbb{E}\left[p_a^1 \sum_{b,c} \theta_c H_{cb}\left\{p_b^1\right\}\right] = \sum_{b,c} \mathbb{E}\left[p_a^1\left\{p_b^1\right\}\right] H_{cb}\theta_c \\
&= \sum_{b,c} V_{ab}\left(H'\right)_{bc}\theta_c = \left(VH'\theta\right)_a
\end{aligned}$$

i.e., indeed, $\mathbb{E}\left[mp^1\right] = VH'\theta = \pi$. $\square$

A.3.3.4   Application to the FX case in multi-country set-up

We now specialize the previous machinery to the FX case. In equilibrium we will indeed have (with $q = (q_i)_{i=1,\ldots,n}$):

$$q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right], \tag{A.43}$$

and $q_0 = -\sum_{i=1}^n q_i$ ensures $\sum_{i=0}^n q_i = 0$. We endogenize this demand, with

$$\Gamma = \gamma V^\alpha, \tag{A.44}$$

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[77]Mathematically, call S the space of random payoffs spanned by (linear combinations of) the traded assets, $\left(p_a^1\right)_{a=1,\ldots,B}$. S is a subset of $L^2(\Omega)$, where Ω is the underlying probability space. $\Psi : S \to S$ is an operator from S to S, while H is a $B \times B$ matrix.

where $V = var\left(e^1\right)$, and $var\left(x\right) = \mathbb{E}\left[x'x\right] - \mathbb{E}\left[x\right]\mathbb{E}\left[x'\right]$ is the variance-covariance matrix of a random vector x. Note that because of (A.29), $var\left(e^1\right) = var\left(x^1\right)$ is independent of e^0. Hence, with this endogenous demand, we have a model that depends on variance, is arbitrage free, and (we believe) sensible.

Let us see how the general asset pricing case applies to the FX case. The basis assets are the currencies, with $p^t = e^t$, θ_a is the position in currency a, and $q_a = \theta_a e_a^0$ is the initial dollar value of the position. The position held in dollars is q_0 (and we still have $e_0^t = 1$ as a normalization). We define

$$D = diag\left(e^0\right), \tag{A.45}$$

so that $q = D\theta$. We take the G matrix to be

$$G = \gamma V^\alpha D, \tag{A.46}$$

for scalars $\gamma > 0$ and $\alpha \geq 0$. Recall that $V = var\left(e^1\right)$ is a matrix. The reader is encouraged to consider the leading case where $\alpha = 1$. In general, V^α is the variance-covariance matrix to the power α: if we write $V = U'\Lambda U$ for U an orthogonal matrix and $\Lambda = diag\left(\lambda_i\right)$ a diagonal matrix, $V^\alpha = U'diag\left(\lambda_i^\alpha\right)U$.

Proposition A.13 *(FX case: Foundation for the financiers' demand (A.22)) With the above foundation, the financiers' equilibrium holding q satisfies:*

$$\mathbb{E}\left[e^1 - e^0\right] = \Gamma q, \tag{A.47}$$

with

$$\Gamma = \gamma V^\alpha,$$

where $\gamma > 0$ and $\alpha \geq 0$ are real numbers, and $V = var\left(e^1\right)$ is the variance-covariance matrix of exchange rates. In other terms, when Γ is invertible, we obtain the Gamma demand (A.22), $q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right]$.

Proof. This is a simple correlate of Proposition A.9. This Proposition yields

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta,$$

Using $p^t = e^t$, $\Gamma \equiv \gamma V^\alpha$, $G \equiv \Gamma D$, $q = D\theta$, we obtain

$$\mathbb{E}\left[e^1 - e^0\right] = G\theta = \Gamma D\theta = \Gamma q.$$

□

It may be useful to check the logic by inspecting what this yields in the Basic Gamma model. There, the outside option of the financiers is given by (A.39) (using $\theta = -q/e_0$, since in the basic Gamma model the dollar value of the yen position is $-q$)

$$\theta'S\theta = \gamma\theta^2 var\left(e_1\right)^\alpha e_0 = \gamma var\left(e_1\right)^\alpha \frac{q^2}{e_0}.$$

The financiers' maximization problem is thus:

$$\max_q V_0 \text{ where } V_0 := \mathbb{E}\left[1 - \frac{e_1}{e_0}\right]q,$$

$$\text{s.t. } V_0 \geq \gamma\,var\left(e_1\right)^\alpha \frac{q^2}{e_0},$$

i.e., the divertable fraction is $\gamma var(e_1)^\alpha \frac{q}{e_0}$. It is increasing in q and the variance of the trade (a "complexity" effect).

The constraint binds, and we obtain:

$$\mathbb{E}\left[1 - \frac{e_1}{e_0}\right] q = \gamma var(e_1)^\alpha \frac{q^2}{e_0},$$

or,

$$\mathbb{E}\left[e_0 - e_1\right] = \gamma var(e_1)^\alpha q, \tag{A.48}$$

which confirms the intuitive properties of this foundation.

A.3.3.5 Application to the CIP and UIP trades

Suppose that the assets are: dollar bonds paying at time 1, yen bonds paying at time 1 (so that their payoff is e_1), and yen futures that pay $e_1 - F$ at time 1, where F is the futures' price. The payoffs (expressed in dollars) are:

$$P_1 = (1, e_1, e_1 - F)',$$

and the equilibrium time-0 price is:

$$P_0 = (1, e_0, 0)',$$

as a futures position requires 0 initial investment.

Suppose that financiers undertake the CIP trade, i.e. they hold a position:

$$\Theta^{CIP} = (e_0, -1, 1)',$$

where they are long the dollar, short the yen, and long the future. To review elementary notions in this language, the initial price is $\Theta^{CIP} \cdot P^0 = 0$. The terminal payoff is $\Theta^{CIP} \cdot P^1 = e_0 - F$, hence, by no arbitrage, we should have $F = e_0$.

The financiers can also engage in the UIP trade; in the elementary UIP trade they are long 1 dollar, and short the corresponding yen amount:

$$\Theta^{UIP} = \left(1, \frac{-1}{e_0}, 0\right)'.$$

Assume that financiers' portfolio is composed of C CIP trades, and q UIP trades:

$$\Theta = C\Theta^{CIP} + q\Theta^{UIP}.$$

We expect the risk premia in this economy to come just from the risk currency part (q), not the CIP position (C). Let us verify this.

In terms of the reduced basis, we have

$$\theta^{CIP} = (e_0 - F, 0)',$$
$$\theta^{UIP} = \left(1, \frac{-1}{e_0}\right)',$$

so that

$$\theta = C\theta^{CIP} + q\theta^{UIP}.$$

Hence, the model confirms that the financiers have 0 exposure to the yen coming from the CIP trade. We

A.20

then have

$$\mathbb{E}\left[e_0 - e_1\right] = \Gamma q,$$

with $\Gamma = \gamma var(e_1)^\alpha$. The CIP trade, causing no risk, causes no risk premia. We summarize the results in the following lemmas.

Lemma A.4 *If the financiers undertake both CIP and UIP trades, only the net positions coming from the UIP trades induce risk premia.*

Lemma A.5 *Assume that $\alpha \geq 1$, or that $V = var(x^1)$ is invertible (and $\alpha \geq 0$). Then, in the FX model risk-less portfolios earn zero excess returns. In particular, CIP holds in the model, while UIP does not.*

Proof: Define $W = \gamma V^{\alpha-1} D$, which is well-defined under the lemma's assumptions. Then, we can write $G = \gamma V^\alpha D$ as $G = VW$, and apply Proposition A.10. $\square$

A.4 Numerical Generalization of the Model

We include here both a generalization of the basic Gamma model of Section 1 that relaxes some of the assumptions imposed in the main body of the paper for tractability, and an example of how to incorporate the financial market structure of our model in a model with pricing frictions in the goods market (incomplete pass trough) .

The generalization of the model in Section 1 has to be solved numerically. Our main aim is to verify, at least numerically, that all the core forces of the basic model carry through to this more general environment. Similarly the model with incomplete pass-trough has to be solved for numerically. Our main aim is to show the reader how to incorporate our financial model in larger macroeconomics models that potentially have extensive features on the productions and goods market side.

For both models we provide a brief numerical simulation. We stress, however, that this is only a numerical example without any pretense of being a full quantitative assessment. A full quantitative assessment, with its need for further channels and numerical complications, while interesting, is the domain of future research.

A.4.1 Numerical Generalization of the Gamma Model

Model Equations   Since the model is a generalization of the basic one, we do not restate, in the interest of space, the entire structure of the economy. We only note here that the model has infinite horizon, symmetric initial conditions (both countries start with zero bond positions), and report below the system of equations needed to compute the solution.

$$R_{t+1} = \frac{\chi_t/Y_{NT,t}}{\beta_t \mathbb{E}_t[\chi_{t+1}/Y_{NT,t+1}]}, \tag{A.49}$$

$$R_{t+1}^* = \frac{\chi_t^*/Y_{NT,t}^*}{\beta_t^* \mathbb{E}_t[\chi_{t+1}^*/Y_{NT,t+1}^*]}, \tag{A.50}$$

$$Q_t = \frac{1}{\Gamma}\mathbb{E}_t\left[\left(\eta\beta_t\frac{Y_{NT,t}/\chi_t}{Y_{NT,t+1}/\chi_{t+1}} + (1-\eta)\beta_t^*\frac{e_t}{e_{t+1}}\frac{Y_{NT,t}^*/\chi_t^*}{Y_{NT,t+1}^*/\chi_{t+1}^*}\right)(e_t R_{t+1} - R_{t+1}^* e_{t+1})\right] \tag{A.51}$$

$$Q_t = f_t e_t - f_t^* - D_t, \tag{A.52}$$

$$D_t = D_{t-1}R_t + (\alpha Q_{t-1} - e_{t-1}f_{t-1})\left(R_t - R_t^*\frac{e_t}{e_{t-1}}\right) + e_t\frac{\xi_t}{\chi_t^*}Y_{NT,t}^* - \frac{\iota_t}{\chi_t}Y_{NT,t}, \tag{A.53}$$

where η is the share of financiers' profits repatriated to the US, and D are the US net foreign assets. This is a system of five nonlinear stochastic equations in five endogenous unknowns $\{R, R^*, e, Q, D\}$. We solve the system numerically by second order approximation. The exogenous variables evolve according to:

$$\ln \iota_t = (1 - \phi_\iota) \ln \iota_{t-1} + \sigma_\iota \varepsilon_{\iota,t}; \qquad \ln \xi_t = (1 - \phi_\xi) \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}, \qquad (A.54)$$

$$f_t = (1 - \phi_f) f_{t-1} + \sigma_f \varepsilon_{f,t}; \qquad f_t^* = (1 - \phi_f) f_{t-1}^* + \sigma_f \varepsilon_{f^*,t}, \qquad (A.55)$$

$$\beta_t = \bar{\beta} \exp(x_t); \qquad \beta_t^* = \bar{\beta} \exp(x_t^*), \qquad (A.56)$$

$$x_t = (1 - \phi_x) x_{t-1} + \sigma_x \varepsilon_{x,t}; \qquad x_t^* = (1 - \phi_x) x_{t-1}^* + \sigma_x \varepsilon_{x^*,t}, \qquad (A.57)$$

where $[\varepsilon_\iota, \varepsilon_\xi, \varepsilon_f, \varepsilon_{f^*}, \varepsilon_x, \varepsilon_{x^*}] \sim N(0, I)$. We assume that all other processes, including the endowments, are constant.

The deterministic steady state is characterized by: $\{\bar{e} = 1, \bar{R} = \bar{R}^* = \bar{\beta}^{-1}, \bar{Q} = \bar{D} = \bar{D}^* = 0\}$.[78] In order to provide a numerical example of the solution, we briefly report here the chosen parameter values. We stress that this is not an estimation, but simply a numerical example of the solutions. We set $\bar{\beta} = 0.985$ to imply a steady state annualized interest rate of 6%. We set the share of financiers' payout to households at $\eta = 0.5$, so that it is symmetric across countries. We set all constant parameters at 1 ($Y_H = Y_L = a = a^* = 1$), except for the value of non-tradables set at 18 ($Y_{NT} = Y_{NT}^* = \chi = \chi^* = 18$), so that they account for 90% of the consumption basket. We set $\Gamma = 0.1$.[79] Finally, we set the shock parameters to: $\phi_\iota = \phi_\xi = 0.0018, \sigma_\iota = \sigma_\xi = 0.0037, \phi_f = 0.0001, \sigma_f = 0.05, \phi_x = 0.0491, \sigma_x = 0.0073$.

We report in Table A.1 below a short list of simulated moments.[80] For a rough comparison, we also provide data moments focusing on the GBP/USD exchange rate and US net exports.

Finally, we provide a numerical example of classic UIP regressions. The regression specification follows:

$$\Delta \ln(e_{t+1}) = \alpha + \beta_{UIP} [\ln(R_t) - \ln(R_t^*)] + \varepsilon_t.$$

The above regression is the empirical analog to the theoretical results in Section 5.1.[81] We find a regression coefficient well below one ($\hat{\beta} = 0.34$), the level implied by UIP. Indeed, on average we strongly reject UIP with an average standard error of 0.42. The regression adjusted R^2 is also low at 0.039. The results are broadly in line with the classic empirical literature on UIP. As a reference, if we perform the same regression on the GBP/USD exchange rate for the period from 1975Q1 to 2012Q2 we find a coefficient of 0.42 (standard error equal to 0.24), which is significantly below the UIP value of 1, and $R^2 = 0.014$.

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[78] Note that the deterministic steady state is stationary whenever $\Gamma > 0$, which we always assume here (i.e $\alpha = 0$ from the main text), as shown in Proposition A.2. Similarly the portfolio of the intermediary is determinate via the assumption that households only save in domestic currency and via the limited commitment problem of the intermediary.

[79] We set this conservative value of Γ based on a thought experiment on the aggregate elasticity of the exchange rate to capital flows. We suppose that an inelastic short-term flow to buy the Dollar, where the scale of the flow is comparable to 1 year worth of US exports (i.e. $f^* = 1$), would induce the Dollar to appreciate 10%. The numbers are simply illustrative, but are in broad congruence with the experience of Israel and Switzerland during the recent financial crisis. Let us revert to the basic Gamma model. Suppose that period 1 is a "long run" during which inflows have already mean-reverted (so that the model equations are: $e_0 - 1 + f^* + Q = 0, e_1 = 1, Q = \frac{1}{\Gamma}(e_0 - e_1)$). Then, we have $e_0 = 1 - \frac{\Gamma}{1+\Gamma} f^*$. Hence, the price impact is $e_0 - 1 = -\frac{\Gamma}{1+\Gamma} \simeq -0.1$. This leads to $\Gamma \simeq 0.1$.

[80] The moments are computed by simulating 500,000 periods. We drop the first 100,000 observations (burn-in period).

[81] To estimate the regression based on model-produced data, we simulate the model for 500,000 periods, dropping the first 100,000, and then sample at random 10,000 data intervals of length 150. The length is chosen to reflect the data span usually available for the modern period of floating currencies (150 quarters). On each data interval, we estimate the above regression. Finally, we average across the regression output from the 10,000 samples.

Table A.1: Numerical Example of Simulated Moments

Moment	Data	Model
$SD\left(\frac{e_{t+1}}{e_t} - 1\right)$	0.1011	0.1317
$\phi(e_{t+1}, e_t)$	0.2442	0.1471
$\bar{R}^c$	0.0300	0.0388
$SD(R_t^c)$	0.1011	0.1310
$SD(nx_t)$	0.0334	0.0240
$\phi(nx_t, nx_{t-1})$	0.0708	0.1618
$SD(R_t)$	0.0479	0.0480
$\phi(R_{t+1}, R_t)$	0.1826	0.1819

Data and model-simulated moments. The first column reports the standard deviation ($SD(\frac{e_{t+1}}{e_t} - 1)$) and (one minus) autocorrelation ($\phi(e_{t+1}, e_t)$) of exchange rates, the average carry trade return ($\bar{R}^c$) and its standard deviation ($SD(R_t^c)$), the standard deviation ($SD(nx_t)$) and (one minus) the autocorrelation coefficient ($\phi(nx_t, nx_{t-1})$) of net exports over GDP for the US, and the standard deviation ($SD(R_t)$) and (one minus) autocorrelation of interest rates ($\phi(R_{t+1}, R_t)$). Data sources: exchange rate moments are for the GBP/USD, the carry trade moments are based on Lettau, Maggiori and Weber (2014) assuming the interest rate differential is 5%, the interest rate moments are based on the yield on the 6-month treasury bill minus a 3-year moving average of the 6-month rate of change of the CPI. All data are quarterly 1975Q1-2012Q2 (150 observations). The reported moments are annualized. Model implied moments are computed by simulating 500,000 periods (and dropping the first 100,000). The carry trade moments are computed selecting periods in the simulation when the interest rate differential is between 4% and 6%.

A.4.2 Frictions in Goods Markets and Financial Markets

In the previous subsection we have maintained the base assumption from the main body of the paper that prices are flexible and goods market frictionless so that the law of one price holds across tradable goods. While this is useful to sharpen the focus on the main financial frictions that are at the core of our analysis, it does not provide a full account of the disconnect of the exchange rate from the goods market. In particular, the quantity of export (import) might react too strongly to movements in the exchange rate. The reader can think of the exchange rate disconnect as coming both from financial market frictions and goods market frictions. We show below how many important goods market frictions can be incorporated in our set-up. Since the procedure is general, and does not depend on the specific details of the goods market model, we proceed at a general level of abstraction. We then specialize the set-up to a particular model of local currency pricing.

Most models of goods market frictions, like models of sticky prices under LCP or PCP, assume complete markets. Thus in solving these models, the asset markets boil down to a *single* risk sharing condition:

$$\left(\frac{C_t}{C_t^*}\right)^\sigma = \mathscr{E}_t. \tag{A.58}$$

This condition implies that consumption ratios across countries are proportional to the real (CPI based) exchange rate.[82] This condition can then be used in conjunction with the rest of the model to compute equilibrium values.

Our set-up can be incorporated readily in such models by replacing Equation (7) with a system of 5

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[82]The condition here is for the case of CRRA preferences with risk aversion coefficient σ and symmetric countries. More generally we have: $\frac{U_c(C_t^*)}{U_c(C_t)} = \frac{\mathscr{E}_t}{\nu}$, where U_c is the marginal utility of consumption and ν the Negishi weight.

equations that fully characterize the imperfect financial markets in our model. These equations are:

$$1 = \mathbb{E}_t \left[\beta_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} R_{t+1} \right], \tag{A.59}$$

$$1 = \mathbb{E}_t \left[\beta_t^* \frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{P_t^*}{P_{t+1}^*} R_{t+1}^* \right], \tag{A.60}$$

$$Q_t = \frac{P_t^*}{\Gamma} \mathbb{E}_t \left[\left(\eta \beta_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} + (1-\eta) \beta_t^* \frac{e_t}{e_{t+1}} \frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{P_t^*}{P_{t+1}^*} \right) \left(e_t R_{t+1} - e_{t+1} R_{t+1}^* \right) \right], \tag{A.61}$$

$$Q_t = P_t^* f_t e_t - f_t P_t - D_t, \tag{A.62}$$

$$D_t = R_t D_{t-1} + \left(\eta Q_{t-1} - e_{t-1} f_{t-1} P_{t-1}^* \right) \left(R_t - R_t^* \frac{e_t}{e_{t-1}} \right) + e_t P_{H,t}^* C_{H,t}^* - P_{F,t} C_{F,t}. \tag{A.63}$$

Notice that these equations are the same (slightly more general) block of equations we used to solve the self contained model in the previous subsection. In this sense these equations are quite portable and can be embedded in larger set-ups. Before we do so, we analyze each equation in turn. Equations (A.59-A.60) are standard Euler equations for a CRRA agent trading nominal risk-free interest rates. Equation (A.61) is the demand equation of the financiers, which in this case we extended to the nominal set-up (Q_t is in nominal dollars). Equation (A.62) is the market clearing condition for the foreign exchange market. Equation (A.63) is the evolution of the US net foreign assets. The last two terms on the right hand side of Equation (A.63) are US net exports in nominal dollars. Note that we are allowing for local currency pricing (i.e. $P_{H,t}^*$ is the nominal price in Yen of US export goods faced by the Japanese in Japan). Finally, the exogenous forcing variables for flow shocks (f, f^*), which are defined in real terms, and discount factor shocks are given by Equations (A.55-A.57).

We report below the first order approximation (log-linearization) to the above system of equations:[83]

$$r_{t+1} = x_t + \mathbb{E}_t[\Delta p_{t+1} + \sigma \Delta c_{t+1}], \tag{A.64}$$
$$r_{t+1}^* = x_t^* + \mathbb{E}_t[\Delta p_{t+1}^* + \sigma \Delta c_{t+1}^*], \tag{A.65}$$
$$\tilde{q}_t = \frac{1}{\Gamma} \mathbb{E}_t[r_{t+1} - r_{t+1}^* - \Delta e_{t+1}], \tag{A.66}$$
$$\tilde{q}_t = f_t - f_t^* - \tilde{d}_t, \tag{A.67}$$
$$\Delta \tilde{d}_t = \beta^{-1} \Delta \tilde{d}_{t-1} + \bar{C}_H^*[\Delta c_{H,t}^* - \Delta c_{F,t} + \Delta e_t + \Delta p_{H,t}^* - \Delta p_{F,t}], \tag{A.68}$$

where $\tilde{q}_t, \tilde{d}_t$ are expressed in deviation from steady state and are in units of real dollars (i.e. $\tilde{Q} \equiv Q/P$), and $\bar{C}_H^*$ is the steady state level of US export quantities.

Contrary to the model in the previous section, the above equations do not describe the entire equilibrium in the present set-up. For example, the dynamics of import and export prices ($\Delta p_{H,t}^* - \Delta p_{F,t}$) will depend on optimal price setting conditions by producers. In order to analyze one interesting example of equilibrium, we embed the above financial market set-up in the new-Keynesian model with sticky prices in local currency

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[83]Since in many nominal models, including the one that we consider here, the price level is not stationary we log-linearize the *real* version of the system of equations around the deterministic steady state. All variables are expressed in percentage deviation from steady state. For variables such as Q_t that have value zero in steady state, we linearize rather than log-linearize.

described in the Handbook of International Economics chapter of Burstein and Gopinath (2015).[84] In the interest of not stating from first principles an extensive model that is not the main focus of our paper, we rather summarize their set-up and refer the interested reader to the original paper.

The model in Burstein and Gopinath (2015) consists of two symmetric countries with infinitely lived agents who consume a composite final good composed of non-tradable and tradable goods. All goods are produced with a technology linear in labor with no productivity shocks and no capital. Households have an isoelastic disutility of labor and wages are flexible. The tradable component of the final good is itself an aggregator of intermediate goods produced domestically and in the foreign economy. Intermediate goods producers are monopolistically competitive and set their prices according to a Calvo mechanism. When selected to re-set prices producers can choose, at the same time, two different prices for the domestic market in domestic currency and for the foreign market in foreign currency (hence the local currency pricing). Agents are subject to a cash in advance constraint that always binds and sets the level of nominal expenditure in each period equal to the exogenous money supply ($P_t C_t = M_t$). The growth rate of the money supply follows an AR(1) process.

We close the model in Burstein and Gopinath (2015) by removing the complete market risk sharing equation (the analogous of Equation (7)) and replacing it with our financial structure (Equations A.64-A.68). In Table A.2 we report a simulated numerical example of the resulting model. We stress that this is an example, without the pretense of being a full quantitative assessment.[85]

The model appears to be able to broadly match major moments of the data. Many moments are similar to those shown in the previous subsection in the case of fully flexible prices. Of particular interest is the correlation between export quantities and nominal exchange rates. Columns 3 and 4 of Table A.2 show that the correlation decreases, while leaving most other macro moments unchanged, as prices in local currency become stickier (κ being the probability of not adjusting the price in a given period). Intuitively, a nominal depreciation of the USD does not lead to a big export increase if US producers keep their price constant in Japanese yen. This is one of the main roles played by the goods market frictions here. A corresponding moment in the data is not readily identifiable given that price adjustment frequencies change substantially across goods and countries often use a combination of LCP and PCP. We thus prefer to show the ability of the model to generate an imperfect correlation, with lower correlation for goods with higher level of stickiness in local currency.[86] The goods market pricing frictions, together with the financial frictions, also help break the complete market risk sharing equation, i.e. the Backus and Smith condition. Backus and Smith (1993) document that consumption growth rates across countries are largely unrelated to real exchange rates. On the contrary, the complete market risk sharing equation (Equation (7)) implies a perfect correlation. Our financial imperfections break the Backus and Smith condition, and interestingly interact with the price setting frictions (which affect the real exchange rate dynamics) to produce a relatively low correlation between consumption and real exchange rates.[87]

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[84]In particular see Sections 7 and 9 in Burstein and Gopinath (2015). The handbook chapter itself builds on the model of Carvalho and Nechio (2011).

[85]We parametrize the model following most closely Carvalho and Nechio (2011). We set $\bar{\beta} = 0.985$ to obtain a 6% steady state rate of interest, the autocorrelation of money growth at 0.55 and its standard deviation at 0.015, the relative risk aversion of the agents (σ) at 3, the Frisch elasticity at 1, the share of non-tradables at 0.5, and the import share in tradables at 0.25, the elasticity of substitution between imported and home intermediate goods at 1.5, and we vary the probability of price adjustment between 0.49 and 0.6. Finally we set $\phi_f = 0.05, \sigma_f = 0.1, \phi_x = 0.0491, \sigma_x = 0.0073, \Gamma = 0.1$. All shocks are orthogonal.

[86]Aggregation of the micro evidence to a symmetric macro model, like the one considered here, is an arduous task. For example, Gopinath and Rigobon (2008) show that for the US export goods tend to predominantly be under PCP but import goods tend to be under LCP, furthermore within each type of pricing the frequency of price adjustment varies substantially across goods.

[87]Indeed Burstein and Gopinath (2015) note that in the model that they analyze under the assumption of complete

Table A.2: Numerical Example of Simulated Moments: Local Currency Pricing

Moment	Data	Model, $\kappa = 0.49$	Model, $\kappa = 0.6$
$SD(\Delta e_t)$	0.101	0.093	0.098
$\phi(e_{t+1}, e_t)$	0.244	0.004	0.005
$SD(nx_t)$	0.033	0.015	0.015
$\phi(nx_{t+1}, nx_t)$	0.071	0.758	0.755
$SD(R_t)$	0.067	0.049	0.049
$\phi(R_{t+1}, R_t)$	0.197	0.373	0.405
$SD(\Delta p_t)$	0.017	0.033	0.032
$\rho(\Delta p_{t+1}, \Delta p_t)$	0.918	0.833	0.786
$\bar{R}^C$	0.030	0.042	0.039
$SD(R_t^C)$	0.101	0.092	0.097
$\rho(\Delta c_t - \Delta c_t^*, \Delta \mathscr{E}_t)$	0	-0.373	-0.033
$\rho(\Delta e_t, \Delta c_{H,t}^*)$	––	0.573	0.392

Data and simulated moments. The first column reports the standard deviation ($SD(\Delta e_t)$) and (one minus) the autocorrelation ($\phi(e_{t+1}, e_t)$)of the nominal exchange rate, the standard deviation ($SD(nx_t)$)and (one minus) autocorrelation ($\phi(nx_{t+1}, nx_t)$) of net exports over GDP for the US, the standard deviation ($SD(R_t)$) and (one minus) the autocorrelation of the nominal interest rate ($\phi(R_{t+1}, R_t)$), the standard deviation ($SD(\Delta p_t)$) and (one minus) the autocorrelation ($\rho(\Delta p_{t+1}, \Delta p_t)$) of CPI inflation, the average carry trade return ($\bar{R}^C$) and its standard deviation ($SD(R_t^C)$), the correlation between the difference in real growth rates of consumption and the growth rate of the real exchange rate ($\rho(\Delta c_t - \Delta c_t^*, \Delta \mathscr{E}_t)$), and the correlation between the growth rate of the nominal exchange rate and the growth rate of the quantity of exports ($\rho(\Delta e_t, \Delta c_{H,t}^*)$). Data sources: exchange rate moments are for the nominal GBP/USD, the nominal interest rate is the 6-month secondary market treasury rate for the US, CPI inflation is the quarter-on-quarter growth of the CPI for the US, the carry trade moments are based on Lettau, Maggiori and Weber (2014) assuming the interest rate differential is 5%, and the lack of correlation between relative consumption growth and real exchange rates is based on Backus and Smith (1993). All data are quarterly 1975Q1-2012Q2 (150 observations). The reported moments are annualized. The model implied moments are computed by simulating 500,000 periods (and dropping the first 100,000). The carry trade moments in the model are computed selecting periods in the simulation when the annualized interest rate differential is between 4% and 6%. κ is the probability of a firm not being able to adjust its price in a given period.

A.5   Proofs

A.5.1   Proofs for the Main Body of the Paper

Proof of Proposition 3   The flow equilibrium conditions in the dollar-yen markets are:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{A.69}$$
$$\xi_1 e_1 - \iota_1 - RQ_0 = 0. \tag{A.70}$$

Summing (A.69) and (A.70) gives the intertemporal budget constraint: $R(\xi_0 e_0 - \iota_0) + \xi_1 e_1 - \iota_1 = 0$. From this, we obtain:

$$e_1 = \xi_1^{-1}(R\iota_0 + \iota_1 - R\xi_0 e_0). \tag{A.71}$$

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markets there is a very tight, and counterfactual, connection between the exchange rate and macro fundamentals (relative consumption) coming from financial markets.

The market clearing in the Dollar / Yen market, $\xi_0 e_0 - \iota_0 + \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right] = 0$, gives:

$$\frac{R^*}{R}\mathbb{E}[e_1] = e_0 + \Gamma(\xi_0 e_0 - \iota_0) = (1 + \Gamma\xi_0)e_0 - \Gamma\iota_0. \tag{A.72}$$

Combining (A.71) and (A.72),

$$\mathbb{E}[e_1] = \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right] - \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R e_0 = \frac{R}{R^*}(1 + \Gamma\xi_0)e_0 - \frac{R}{R^*}\Gamma\iota_0,$$

i.e.

$$\begin{aligned}
e_0 &= \frac{\frac{R}{R^*}\Gamma\iota_0 + \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right]}{\frac{R}{R^*}(1 + \Gamma\xi_0) + \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R} = \frac{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right] + \Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{R}\xi_1^{-1}\iota_1\right]}{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right] + \Gamma\right)\xi_0 + 1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\iota_0}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}.
\end{aligned}$$

We can now calculate e_1. We start from its expected value:

$$\begin{aligned}
\frac{R^*}{R}\mathbb{E}[e_1] &= (1 + \Gamma\xi_0)e_0 - \Gamma\iota_0 = (1 + \Gamma\xi_0)\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1} - \Gamma\iota_0 \\
&= \frac{\left\{(1 + \Gamma\xi_0)\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right) - \Gamma\left[\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1\right]\right\}\iota_0 + (1 + \Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\right]\iota_0 + (1 + \Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1} = \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\xi_0\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}.
\end{aligned}$$

To obtain the time-1 innovation, we observe that $e_1 = \frac{1}{\xi_1}(R\iota_0 + \iota_1 - R\xi_0 e_0)$ implies:

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R(\iota_0 - \xi_0 e_0)\left\{\frac{1}{\xi_1}\right\}.$$

As:

$$\iota_0 - \xi_0 e_0 = \iota_0 - \xi_0 \frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1} = \frac{\iota_0 - \mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1},$$

we obtain:

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R\frac{\iota_0 - \mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1}\left\{\frac{1}{\xi_1}\right\}.$$

We next derive the value of Γ. Notice that we can write the above equation as:

$$\{e_1\} = \varepsilon + \frac{1}{a + \Gamma}\eta,$$

$$\varepsilon \equiv \left\{\frac{\iota_1}{\xi_1}\right\},$$

$$\eta \equiv \left(\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1} \frac{\iota_1}{R} \right] \right) \frac{1}{\xi_0} \left\{ \frac{1}{\xi_1} \right\},$$

$$a \equiv \mathbb{E}\left[\frac{R^*}{\xi_1} \left(\xi_0 + \frac{\xi_1}{R^*} \right) \right] \frac{1}{\xi_0}.$$

Then,

$$var(e_1) = \sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{(a+\Gamma)^2}.$$

Letting $G(\Gamma)$ be

$$G(\Gamma) \equiv \Gamma - \gamma \left(\sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{(a+\Gamma)^2} \right)^\alpha, \tag{A.73}$$

then Γ is defined as

$$G(\Gamma) = 0. \tag{A.74}$$

When $\alpha = 0$, we get the basic Gamma model. When $\alpha = 1$, we have a polynomial of degree 3 in Γ. When there is no noise and $\alpha > 0$, $\Gamma = 0$. In general, it is still amenable to computation: there is a unique positive solution of $G(\Gamma)$ (as $G(\Gamma)$ is increasing in Γ, and $G(0) < 0$, $\lim_{\Gamma \to \infty} G(\Gamma) = \infty$).

Proof of Lemma 3 In the decentralized allocation, the consumer's intra-period consumption, Equation (5), gives the first order conditions:

$$p_{NT} C_{NT} = \frac{\chi}{\lambda}; \qquad p_{NT}^* C_{NT}^* = \frac{\chi^*}{\lambda^*};$$

$$p_H C_H = \frac{a}{\lambda}; \qquad \frac{p_H}{e} C_H^* = \frac{\xi}{\lambda^*}; \tag{A.75}$$

$$e p_F^* C_F = \frac{\iota}{\lambda}; \qquad p_F^* C_F^* = \frac{a^*}{\lambda^*}.$$

so that

$$e = \frac{\frac{C_H^* \lambda^*}{\xi}}{\frac{C_H \lambda}{a}}.$$

Suppose that the Negishi weight is ν. The planner maximizes $U + \nu U^*$ subject to the resource constraint; hence, in particular $\max_{C_H + C_H^* \le Y_H} a \ln C_H + \nu \xi \ln C_H^*$, which gives the planner's first order condition $\frac{a}{C_H} = \frac{\nu \xi}{C_H^*}$. Hence, in the first best exchange rate satisfies:

$$e_t^{FB} = \nu \frac{\lambda_t^*}{\lambda_t} = \nu \frac{p_{NT} C_{NT,t}/\chi_t}{p_{NT}^* C_{NT,t}^*/\chi_t^*}.$$

In the basic case of Lemma 3, we have $\lambda_t = \lambda_t^* = 1$, so $e_t^{FB} = \nu$. Note that this is derived under the assumption of identical discount factor $\beta = \beta^*$. $\square$

Proof of Proposition 7 We first prove a Lemma.

Lemma A.6 *In the setup of Proposition 3, e_0 is increasing in ι_t and R^* and decreasing in ξ_t and R; $\frac{\partial e_0}{\partial \iota_0}$ increases in Γ. In addition, e_0 increases in Γ if and only the US is a natural net debtor at time 0^+, i.e. $N_{0^+} \equiv \xi_0 e_0 - \iota_0 < 0$.*

Proof: The comparative statics with respect to ι_t, ξ_0, and R are simply by inspection. We report here the less obvious ones:

$$\frac{\partial e_0}{\partial \xi_1} = \frac{\mathbb{E}\left[\frac{e_0\xi_0 - \iota_0 - \frac{\iota_1}{R}}{\xi_1^2}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = -\frac{\mathbb{E}\left[\frac{e_1}{R\xi_1}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} < 0,$$

where we made use of the state-by-state budget constraint $e_0\xi_0 - \iota_0 + \frac{e_1\xi_1 - \iota_1}{R} = 0$. To be very precise, a notation like $\frac{\partial e_0}{\partial \xi_1}$ is the sensitivity of e_0 to a small, deterministic increment to random variable ξ_1.

$$\frac{\partial e_0}{\partial R^*} = \frac{1}{R^{*2}}\frac{e_0 - \Gamma Q}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = \frac{1}{RR^*}\frac{\mathbb{E}[e_1]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} > 0,$$

where we made use of the financiers' demand equation, $\Gamma Q_0 = \mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right]$, and the flow equation, $\xi_0 e_0 - \iota_0 + Q_0 = 0$.

We also have,

$$\frac{\partial e_0}{\partial \Gamma} = -N_{0^+}\frac{1}{1 + R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + \xi_0\Gamma} < 0,$$

where we made use of the definition $N_{0^+} = e_0\xi_0 - \iota_0$. This implies:

$$\frac{\partial^2 e_0}{\partial \Gamma \partial \iota_0} = \frac{1}{\left(R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0\right)^2} > 0.\square$$

This implies all the points of Proposition 7 with two exceptions. The effects with respect to interest rate changes, both domestic and foreign, hold for f, f^* sufficiently small. Finally, we focus on the impact of f^*. Simple calculations yield:

$$\frac{\partial e_0}{\partial f^*} = -\frac{\Gamma}{R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0} < 0.$$

We notice that the comparative statics with respect to f are less clear-cut, because f affects the value of $\Gamma\widetilde{\xi}_1$, and hence affects risk-taking. However, we have $\frac{\partial e_0}{\partial f} > 0$ for typical values (e.g. $R = R^* = 1, \widetilde{\xi}_0 = \widetilde{\xi}_1$). $\square$

Proof of Lemma 5 and Proposition 8 The economy is described by:

$$e_0 - 1 - qe_0 + Q_0 = 0; \qquad e_1 - \iota_1 + qe_1 - Q_0 = 0.$$

Using Proposition 6 (with $\widetilde{\xi}_0 = 1 - q, \widetilde{\xi}_1 = 1 + q$), we have:

$$e_0(q) = 1 + \frac{\Gamma(q + q^2)}{2 + \Gamma(1 - q^2)}, \tag{A.76}$$

$$e_1(q) = 1 + \frac{\Gamma(-q + q^2)}{2 + \Gamma(1 - q^2)} + \frac{\varepsilon}{1 + q}. \tag{A.77}$$

where we define $\varepsilon \equiv \iota_1 - 1$, the innovation to ι_1 (recall $\mathbb{E}\iota_1 = 1$). This implies Lemma 5.

The intervention's impact on the *average* exchange rate is only second order: it creates a depreciation at time 0, and an appreciation at time 1.

Recall that we assume $a_t^* = \iota_t$ to keep the countries symmetric in their demands of tradables, and simplify the algebra. Given equation (A.75) and $\lambda_t = \lambda_t^* = 1$, we can define $s_t \equiv \frac{C_{H,t}}{C_{H,t}+C_{H,t}^*} = \frac{C_{F,t}}{C_{F,t}+C_{F,t}^*}$ to be the real US share of consumption in both US or Japanese tradables. This share is equal to:

$$s_t = \frac{1}{1+e_t}. \tag{A.78}$$

By definition, US consumption of good g is $C_{g,t} = s_t Y_{g,t}$, where $Y_{g,t}$ is the world production of good g. US welfare at time t is (dropping the non-tradables endowment term, and correspondingly setting $\chi = 0$ for algebraic convenience):

$$U_t = a_t \ln C_{H,t} + \iota_t \ln C_{F,t} = a_t \ln (s_t Y_{H,t}) + \iota_t \ln (s_t Y_{F,t})$$
$$= (a_t + \iota_t) \ln s_t + a_t \ln Y_{H,t} + \iota_t \ln Y_{F,t},$$

and intertemporal US welfare is (using $a_t = 1$):

$$U(q) = \mathbb{E}[U_0 + U_1] = \mathbb{E} \sum_{t=0}^{1} ((1+\iota_t) \ln s_t + \ln Y_{H,t} + \iota_t \ln Y_{F,t}).$$

US production of tradables at time 0 is (by equation (25)): $Y_{H,0} = \min\left(\frac{1+e_0}{\overline{p}_H}, L\right)$, so $Y_{H,0}$ increases in e for $e \in [0, \overline{e}_0]$, with $\overline{e}_0 \equiv \overline{p}_H L - 1$. Hence:

$$U(q) = \ln \min \left(\frac{1+e_0(q)}{\overline{p}_H}, L\right) + h(q) + K, \tag{A.79}$$

where $h(q) \equiv \mathbb{E} \sum_{t=0}^{1} (1+\iota_t) \ln s_t$ captures the allocational distortions, and $K = 3 \ln L$ is a constant independent of policy (as prices are flexible at $t = 1$ in the US). Hence, US welfare goes up when its share s_t is higher and when world production is higher ($Y_{H,0}$ higher).

We shall see that the intervention on a scale q, by inducing a time-0 Dollar devaluation (i) improves by a first order effect the production at time 0, and (ii) leads only to a second order loss in the total share of consumption $h(q)$. Hence, the intervention is desirable. In addition, it is desirable also for Japan: Japan benefits from the increase in world production, and experiences only a second order change from the intertemporal distortion. Let us see this analytically.

We define, by analogy with (A.79):

$$V(q) \equiv \ln \frac{1+e_0(q)}{\overline{p}_H} + h(q) + K,$$

so that $V(q) = U(q)$ when $\frac{1+e_0(q)}{\overline{p}_H} \leq L$, i.e. in the unemployment region. We have: $V'(0) = \frac{e_0'(0)}{1+e_0(0)} + h'(0)$ and $e_0(0) = 1$. Taking the derivative of (A.76) and (A.77) at $q = 0$ gives:

$$e_0'(0) = \frac{\Gamma}{2+\Gamma}, \tag{A.80}$$

$$e_1'(0) = -\frac{\Gamma}{2+\Gamma} - \varepsilon. \tag{A.81}$$

Hence:

$$V'(0) = \frac{\Gamma}{2(2+\Gamma)} + h'(0).$$

Recalling $\iota_1 = 1 + \varepsilon$ we then compute:

$$
\begin{aligned}
h'(0) &= \mathbb{E} \sum_{t=0}^{1} (1 + \iota_t) \left. \frac{d \ln s_t}{dq} \right|_{q=0} \\
&= -2 \frac{e_0'(0)}{1 + e_0(0)} - \mathbb{E} \left[(2 + \varepsilon) \frac{e_1'(0)}{1 + e_1(0)} \right] \text{ by (A.78)} \\
&= -2 \frac{\frac{\Gamma}{2+\Gamma}}{2} - \mathbb{E} \left[(2 + \varepsilon) \frac{\frac{-\Gamma}{2+\Gamma} - \varepsilon}{2 + \varepsilon} \right] \\
&= 0.
\end{aligned}
$$

This confirms that intertemporal distortions induced are second order.

Given $V'(0) > 0$, by continuity, there exists a $q^{\max}$ such that $V'(q) > 0$ for $q \in [0, q^{\max})$. We call $\bar{q}(e_0)$ the inverse of the function $e_0(q)$. Since $e_0(q)$ is an increasing function (for $q \geq 0$), so is $\bar{q}(e_0)$. Recall also that the least-devalued exchange rate that ensures full employment is $\bar{e}_0 = \bar{p}_H L - 1$. Define $\bar{p}_H^{\max}$ such that $\bar{q}(\bar{p}_H^{\max} L - 1) = q^{\max}$. Finally, recall that $p_{H,0}^{\circ} = \frac{2}{L}$ is the flexible equilibrium price of US tradable. The assumption that there is unemployment initially is equivalent to $\bar{p}_H > p_{H,0}^{\circ}$.

Take an initial price $\bar{p}_H \in (p_{H,0}^{\circ}, \bar{p}_H^{\max})$. To eliminate unemployment caused by $\bar{p}_H$, the government can perform the intervention $q^{opt} \equiv \bar{q}(\bar{p}_H L - 1)$. Now, given that $\bar{q}(\cdot)$ is increasing and $\bar{p}_H < \bar{p}_H^{\max}$, we have $q^{opt} < q^{\max}$. Given that $V'(q) > 0$ for $q \in [0, q^{\max})$, and that $U'(q) = V'(q)$ for $q \in [0, q^{opt})$, we have $U'(q) > 0$ for $q \in [0, q^{opt})$. This means that welfare is increasing in the size of the intervention, in the range $q \in [0, q^{opt})$.

Japanese welfare. It is intuitive that Japanese welfare will also increase when the US government's FX intervention devalues the Dollar: Japan enjoys a first-order gain from the increase in US production and experiences only a second-order change from the intertemporal distortion (at least, omitting for now potential Jensen's terms). Of course, this hinges on the assumption that prices are flexible in Japan. We provide here the more formal arguments.

Defining s_t^* to be the Japanese share of tradables at t,

$$
s_t^* = 1 - s_t = \frac{e_t}{1 + e_t},
$$

Japanese utility at time t is (using $a_t^* = \iota_t$, $\xi_t = 1$):

$$
\begin{aligned}
U_t^* &= a_t^* \ln C_{H,t}^* + \xi_t \ln C_{F,t}^* = \iota_t \ln s_t^* Y_{H,t} + \ln s_t^* Y_{F,t} \\
&= (1 + \iota_t) \ln s_t^* + \iota_t \ln Y_{H,t} + \ln Y_{F,t}.
\end{aligned}
$$

Hence, Japanese welfare is

$$
\begin{aligned}
U^*(q) &= \mathbb{E} [U_0^* + U_1^*] = \mathbb{E} \sum_{t=0}^{1} \left((1 + \iota_t) \ln s_t^* + \iota_t \ln Y_{H,t} + \ln Y_{F,t} \right) \\
&= \ln \min \left(\frac{1 + e_0(q)}{\bar{p}_H}, L \right) + h^*(q) + K,
\end{aligned}
$$

defining $h^*(q) \equiv \mathbb{E} \sum_{t=0}^{1} (1 + \iota_t) \ln s_t^*$. We define, as for the US case, $V^*(q) \equiv \ln \frac{1 + e_0(q)}{\bar{p}_H} + h^*(q) + K$. That

implies:

$$V^{*'}(0) = \frac{e'_0(0)}{1+e_0(0)} + h^{*'}(0)$$

$$= \frac{\Gamma}{2(2+\Gamma)} + h^{*'}(0).$$

We next calculate $h^{*'}(0)$. We start with $\ln s_t^* = \ln \frac{e_t}{1+e_t} = \ln e_t - \ln(1+e_t)$, so

$$\left. \frac{d \ln s_t^*}{dq} \right|_{q=0} = e'_t(0) \left[\frac{1}{e_t(0)} - \frac{1}{1+e_t(0)} \right] = \frac{e'_t(0)}{e_t(0)(1+e_t(0))}$$

and:

$$h^{*'}(0) = \mathbb{E} \sum_{t=0}^{1} (1+\iota_t) \left. \frac{d \ln s_t^*}{dq} \right|_{q=0}$$

$$= \mathbb{E} \sum_{t=0}^{1} (1+e_t(0)) \frac{e'_t(0)}{e_t(0)(1+e_t(0))} \text{ using } \iota_t = e_t(0),$$

$$= \mathbb{E} \sum_{t=0}^{1} \frac{e'_t(0)}{e_t(0)}$$

$$= e'_0(0) + \mathbb{E} \frac{-e'_0(0) - \varepsilon}{1+\varepsilon} \text{ using (A.80)-(A.81) and } e_1(0) = 1+\varepsilon$$

$$= (e'_0(0) - 1) \mathbb{E} \frac{\varepsilon}{1+\varepsilon} = \left(\frac{\Gamma}{2+\Gamma} - 1 \right) \mathbb{E} \frac{\varepsilon}{1+\varepsilon}$$

$$= -\frac{2}{2+\Gamma} \mathbb{E} \frac{\varepsilon}{1+\varepsilon}$$

$$\geq 0$$

because the function $\frac{x}{1+x}$ is concave in the domain $x > -1$, and $\mathbb{E}\varepsilon = 0$, so that $\mathbb{E}\frac{\varepsilon}{1+\varepsilon} \leq 0$.
    Summing up, we have:

$$V^{*'}(0) = \frac{\Gamma}{2(2+\Gamma)} + h^{*'}(0) \geq \frac{\Gamma}{2(2+\Gamma)} > 0$$

Given $V^{*'}(0) > 0$, by continuity, there is a $q^{\max,*}$ such that $V^{*'}(q) > 0$ for $q \in [0, q^{\max,*})$. Hence (by the reasoning done above for the US consumer), the Proposition also holds for Japanese welfare (i.e., welfare in both US and Japan increases as q increases, for $q \in [0, q^{opt}]$), provided that the initial distortion $\overline{p}_H$ is not too great. More specifically, we should have $\overline{p}_H \in (p_H^\circ, \overline{p}_H^{\max})$, where we define $\overline{p}_H^{\max}$ to be the price such that $\overline{q}(\overline{p}_H^{\max}L - 1) = \min(q^{\max}, q^{\max,*})$. $\square$

Proof of Proposition 9 The proof is quite similar to that of Proposition 8. Given the countries are symmetric in their taste for foreign tradable goods ($a_t = \xi_t = 1$, $a_t^* = \iota_t$), the share of US consumption for both the US and Japanese tradable good is $s_t = \frac{1}{1+e_t}$.

Intertemporal US welfare is:[88]

$$U = \mathbb{E}\left[U_0 + U_1\right] = \mathbb{E}\sum_{t=0}^{1}\left(\iota_t + a_t\right)\ln s_t + a_t \ln Y_{H,t} + \iota_t \ln Y_{F,t},$$

i.e.

$$U(\tau) = \ln\min\left(\frac{1+e_0(\tau)}{\bar{p}_H}, L\right) + h(\tau) + K, \tag{A.82}$$

where K is a constant independent of policy, and $h(\tau)$ captures the sum of the allocational distortions:

$$h(\tau) \equiv \mathbb{E}\sum_{t=0}^{1}\left(1+\iota_t\right)\ln s_t = \mathbb{E}\sum_{t=0}^{1}\left(1+\iota_t\right)\ln\frac{1}{1+e_t(\tau)}.$$

We define:

$$V(\tau) = \ln\frac{1+e_0(\tau)}{\bar{p}_H} + h(\tau) + K,$$

the counterpart of $U(\tau)$, without the "min" sign ($V(\tau)$ is welfare, assuming that the economy is below full employment).

We recall:

$$e_0(\tau) = \frac{\left(1 + \frac{\Gamma}{1-\tau}\right)\iota_0 + \mathbb{E}\left[\iota_1\right]}{2 + \frac{\Gamma}{1-\tau}},$$

$$e_1(\tau) = \frac{\iota_0 + \left(1 + \frac{\Gamma}{1-\tau}\right)\mathbb{E}\left[\iota_1\right]}{2 + \frac{\Gamma}{1-\tau}} + \{\iota_1\},$$

which implies

$$e_0'(0) = \Gamma\frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} = -e_1'(0),$$

so that

$$h'(0) = -\mathbb{E}\sum_{t=0}^{1}\left(1+\iota_t\right)\frac{e_t'(0)}{1+e_t(0)},$$

$$h'(0) = -e_0'(0)D,$$

$$D \equiv \mathbb{E}\left[\frac{1+\iota_0}{1+e_0(0)} - \frac{1+\iota_1}{1+e_1(0)}\right]$$

The marginal impact welfare of a small tax is:

$$V'(0) = \frac{e_0'(0)}{1+e_0(0)} + h'(0) = e_0'(0)\left[\frac{1}{1+e_0(0)} - D\right].$$

We observe that when $\mathbb{E}\iota_1 = \iota_0$, then $e_0(0) = \iota_0$ and $e_1(0) = \iota_1$, so that $\frac{1+\iota_0}{1+e_0(0)} - \frac{1+\iota_1}{1+e_1(0)} = 0$ and $D = 0$. By continuity, when $|\mathbb{E}\iota_1 - \iota_0|$ is small enough, D is close to 0. Hence, we assume that $|\mathbb{E}\iota_1 - \iota_0|$ is small enough, so that we have $\frac{1}{1+e_0(0)} - D > 0$. That implies $V'(0) > 0$.

---

[88]The analysis for Japanese welfare is entirely symmetric: the same small tax creates a first-order improvement in Japanese welfare because it increases US tradable output and hence the Japanese equilibrium consumption of US tradable goods, but only creates a smaller distortion in the intertemporal Japanese consumption shares.

A.33

Given $V'(0) > 0$, by continuity, there is a $\tau^{\max}$ such that $V'(\tau) > 0$ for $\tau \in [0, \tau^{\max})$.

We call $\overline{\tau}(e_0)$ the inverse of the function $e_0(\tau)$. Define $\overline{p}_H^{\max}$ such that $\overline{\tau}(\overline{p}_H^{\max}L - 1) = \tau^{\max}$. Then, for all $\overline{p}_H \in [p_{H,0}^\circ, \overline{p}_H^{\max})$, we have $U'(\tau) > 0$ for $\tau \in [0, \overline{\tau}(\overline{p}_H - 1)]$. This means that welfare is increasing in the size of the intervention. The optimal intervention in that range is $\tau^{opt} = \overline{\tau}(\overline{p}_H - 1)$, the smallest (positive) tax that restores full employment.

The part on Japanese welfare is proven by exactly the same arguments as the arguments above, and those of the Japanese welfare part at the end of the proof of Proposition 8.$\square$

Proof of Proposition 10   Suppose that $m_0^{*'}$ deviates from its original value, 1, keeping m_1^* constant. Then, $R^{*'} = 1/m_0^{*'}$, hence (by Proposition 6) $e_0' = 1/m_0^{*'}$, while e_1 is unchanged at 1. Japanese demand for US exports is unchanged because $e_0' m_0^{*'} = 1$. Hence, the US export market is still in equilibrium with the US at full employment ($Y_{H,0} = L$). There is no need for policy intervention to ensure full employment. $\square$

Proof of Proposition 11   **The case of short-lasting rigidity.** We follow the notations and procedure of the proof of Proposition 8. To make the proof readable, we simply consider the case with $\iota_1 \equiv 1$. One can restate the arguments of the proof of Proposition 8 for the more general case.

We first derive the exchange rate. The pseudo-imports and exports are:

$$\widetilde{\iota}_0 = m_0; \qquad \widetilde{\iota}_1 = 1;$$
$$\widetilde{\xi}_0 = 1 - q; \qquad \widetilde{\xi}_1 = 1 + q.$$

and the gross interest rates are: $R = \frac{1}{m_0}$ and $R^* = 1$.

The exchange rate is (using Proposition 3 and Proposition 6):

$$e_0 = \frac{\mathbb{E}\left[\frac{\widetilde{\iota}_0 + \frac{\widetilde{\iota}_1}{R}}{\widetilde{\xi}_1}\right] + \frac{\Gamma \widetilde{\iota}_0}{R^*}}{\mathbb{E}\left[\frac{\widetilde{\xi}_0 + \frac{\widetilde{\xi}_1}{R^*}}{\widetilde{\xi}_1}\right] + \frac{\Gamma \widetilde{\xi}_0}{R^*}} = m_0 \frac{2 + \Gamma(1+q)}{2 + \Gamma(1-q^2)};$$

$$e_1 = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1}\left(\widetilde{\iota}_0 + \frac{\widetilde{\iota}_1}{R}\right)\right] + \Gamma \widetilde{\xi}_0 \mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1}\frac{\widetilde{\iota}_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1}\left(\widetilde{\xi}_0 + \frac{\widetilde{\xi}_1}{R^*}\right)\right] + \Gamma \widetilde{\xi}_0} = \frac{1}{m_0} m_0 \frac{2 + \Gamma(1-q)}{2 + \Gamma(1+q)(1-q)}$$

$$= \frac{2 + \Gamma(1-q)}{2 + \Gamma(1-q^2)}.$$

with $\{e_1\} = 0$. Hence,

$$e_0 = m_0(1 + \eta_0); \quad e_1 = 1 + \eta_1,$$

where $\eta_0 = \frac{\Gamma(q+q^2)}{2+\Gamma(1-q^2)}$ and $\eta_1 = \frac{\Gamma(-q+q^2)}{2+\Gamma(1-q^2)}$ are the expressions we found in the proof of Proposition 8.

Output is: $Y_{H,t} = \min\left(\frac{a_t m_t + \xi_t e_t m_t^*}{\overline{p}_H}, L\right) = \min\left(\frac{m_0 + e_0}{\overline{p}_H}, L\right)$, i.e.,

$$Y_{H,t} = \min\left(\frac{m_0(2 + \eta_0)}{\overline{p}_H}, L\right).$$

A.34

US welfare is given by (A.79), augmented for the cost of monetary distortions:

$$U(q) = \ln\min\left(\frac{1+e_0(q)}{\overline{p}_H}, L\right) + h(q) - g(m_0) + K, \tag{A.83}$$

where $K = 3\ln L$ is a constant, and $h(q) \equiv 2\mathbb{E}\sum_{t=0}^{1}\ln s_t$ is the sum of the allocational distortions, with s_t is the share of the home good going to the US: $s_t = \frac{1}{1+\frac{e_t}{m_t}}$, i.e.

$$s_t = \frac{1}{2+\eta_t}.$$

Let us compute a few expressions that will be useful in later derivations. We will use the Taylor expansions below, when q is small, i.e. when η_0 and η_1 are small:

$$\eta_0 = \frac{\Gamma}{2+\Gamma}q + O(q^2),$$

which implies

$$q = \frac{2+\Gamma}{\Gamma}\eta_0 + O(\eta_0^2),$$

$$\begin{aligned}
\eta_1 + \eta_0 &= \frac{2\Gamma q^2}{2+\Gamma(1-q^2)} = \frac{2\Gamma}{2+\Gamma}q^2 + O(q^4) \\
&= \frac{2\Gamma}{2+\Gamma}\left(\frac{2+\Gamma}{\Gamma}\eta_0 + O(\eta_0^2)\right)^2 + O(q^4) = \frac{2(2+\Gamma)}{\Gamma}\eta_0^2 + O(\eta_0^3) \\
&= 2\left(\frac{2}{\Gamma}+1\right)\eta_0^2 + O(\eta_0^3).
\end{aligned}$$

We now calculate the distortion term:

$$\begin{aligned}
-\frac{h}{2} &= -\sum_{t=0}^{1}\ln s_t = -\sum_{t=0}^{1}\ln\frac{1}{2+\eta_t} = \ln\left[(2+\eta_0)(2+\eta_1)\right] \\
&= \ln\left[4 + 2(\eta_0+\eta_1) + \eta_0\eta_1\right] \\
&= \ln\left[4 + 2\left(2\left(\frac{2}{\Gamma}+1\right)\eta_0^2 + O(\eta_0^3)\right) + \eta_0\left(-\eta_0 + O(\eta_0^2)\right)\right] \\
&= \ln\left[4 + \left(\frac{8}{\Gamma}+3\right)\eta_0^2 + O(\eta_0^3)\right] \\
&= \ln 4 + \left(\frac{2}{\Gamma}+\frac{3}{4}\right)\eta_0^2 + O(\eta_0^3),
\end{aligned}$$

hence

$$h = h(\eta_0, \Gamma) = -2\ln 4 - \left(\frac{4}{\Gamma}+\frac{3}{2}\right)\eta_0^2 + O(\eta_0^3). \tag{A.84}$$

We can now reconsider welfare (A.83). For the purposes of the impact of Γ on the choice of monetary versus FX policy, it is enough to consider the subproblem of achieving a given employment level Y_H, without optimizing for Y_H. The problem is to minimize distortions subject to achieving the desired $Y_H \leq L$:

$$\min_{\eta_0, m_0} h(\eta_0) - g(m_0) \text{ s.t. } m_0(2+\eta_0) = \overline{p}_H Y_H.$$

A.35

This yields $\eta_0 = \eta_0(m_0) \equiv \frac{\overline{p}_H Y_H}{m_0} - 2$, so that the optimal monetary intervention is:

$$\max_{m_0} W(m_0) = h(\eta_0(m_0)) - g(m_0),$$

so that $W_{m_0} = 0$ and $W_{m_0 m_0} < 0$ at the optimum.

We want to ask: "do we have less reliance on monetary policy (rather than FX intervention) when Γ increases?". The answer is yes if $\frac{\partial m_0}{\partial \Gamma} < 0$. Accordingly, we calculate:

$$
\begin{aligned}
sign\left(\frac{\partial m_0}{\partial \Gamma}\right) &= sign\left(\frac{-W_{m_0 \Gamma}}{W_{m_0 m_0}}\right) \text{ as } W_{m_0} = 0 \text{ implies } \frac{\partial m_0}{\partial \Gamma} = \frac{-W_{m_0 \Gamma}}{W_{m_0 m_0}} \\
&= sign\left(W_{m_0 \Gamma}\right) \text{ as } W_{m_0 m_0} < 0 \\
&= sign\left(h_{\eta_0 \Gamma} \eta_0'(m_0)\right) = -sign\left(h_{\eta_0 \Gamma}\right) \text{ since } \eta_0'(m_0) < 0 \\
&< 0,
\end{aligned}
$$

since equation (A.84) implies $h_{\eta_0 \Gamma} > 0$ for a small enough, positive intervention.

Case of the long-lasting rigidity. Here we simply sketch the argument, given that the detailed analytical forces have been already made explicit in the above proof and the proof of Proposition 8. An FX intervention won't improve welfare for the following reason: a high intervention to buy Yen today makes the Yen appreciate today (which helps the US) but makes it (by the "boomerang" effect) depreciate tomorrow, by the same amount. Hence, there has been no net help: the welfare impact of q policies is 0 on the unemployment front, and negative on the intertemporal distortion front. Hence, on net, a q policy has negative impact.

Hence, only monetary policy helps, in the usual way: the government wants to partially inflate, i.e. increase m_0 and m_1 (even though there's a cost g of doing so). $\square$

Proof of Proposition 12

$$
\begin{aligned}
\overline{R}^c &= \frac{\mathbb{E}\left[\frac{R^*}{R} e_1 - e_0\right]}{e_0} = \frac{-\Gamma Q_0}{e_0} \\
&= -\Gamma\left(\frac{\iota_0 - e_0}{e_0}\right) = \Gamma\left(1 - \frac{\iota_0}{e_0}\right).
\end{aligned}
$$

Recall that:

$$e_0 = \frac{(R^* + \Gamma)\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1]}{R^* + \Gamma + 1},$$

so that we conclude:

$$\overline{R}^c = \Gamma\left(1 - \iota_0 \frac{R^* + \Gamma + 1}{(R^* + \Gamma)\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1]}\right).$$

which, rearranged, gives the announced expression.

Derivation of 3-period economy exchange rates We will use the notation:

$$\mathscr{R}^* \equiv \frac{R^*}{R}.$$

Recall that we assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. Therefore, from the flow demand equation for $t = 1$, $e_1 - \iota_1 + Q_1 = 0$, and the financiers' demand, $Q_1 = \frac{e_1 - \mathscr{R}^* \mathbb{E}[e_2]}{\Gamma_1}$, we get an

expression for e_1:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathscr{R}^* \mathbb{E}_1 [e_2]}{\Gamma_1 + 1}.$$

The flow demand equation for $t = 2$ gives $e_2 = \iota_2$, so we can rewrite e_1 as:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathscr{R}^* \mathbb{E}_1 [\iota_2]}{\Gamma_1 + 1}.$$

Similarly for e_0, we have

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathscr{R}^* \mathbb{E}_0 [e_1]}{\Gamma_0 + 1},$$

and we can use our expression for e_1 above to express e_0 as:

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathscr{R}^* \mathbb{E}_0 \left[\frac{\Gamma_1 \iota_1 + \mathscr{R}^* \iota_2}{\Gamma_1 + 1} \right]}{\Gamma_0 + 1}. \square$$

Proof of Proposition 13 We have already derived Claim 1. For Claim 2, we can calculate, from the definition of carry trade returns ($R^c \equiv \frac{R^*}{R} \frac{e_1}{e_0} - 1$) and equation (28):

$$\overline{R}^c = (\mathscr{R}^* - 1) \Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathscr{R}^*}{\overline{\Gamma}_1 (\Gamma_0 + \mathscr{R}^*) + \Gamma_0 + (\mathscr{R}^*)^2} > 0.$$

Hence, the expected carry trade return is positive.

For Claim 3, recall that a function $\frac{ax+b}{cx+d}$ is increasing in x iff $\Delta^x \equiv ad - bc > 0$. For Γ_0,

$$\Delta^{\Gamma_0} = \left(1 + \overline{\Gamma}_1 + \mathscr{R}^*\right) \left(\overline{\Gamma}_1 \mathscr{R}^* + (\mathscr{R}^*)^2\right) > 0,$$

which proves $\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0$.

For $\overline{\Gamma}_1$, the discriminant is

$$\frac{\Delta^{\overline{\Gamma}_1}}{(\mathscr{R}^* - 1) \Gamma_0} = \Gamma_0 + (\mathscr{R}^*)^2 - (1 + \mathscr{R}^*)(\Gamma_0 + \mathscr{R}^*) = -\mathscr{R}^*(1 + \Gamma_0) < 0,$$

so that $\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0$.

Finally, for $\mathscr{R}^*$, we simply compute:

$$\frac{\partial \overline{R}^c}{\partial \mathscr{R}^*} = \frac{\Gamma_0 (1 + \Gamma_0)(1 + \overline{\Gamma}_1)\left(2\mathscr{R}^* + \overline{\Gamma}_1\right)}{\left(\Gamma_0 (1 + \overline{\Gamma}_1) + \overline{\Gamma}_1 \mathscr{R}^* + (\mathscr{R}^*)^2\right)^2} > 0. \square$$

Proof of Proposition 14 The regression corresponds to: $\beta_{\text{UIP}} = \frac{-\partial}{\partial R^*} \mathbb{E} \left[\frac{e_1}{e_0} - 1 \right]$. For simplicity we calculate this derivative at $R = R^* = \mathbb{E}\iota_t = 1$, and with deterministic $\Gamma_1 = \overline{\Gamma}_1$. Equation (28) yields, for those values but keeping R^* potentially different from 1:

$$e_0 = \frac{\Gamma_0 + R^* \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}}{\Gamma_0 + 1}; \qquad \mathbb{E}e_1 = \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}.$$

A.37

Calculating $\beta_{\mathrm{UIP}} = \frac{-\partial}{\partial R^*}\mathbb{E}\left[\frac{e_1}{e_0} - 1\right] = \frac{-\partial}{\partial R^*}\frac{\mathbb{E}e_1}{e_0}$ gives:

$$\beta_{\mathrm{UIP}} = \frac{1+\bar{\Gamma}_1 - \Gamma_0}{(1+\Gamma_0)\left(1+\bar{\Gamma}_1\right)}.$$

Hence, $\beta_{\mathrm{UIP}} \leq \frac{1+\bar{\Gamma}_1}{(1+\Gamma_0)\left(1+\bar{\Gamma}_1\right)} = \frac{1}{1+\Gamma_0} < 1.$ $\square$

A.5.2 Proofs for Appendix A.1

Proof of Proposition A.1 *Derivation of equation (A.1).* Assume $\xi_t = 1$. We start from the flow equation, $NX_t - RQ_{t-1} + Q_t = 0$, and $Q_t = \frac{\mathbb{E}_t[e_t - e_{t+1}]}{\Gamma}$. Hence:

$$\Gamma\left(e_t - \iota_t\right) - R\left(e_{t-1} - e_t\right) + e_t - \mathbb{E}_t[e_{t+1}] = 0,$$

i.e.

$$\mathbb{E}_t[e_{t+1}] - (1+R+\Gamma)e_t + Re_{t-1} + \Gamma\iota_t = 0.$$

The characteristic equation of the (homogenous version) of this second order difference equation is:

$$g(X) \equiv X^2 - (1+R+\Gamma)X + R = 0, \tag{A.85}$$

and the solutions are:

$$\lambda = \frac{(1+R+\Gamma) - \sqrt{(1+R+\Gamma)^2 - 4R}}{2}; \quad \lambda' = \frac{R}{\lambda}.$$

We define $\Lambda = \frac{1}{\lambda'}$, i.e. $\Lambda = \frac{\lambda}{R}$. The geometry of the solutions of a quadratic equation indicates (via $\lambda < \lambda'$ and $g(1) < 0 < g(0)$) that we have $0 < \lambda < 1 < \lambda'$, hence $\Lambda < \frac{1}{R}$.

It is well-known that the solution is of the type $e_t = AQ_{t-1} + \mathbb{E}_t\left[\sum_{s=t}^{\infty}\left(B\left(\lambda'\right)^{t-s} + C\lambda^{t-s}\right)\iota_s\right]$, for some constants A, B, C. Because $\lambda < 1$, we need $C = 0$, otherwise the sum would diverge. Hence, we can write:

$$e_t = AQ_{t-1} + B\mathbb{E}_t\left[\sum_{s=t}^{\infty}\Lambda^{s-t}\iota_s\right].$$

Also, when $Q_{t-1} = 0$ and $\iota_s = \iota$ for all $s \geq t$, we must have $e_t = \iota$ (indeed, then e_s is constant, so that $Q_s = 0$ and the dollar flow equation gives $e_s - \iota = 0$). That gives $1 = B\sum_{s=t}^{\infty}\Lambda^{s-t} = \frac{B}{1-\Lambda}$, i.e. $B = 1 - \Lambda$. Finally, careful examination of the boundary condition at 0 (or use of the analogy $\widetilde{\iota}_0 \equiv \iota_0 + Q_0^-$) gives $AQ_{-1} = BQ_0^-$, with $Q_0^- \equiv RQ_{-1}$. We conclude:

$$e_t = \mathbb{E}_t\left[\sum_{s=t}^{\infty}\Lambda^{s-t}(1-\Lambda)\iota_s\right] + (1-\Lambda)RQ_{t-1}.$$

Another proof is possible, relying on the machinery in Blanchard and Kahn (1980). We sketch that alternative proof here. In terms of the Blanchard-Kahn notation, the system is

$$\mathbb{E}_t\begin{pmatrix}Q_t \\ e_{t+1}\end{pmatrix} = \begin{pmatrix}R & -1 \\ -\Gamma R & 1+\Gamma\end{pmatrix}\begin{pmatrix}Q_{t-1} \\ e_t\end{pmatrix} + \begin{pmatrix}1 \\ -\Gamma\end{pmatrix}\iota_t$$

and the eigenvalues of the matrix are $\lambda_1 = \lambda$, $\lambda_2 = 1/\Lambda$. Then the value of e_t comes from the last equation

A.38

p.1309 of their paper, with $\mu \equiv (\lambda_1 - a_{11})\gamma_1 - a_{12}\gamma_2$ in their notation. $\square$

Derivation of (A.2). It is enough to prove (A.2) for $t = 0$. First, the term f_s^* comes simply from using $\widetilde{\iota}_s = \iota_s - f_s^*$ and using (A.1). The more involved term concerns the interest rates. We note $\rho_t \equiv \frac{R_{t+1}^*}{R_{t+1}} - 1$ the interest rate differential. We perform a Taylor expansion in it. The financiers' demand satisfies:

$$\Gamma Q_t = \mathbb{E}_t\left[e_t - e_{t+1}\frac{R_t^*}{R_t}\right]$$
$$= \mathbb{E}_t\left[e_t - e_{t+1}(1+\rho_t)\right] = \mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t\mathbb{E}_t\left[e_{t+1}\right]$$
$$= \mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t e_* + o(\rho_t)$$

where we approximate $\mathbb{E}_t[e_{t+1}]$ by the steady state value, e_*. For instance, $e_* = \frac{\iota_*}{\xi_*}$. Then, the Dollar-Yen balance equation $NX_t - RQ_{t-1} + Q_t = 0$ becomes (dropping for now the $o(\rho_t)$ terms):

$$e_t - \iota_t - R\frac{\mathbb{E}_{t-1}\left[e_{t-1} - e_t\right] - \rho_{t-1}e_*}{\Gamma} + \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t e_*}{\Gamma} = 0,$$

hence we have the same system as before, replacing ι_t by

$$\widetilde{\iota}_t \equiv \iota_t + \frac{e_*}{\Gamma}(\rho_t - R\rho_{t-1}),$$

using the convention $\rho_{-1} = 0$ when we calculate e_0 to simplify the boundary conditions. Indeed, we then have:

$$e_t - \widetilde{\iota}_t - R\frac{\mathbb{E}_{t-1}\left[e_{t-1} - e_t\right]}{\Gamma} + \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right]}{\Gamma} = 0.$$

Hence, equation (A.1) gives (when $RQ_{t-1} = 0$)

$$e_0 = (1-\Lambda)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\widetilde{\iota}_s\right] = (1-\Lambda)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\iota_s\right] + H$$

$$H = (1-\Lambda)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\frac{e_*}{\Gamma}(\rho_s - R\rho_{s-1})\right]$$

$$= (1-\Lambda)\frac{e_*}{\Gamma}\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s(1-R\Lambda)\rho_s\right] \text{ by rearranging (by "Abel transformation")}$$

$$= \frac{(1-\Lambda)(1-R\Lambda)}{\Gamma}e_*\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\rho_s\right],$$

and

$$\frac{(1-\Lambda)(1-R\Lambda)}{\Gamma} = \frac{\left(1-\frac{\lambda}{R}\right)(1-\lambda)}{\Gamma} \text{ using } \Lambda = \frac{\lambda}{R}$$
$$= \frac{\lambda^2 - (1+R)\lambda + R}{R\Gamma}$$
$$= \frac{\Gamma\lambda}{R\Gamma} \text{ as } \lambda \text{ satisfies (A.85)}$$
$$= \Lambda,$$

A.39

so

$$H = \Lambda e_* \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \Lambda^s \rho_s \right],$$

Hence,

$$e_0 = (1 - \Lambda) \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \Lambda^s \iota_s \right] + e_* \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \Lambda^{s+1} \rho_s \right]$$

$$= (1 - \Lambda) \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \Lambda^s \iota_s \right] + e_* \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \Lambda^{s+1} \frac{R_s^* - R_s}{R_s} \right],$$

up to higher-order terms in $R_s^* - R_s$. $\square$

Finally, we verify that the f_s^* impact is present only when $\Gamma > 0$. The net present value of the noise traders' demand is 0: $\sum_{s=t}^{\infty} \frac{1}{R^{s-t}} f_s = -F_t^{*-}$, where F_t^{*-} is the holding of the f_s^* traders at the beginning of time t. As the US net foreign assets are $N_t = Q_t + F_t^*$, when $\Gamma = 0$ (so that $\Lambda = \frac{1}{R}$) the expression reduces (when $\iota_t = \iota_*$, $r_t^* - r_t = 0$ for simplicity):

$$e_t = e_* + \mathbb{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{R} \right)^{s-t} \left[\left(1 - \frac{1}{R} \right) (-f_s^*) \right] + \left(1 - \frac{1}{R} \right) Q_t^-$$

$$= e_* + \left(1 - \frac{1}{R} \right) (Q_t^- + F_t^{*-}) = e_* + \left(1 - \frac{1}{R} \right) N_t,$$

hence the actions of the noise traders don't have an effect when $\Gamma = 0$.

Proof of Proposition A.2 Using Proposition A.1, we have:

$$\mathbb{E}_{t-1} e_t = 1 + (1 - \Lambda) R Q_{t-1}.$$

On the other hand, the equation of motion $NX_t - RQ_{t-1} + Q_t$ gives: $\mathbb{E}_0 e_t - 1 - R\mathbb{E}_0 Q_{t-1} + \mathbb{E}_0 Q_t = 0$, so $\mathbb{E}_0 Q_t = \Lambda R \mathbb{E}_0 Q_{t-1} = \lambda \mathbb{E}_0 Q_{t-1}$ and

$$\mathbb{E}_0 Q_{t-1} = \lambda^t Q_{-1}$$

The debt due at t is $Q_t^- = RQ_{t-1}$; so $\mathbb{E}_0 Q_t^- = \lambda^t Q_0^-$, and

$$\mathbb{E}_0 e_t - 1 = (1 - \Lambda) R \mathbb{E}_0 Q_{t-1} = (1 - \Lambda) \mathbb{E}_0 Q_t^- = (1 - \Lambda) \lambda^t Q_0^-$$

$\square$

Appendix References

Backus, David K., and Gregor W. Smith. 1993. "Consumption and real exchange rates in dynamic economies with non-traded goods." *Journal of International Economics*, 35(3-4): 297–316.

Basak, Suleyman, and Anna Pavlova. 2013. "Asset prices and institutional investors." *The American Economic Review*, 103(5): 1728–1758.

Blanchard, Olivier, and Charles Kahn. 1980. "The solution of linear difference models under rational expectations." *Econometrica*, 1305–1311.

Burstein, Ariel, and Gita Gopinath. 2015. "Chapter 7 - International Prices and Exchange Rates." In *Handbook of International Economics*. Vol. 4 of *Handbook of International Economics*, , ed. Kenneth Rogoff Elhanan Helpman and Gita Gopinath, 391 – 451. Elsevier.

Carvalho, Carlos, and Fernanda Nechio. 2011. "Aggregation and the PPP Puzzle in a Sticky-Price Model." *American Economic Review*, 101(6): 2391–2424.

Gopinath, Gita, and Roberto Rigobon. 2008. "Sticky Borders." *The Quarterly Journal of Economics*, 123(2): 531–575.

Lettau, Martin, Matteo Maggiori, and Michael Weber. 2014. "Conditional risk premia in currency markets and other asset classes." *Journal of Financial Economics*, 114(2): 197 – 225.