Government Policy with Time Inconsistent Voters

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Abstract

Behavioral economics presents a "paternalistic" rationale for government intervention. Current literature focuses on benevolent government. This paper introduces politicians who may indulge/exploit these behavioral biases. We present an analysis of the novel features that arise when the political process is populated by voters who may be time inconsistent, a' la Phelps and Polak (1968) and Laibson (1997). Time inconsistent voters exhibit demand for commitment. We show that electorally accountable politicians may choose policies that interfere with individuals' desire to commit, and that government may not be very effective in satisfying the demand for commitment.

1 Introduction

An important and influential approach to government policy has grown out of the field of behavioral economics.[1] A number of contributors to this area argue that some form of government policy interventions can be justified by "paternalistic attitudes" even in cases outside the realm of the textbook approach to public policy, i.e., even absent externalities, public goods, and asymmetric information.[2] In this context, a paternalistic government is viewed as a benevolent social planner who designs policy to help agents make better decisions according to their own interests.[3]

This paper presents a simple model, where, rather than a benevolent planner, policy is determined via the political process. Our approach can essentially be viewed as "behavioral

[1] Camerer et al (2004) contains a number of "second generation" contributions to behavioral economics. See also Thaler and Sunstein (2009).

[2] See for instance Camerer et al. (2003) and Thaler and Sunstein (2009).

[3] This form of paternalism is controversial, partly because it drastically deviates from standard normative economics. For contrasting points of view on this issue, see various contributions in the edited volume by Caplin and Schotter (2008).

economics meets political economy." We follow the public choice tradition of considering how public policy is determined in an environment where there is no social planner: politicians have selfish objectives such as gaining re-election. Several questions naturally emerge in this context. In particular, in environments where voters suffer from behavioral biases, will politicians seeking election exploit/indulge voters' behavioral distortions? Are behavioral distortions amenable to aggregation into collective action? What are the implications for the constitutional scope of government activity?

These questions can in principle be addressed in several environments, depending on the specific behavioral distortion, or the political process under consideration. For instance, it would seem fruitful to introduce political economy considerations in economies populated by a variety of 'behavioral agents,' that is, agents suffering from distortions in beliefs, framing, and a variety of other biases that have been considered in the literature arguing in favor of paternalistic policies. In order to illustrate some of the forces introduced by collective action, the current paper focuses on the widely studied case of time inconsistency: agents have preferences that display present-bias or quasi-hyperbolic discounting a' la Phelps-Polak (1968) and Laibson (1997). It is well known that these preferences can lead to reversals that are not consistent with standard models of exponential discounting.[4'5]

Self-control problems can lead to procrastination – doing things too late, preproperation – doing things too early (see O'Donoghue-Rabin 1999), insufficient savings for retirement (Laibson, Repetto, and Tobacman 1997), harmful obesity and addictions (Gul-Pesendorfer 2007, O'Donoghue-Rabin 2000), etc. These self-control problems also generate a demand for commitment (rehab clinics, illiquid assets with costly withdrawal, etc.) that cannot arise with exponential discounting.

In this paper we build a model of **fiscal irresponsibility and public debt.** Some of the behavioral economics literature focuses on inefficiently low savings under *laissez faire*. In order to understand how government policy may affect national savings, it is important to understand how political incentives for debt are affected by voters' time inconsistency. To study this issue, we embed politically determined government transfers in a highly stylized consumption-savings problem. We endow agents with ample commitment options by means of access to illiquid assets. Agents use illiquid assets to constrain their future selves' con-

[4]Frederick et al. (2002) surveys the experimental evidence on time discounting. Della Vigna (2009) surveys evidence from the field.

[5]Some of the issues related to time inconsistent preferences that we highlight can be represented as problems of self-control, and can also be studied in models of temptation and self-control a' la Gul-Pesendorfer (2001, 2004, 2007). The qualitative implications are similar, and in this paper we focus on the quasi-hyperbolic model.

sumption plans. The environment is designed to ensure that, absent government intervention, agents can guarantee their commitment path of consumption. We introduce government intervention by allowing office-seeking candidates to offer deficit-financed transfers to voters, subject to a maximal debt constraint. We show that for moderate debt constraints, in equilibrium, candidates choose the maximal debt, but voters are able to undo this by rebalancing their portfolios ex ante: a modified Ricardian equivalence result. When debt limits are high, however, government debt completely undermines individuals' ability to commit.

We then introduce distortions induced by government debt and show that, when the *marginal* distortions are not too high relative to the present bias of the decisive voter, equilibrium debt can still be high, leading to high *total* distortions. The logic is the following. Because debt is determined by voters' collective choices, individual saving decisions in prior periods have no impact on debt. In a three period economy, for instance, each individual voter has a *private* incentive to try to undo expected second period debt by an appropriate mix of liquid and illiquid assets, saving less for period 2 and more for period 3. But this individual optimization will, *in the aggregate*, generate demand for transfers in the second period, leading to a collective choice of even higher debt. Thus, portfolio decisions in period 1 produce collective demand for debt in the second period, even when debt is distortionary. We also show that this vicious cycle is not present in the context of an individual being offered liquid assets to undo her prior commitments (as, for instance, in Gottlieb 2008). In private credit arrangements, an individual understands that her first period choices affect her own choices in the second period. In contrast, this link between individual choices in the first period and collective choices in the second is absent in the case of government debt. This analysis offers a new rationale for balanced budget rules in constitutions.

We also show that, when the population is not too heterogeneous, first period welfare of all agents is highest if none of the agents has access to illiquid assets and hence no-one has any ability to commit to later consumption. This is because, in this scenario, no government debt is accumulated in equilibrium. Of course, for any *fixed* level of government debt, first-period selves of these agents are worse off because of the inability to commit. The inefficiency arises as a consequence of the feedback between the demand for illiquid assets in the first period and the demand for debt-financed transfers in the second period. This result provides a different interpretation of the policy recommendations from prior literature that suggest a beneficial effect of policies facilitating savings commitments (see for instance Laibson 1998).

Our paper also contributes to the political economy literature on government debt. We offer a novel explanation of debt in an environment where previously investigated forces determining debt are inoperative. This approach can potentially be useful for thinking about austerity plans that may be suggested by analysts or international organizations such as the IMF to remedy unsustainable fiscal situations. For instance, in some contexts, one can borrow insights from behavioral economics such as the idea of "save more tomorrow" proposed by Thaler and Benartzi (2004) to argue that delayed austerity may be precisely the kind of reform that may be beneficial, despite having been condemned as a "timid reform."

2 Related Literature

Some authors (Benjamin and Laibson 2003, Caplan 2007, Glaeser 2006, Rizzo and Whitman 2009 a, b) have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glaeser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models. None of these papers considers time inconsistent agents. Bendor et al. (2011) present models based on aspiration-based learning to examine a wide variety of political phenomena. Hwang and Mollerstrom (2012) study political reforms with time inconsistent voters and show that gradualism emerges in equilibrium as a consequence of time inconsistency. They also show that election of a patient agenda setter can arise in equilibrium.[6]

Our paper is also related to the literature on the political economy of government debt. Some of that literature explains debt as the outcome of a struggle between different groups in the population who want to gain more control over resources. The reason debt is accumulated is that the group that is in power today may not be in power tomorrow and debt is a way to take advantage of this temporary power. For instance, Cukierman and Meltzer (1989) and Song, Storesletten, and Zilibotti (2010) argue that debt is then a tool to redistribute resources across generations. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) argue that debt is a way to tie the hands of future governments which have different preferences from the current one. In Tabellini and Alesina (1990) voters choose the composition of public spending in an environment where the median voter theorem applies. If the median voter remains the same in both periods the equilibrium involves budget balance. If the median voter tomorrow has different preferences, the current median voter may choose to run a budget debt to take advantage of his temporary power and tie the hands of the future government. The equilibrium may also involve a budget surplus because there is an "insurance" component that links the two periods as well: a surplus tends to equalize the median voter's utility in the two periods. Tabellini and Alesina

[6] Ortoleva and Snowberg (2012) look at the potential effects of over-confidence on electoral outcomes.

give conditions such that debt will be incurred and show that increased polarization leads to larger debt levels. Battaglini and Coate (2008) present a dynamic model of taxation and debt, where a rich policy space is considered within a legislative bargaining environment. Velasco (1996) suggests a model where government resources are a "common property" out of which interest groups can finance their own consumption. Debt arises in his model as a consequence of a dynamic "common pools" problem. Lizzeri (1999) presents a model of debt as a tool of redistributive politics.[7]

In all these models voters are time consistent. Krusell, Kurusçu, and Smith (2002, 2010) examine government policy for agents who suffer self-control problems. Krusell, Kurusçu, and Smith (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (or unjust) world and ultimately affect redistributive politics.

3 A Model of Fiscal Irresponsibility

3.1 Economy

3.1.1 Preferences

We first consider a particularly simple three period model to highlight the basic idea in a particularly stark fashion.

There is a measure 1 of voters who live for three periods. To make things particularly simple, assume that in period 1 voters have a wealth k from which to finance consumption over three periods. No endowment is available in the other two periods.[8] As in Laibson (1997), preferences over consumption sequence c_1, c_2, c_3 are given by

$$\begin{aligned} &U_1\left(c_1, c_2, c_3\right) = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3), \\ &U_2\left(c_2, c_3\right) = u(c_2) + \beta\delta u(c_3), \\ &U(c_3) = u(c_3), \end{aligned} \tag{1}$$

[7] Tabellini (1991) also illustrates how debt and social security differ as distributional instruments in an overlapping generations environment.

[8] This can also be interpreted as a model with positive endowments in all periods but with consumer able to borrow against future endowments in period 1.

where u is a continuous and strictly concave utility function. We also assume that the utility function is three times continuously differentiable. For the moment we assume that all agents are identical. We later consider the effects of heterogeneity. For expositional simplicity, and since our main focus is on the impacts of time inconsistency, we assume that $\delta = 1$.

It is well-known that agents with quasi-hyperbolic preferences suffer from time inconsistency, and therefore exhibit demand for commitment. Let $c_1^*(\beta)$, $c_2^*(\beta)$, and $c_3^*(\beta)$ denote the optimal consumption sequence with commitment in period 1. Namely, $c_1^*(\beta)$, $c_2^*(\beta)$, and $c_3^*(\beta)$ maximize $U_1(c_1, c_2, c_3)$ subject to $c_1 + c_2 + c_3 \leq k$. Let $c_1^U(\beta)$, $c_2^U(\beta)$, and $c_3^U(\beta)$ denote the optimal consumption sequence without commitment. Denote by $s_1^U(\beta)$ and $s_2^U(\beta)$ savings absent commitment. Without commitment, $c_2^U(s_1, \beta)$ maximizes $U_2(c_2, c_3)$ subject to $c_2 + c_3 \leq s_1$. $c_1^U(\beta)$ maximizes $U_1\left(c_1, c_2^U(s_1, \beta), s_1 - c_2(s_1, \beta)\right)$. Thus, $c_2^U(\beta) = c_2^U\left(k - c_1^U(\beta), \beta\right)$ and $c_3^U(\beta) = k - c_2^U(\beta) - c_1^U(\beta)$.

To highlight the demand for commitment, consider any $\beta < 1$, and the commitment consumption sequence $c_1^*(\beta)$, $c_2^*(\beta)$, $c_3^*(\beta)$. This sequence must satisfy

$$u'(c_2^*(\beta)) = u'(c_3^*(\beta))$$

which implies that

$$u'(c_2^*(\beta)) > \beta u'(c_3^*(\beta)).$$

Thus, in period 2, the agent would like to transfer resources from the third period to the second to obtain a consumption that is strictly higher than $c_2^*(\beta)$.

The following Lemma provides a preliminary fact about the effects of commitment on consumption.[9]

Lemma 1

Commitment leads to lower second period consumption: $c_2^*(\beta) < c_2^U(\beta)$.

3.1.2 Financial structure and commitment

We assume that in period 1 voters can choose to invest in liquid or illiquid assets. Assume all liquid and illiquid assets have the same exogenous rate of return of zero.[10] Illiquid assets are two-period securities that cannot be sold in period 2. Liquid assets are one period securities. Absent government intervention in period 2, by appropriate choice of the mix of liquid and

[9] Part (ii) and the assumption on u''' will only be used for the discussion of the case pertaining to heterogeneous preferences.

[10] Our main results do not depend on this partial equilibrium assumption.

illiquid assets, a voter can commit to any desired consumption stream for periods 2 and 3. We denote savings in one period assets, in periods 1 and 2, by $s_{1,2}$ and $s_{2,3}$ respectively, and savings in illiquid assets (relevant for period 1,in which illiquid assets are to become available in period 3) by $s_{1,3}$.

The interplay between agents' desire to commit in period 1 and government decisions in period 2 is a key effect in our model. Allowing for imperfect commitment generates some interesting effects but the major forces are similar. We will return later to the consequences of allowing for differences in returns of liquid and illiquid assets, as well as of allowing the government to subsidize illiquid assets (for instance, as in the case of retirement plans).

3.2 Polity

We now introduce a government that takes actions in periods 2 and 3.[11] There are two candidates running for office. Candidates are office motivated: they receive some positive benefit from electoral victory and hence choose electoral platforms to maximize the probability of winning.[12]

It will soon be clear that candidates' time preferences play no role, and they need not be the same candidates in the two periods. There are simple majoritarian elections in periods 2 and 3. In period 2, each candidate offers a platform given by (y, t) where y is a per capita transfer and t is a lump-sum tax. Let $d = y - t$ denote per capita government debt in period 2. When taxes are non distortionary, all that matters is debt. If taxes are distortionary, in this model there is no reason to have positive contemporaneous taxes. Thus, from now on, we assume that transfers are debt financed so we equate debt and transfers. As a tie-breaking rule, we assume that whenever individuals are indifferent between the two candidates, they vote for either with equal probability.[13]

In what follows, we first consider a benchmark in which debt is non-distortionary and then move on to the case of distortionary debt. We assume debt is financed by foreign lenders at zero interest rate (tantamount to assuming a small open economy),[14] to be repaid by third-

[11] We consider the effects of first period elections later.

[12] It is natural to begin the analysis with standard "Downsian" candidates. We discuss the question of ideologically motivated or virtuous candidates below.

[13] This is akin to assuming that agents have lexicographic preferences that: a. respond to policy first, and upon indifference, to the identity of the candidate; and b. are uniformly distributed with respsect to the preferred candidate.

[14] The main forces present in our model would remain even if we considered interest rate determination in a closed economy. General equilibrium effects are subtle, however, when agents are quasi-hyperbolic (see Krussel et al. 2002).

period revenues raised by lump-sum taxes.[15] We wish to study the effects of constitutionally imposed borrowing limits on the government; let $\overline{d}$ denote the per capita value of this limit.

In period 1 an agent with present bias β, who predicts equilibrium per-capita debt levels of d, chooses savings intended for period 2, denoted by s_{12} and for period 3, denoted by s_{13}, to solve

$$\max_{s_{12},s_{13}} u\left(c_1\right) + \beta u\left(s_{12} + d - s_{23}\right) + \beta u\left(s_{13} + s_{23} - d\right).$$

Note that, since there is a large number of voters, this agent takes as given the second period debt when making first-period choices.[16]

In period 2 a voter with preference parameter β chooses savings s_{23} to solve

$$\max_{s_{23}} u\left(s_{12} + d - s_{23}\right) + \beta u\left(s_{13} + s_{23} - d\right).$$

The resulting optimal consumption sequence is denoted $c_1\left(d\right), c_2\left(d\right), c_3\left(d\right)$. Suppose that candidate A chooses a debt d_A and candidate B chooses debt d_B. Then the voter votes for A in period 2 whenever $u\left(c_2\left(d_A\right)\right) + \beta u\left(c_3\left(d_A\right)\right) > u\left(c_2\left(d_B\right)\right) + \beta u\left(c_3\left(d_B\right)\right)$.

4 Equilibrium Debt and its consequences

We now charachterize equilibrium in this world with time-inconsistent agents for all possible constraints on debt accumulation. We first discuss the case of zero distortions from debt and then move on to consider the effects of distortions. This will allow us to address how some policy/constitutional issues are informed by taking a political economy approach to debt accumulation.

4.1 Ricardian Equivalence with Time Inconsistent Voters

In this section we analyze the effects of government debt on private consumption levels. As it turns out, the magnitude of the cap on debt is crucial in terms of outcomes: low caps allow agents to consume their optimal commitment levels, while larger caps lead to distorted levels of consumption.

[15] For most of our analysis, period 3 elections are vacuous. We return to the effects of additional periods in Section 6.

[16] The belief by agents that their individual savings behavior does not affect second-period debt is clearly correct for the case assumed here with a continuum of voters. With a finite electorate it would be possible to construct different equilibria but these would not be robust to adding some forms of noise in the second period (e.g., noise in second-period turnout). Roughly, what is required is that in the first period no agent believes that she is likely to be pivotal in the second period.

The following result characterizes equilibria for all possible debt limits. There is always an incentive for politicians to promise debt-financed transfers but the consequences of such debt on agents' equilibrium consumption depend on how tight debt limits are.

Proposition 1 (Incomplete Ricardian Equivalence) (i) If $\overline{d} \leq c_2^*(\beta)$ then both candidates offer platforms with debt $\overline{d}$. Equilibrium consumption is $(c_1^*(\beta), c_2^*(\beta), c_3^*(\beta))$.

(ii) If $c_2^*(\beta) < \overline{d} < c_2^U(\beta)$ then both candidates offer platforms with debt $\overline{d}$. In equilibrium, second-period consumption is $c_2 = \overline{d}$.

(iii) If $\overline{d} \geq c_2^U(\beta)$ then any d *such that* $c_2^U(\beta) \leq d \leq k$ is part of an equilibrium. Equilibrium consumption is $\left(c_1^U(\beta), c_2^U(\beta), c_3^U(\beta)\right)$.

Proof. (i) Assume by way of contradiction that, in equilibrium, a debt $d^* < \overline{d}$ is implemented. If this is the case, then a voter can implement the commitment sequence of consumption $c_1^*(\beta)$, $c_2^*(\beta)$, $c_3^*(\beta)$ by choosing $s_{12}(\beta) = c_2^*(\beta) - d^*$, and $s_{13}(\beta) = c_3^*(\beta) + d^*$. This is feasible since $d^* < \overline{d} < c_2^*(\beta)$. Hence, these are the optimal choices for the voter. But, by definition of $c_2^*(\beta)$, $c_3^*(\beta)$, $u'(c_2^*(\beta)) > \beta u'(c_3^*(\beta))$, and therefore, in period 2 all voters would vote for a candidate who offered a slightly higher debt. Thus, the only debt that can be an equilibrium is $\overline{d}$. Given a debt of $\overline{d}$, in period 1, each voter chooses $s_{12}(\beta) = c_2^*(\beta) - \overline{d}$, $s_{13}(\beta) = c_3^*(\beta) + \overline{d}$. Given these saving choices, none of the voters would vote for a candidate that offered a lower debt in the second period, proving that debt and this sequence of consumption constitute a unique equilibrium.

(ii) Assume by way of contradiction that, in equilibrium, a debt $d^* < \overline{d}$ is implemented. As in part (i), voters choose savings to restore commitment as much as possible. Assume that $c_2^*(\beta) < d^*$ (otherwise, the proof of part (i) applies). The agent maximizes

$$\begin{aligned} & u(c_1) + \beta u(c_2) + \beta u(k - c_1 - c_2) \\ s.t. \quad c_2 \ &\geq \ d^*. \end{aligned}$$

The first order conditions yield

$$u'(c_1) = \beta u'(k - c_1 - d^*) > u'(c_2) = u'(d^*)$$

because $d^* > c_2^*(\beta)$ (recall: $u'(c_2^*) = u'(c_3^*)$). This means that the agent sets $s_{12} = 0$ because second-period consumption is already higher than desired by the first-period self. However, since $d^* < c_2^U(\beta)$, $u'(d) > \beta u'(c_3)$. Thus, in period 2 all voters would vote for higher debts contradicting the assumption that d is an equilibrium debt level. Finally, to conclude that a debt of $\overline{d}$ is indeed part of an equilibrium, observe that, given $\overline{d}$, by similar reasoning, the optimal saving choices of all voters would lead to $u'\left(\overline{d}\right) > \beta u'(c_3)$. Thus, no voters would vote for lower debts.

(iii) We first show that the claimed outcomes are part of an equilibrium. Given any candidate equilibrium debt $k > d^* \geq c_2^U(\beta)$ that is expected by voters in period 1, an optimal policy by a voter of type β in period 1 is a choice of $s_{12} = 0$ and $s_{13}(\beta) = c_3^U(\beta) - \left(d^* - c_2^U(\beta)\right)$. In addition, given d^*, in equilibrium, $s_{23}(\beta) = d^* - c_2^U(\beta)$ is to be saved in period 2 for period 3. Given this policy, by the definition of $c_2^U(\beta), c_3^U(\beta)$, we have

$$u'\left(c_2^U(\beta)\right) = \beta u'\left(c_3^U(\beta)\right)$$

giving no incentive to any period-2 self to change her savings plan away from s_{23}. Suppose now that the period-1 self were to change (e.g., increase) s_{13}. Then, the period-2 self would make an offsetting change (reduction) in s_{23} to restore period 2 optimality. Any change in s_{12} would similarly be offset (recall that since $d^* \geq c_2^U(\beta)$, even if $s_{12} = 0$, the period-2 self can unilaterally choose $c_2^U(\beta)$). Thus, the period-1 self has no incentive to deviate.[17]

Given these policies for the voters, consider a deviation to $d < d^*$ in period 2. As long as the deviation is small ($d \geq \max c_2^U(\beta)$), all voters are indifferent (they can just make an offsetting reduction in s_{23} to restore the desired consumption sequence). If the deviation is large ($d < c_2^U(\beta)$), then voters who can no longer make such offsetting reduction in s_{23}. All voters would therefore vote against a candidate offering such a deviation. A deviation to $d > d^*$ would leave all voters indifferent because they could make offsetting changes in s_{23}.

Consider now a candidate equilibrium debt $d^* < \max c_2^U(\beta)$. Such an expected debt would constrain period-2 consumption for a positive mass of voters, leading to victory in period 2 for a candidate offering $d > d^*$. ■

To gain an intuition for this result, it is useful to consider a sequence of deviations from zero debt to the equilibrium level of debt. As a first step, suppose that individuals expect zero debt. Then, equilibrium outcomes would coincide with those in an economy with no government involvement, with all agents committing to $(c_1^*(\beta), c_2^*(\beta), c_3^*(\beta))$. However, this cannot constitute an equilibrium because then, in period 2, all agents would find themselves constrained and would therefore vote for a candidate that offered positive debt. As a second step, consider increasing debt from step 1 (i.e., zero debt) and check if satisfying this initial demand for debt is sufficient to reach an equilibrium. Namely, let us set debt d_1 such that

$$u'(c_2^* + d_1) = \beta u'(c_3^* - d_1).$$

[17] There are multiple ways for the period-1 self to implement the uncommitted sequence, involving increasing s_{12} and s_{23} by the same amounts with offsetting reductions to s_{13}. All these are weakly dominated by the proposed sequence in the text.

This is the level of debt that is the equilibrium of the period-2 election *given* that savings are determined by individuals expecting zero debt and committing to their desired sequence of consumption. This clearly is not an equilibrium either: if agents expect debt d_1, they react by reducing $s_{12}(d_1)$ and increasing $s_{13}(d_1)$ to restore the commitment allocation. We can proceed to find the equilibrium (higher) second-period debt that will be demanded by voters given the lower savings for the second period. It is easy to see that this leads to $d_2 = 2d_1$.

When will this process stop? If the debt limit is binding, the process continues until debt hits the debt limit (parts (i) and (ii) of the proposition). If the debt limit is loose, the process continues until commitment is fully unraveled (case (iii)) because s_{12} cannot go below zero.

In part (i) of this proposition, when the debt limit is low, voters can anticipate government debt and reduce savings intended for period 2 to restore the desired (commitment) sequence of consumption. In a sense, the low debt cap provides a form of commitment by the government not to succumb to individuals' revised preferences in later periods.[18] Because of this saving behavior in period 1, given any anticipated debt level in the feasible range, all voters would like even higher debt in order to consume more in period 2 (they are endogenously liquidity constrained in period 2).

In contrast, in part (ii), equilibrium debt is sufficiently high that agents are no longer able to restore their desired commitments consumption sequence in period 1, but in period 2 voters are still constrained so they vote for candidates who offer maximal debt. Clearly, in the scenario depicted in Proposition 1, debt is no longer neutral, but we postpone the discussion of lack of neutrality.

In the case of part (iii), the debt cap $\overline{d}$ is large. In such cases, the government can no longer commit not to indulge agents' period 2 preferences and consumption is distorted relative to the optimal commitment levels for all agents. This result shows that, even when there are no distortions, if constraints on government action are loose, then government policy is distortionary because it interferes with individuals' ability to commit. Debt allows the government to undo the private commitments chosen by the voters in the prior period. Thus, the government acts as an enabler of the voters, substituting fiscal irresponsibility for private irresponsibility. Private commitments are not sufficient to induce consumption commitments: state commitment (such as tighter balanced budget constitutions) are essential. All period 1 selves (corresponding to all β's) are made better off by tighter limits that lower $\overline{d}$ so as to restore agents' abilities to commit.[19]

[18] Notice that this result relies on the fact that agents foresee perfectly their susceptibility to temptations. We relax this assumption of perfect foresight in the following section.

[19] Notice that we implicitly assume that individuals cannot commit themselves into debt (they can assure

We now comment on Ricardian equivalence with time inconsistent voters. Clearly there is no general (global) Ricardian equivalence since, in different regions for the debt limit, consumption is different. However, there is a "local" version of Ricardian equivalence for sufficiently low debt limits. Furthermore, there is no "contemporaneous" Ricardian equivalence in cases (i) and (ii) of Proposition 1: if there is a surprise increase in the debt limit (and/or debt) in period 2, then agents are not able to undo this by contemporaneous changes in their savings because they succumb to self-control problems. However, when debt limit is not too high (case (i)), voters can anticipate government debt and reduce savings intended for period 2 to restore the desired consumption sequence, which we view as a local Ricardian equivalence. When the debt limit is loose (case (iii)), commitment is undone but, again, we obtain local Ricardian equivalence even with contemporaneous changes: since agents are no longer able to commit, second period consumption is optimal for the second-period self and Ricardian equivalence obtains.

Even with no distortions, if constraints on government action are loose, then government policy is distortionary: it interferes with individuals' ability to commit. Debt allows government to undo the private commitments chosen by voters in prior periods. Government acts as an enabler for the voters, substituting fiscal irresponsibility for private irresponsibility. Private commitments are not sufficient to induce consumption commitments: state commitments (such as tighter balanced budget constitutions) are essential. Period-1 selves are made better off by lower debt limits that restore commitment.

These results may seem closely related to the inefficiency of competitive credit markets when consumers are time inconsistent: even if consumers can buy illiquid assets to attempt to commit to a future consumption path, intermediaries such as credit card companies have the incentive to enter the market, leading to an undoing of commitment.[20] However, the force underlying these results is quite different, and can lead to more dramatic inefficiencies. In order to see this we must move to a world with distortions. We discuss the comparison with private debt explicitly in Section 5.1.

a minimal wealth of zero in period 2). Were they able to commit themselves to a personal debt of up to d_P, the results of the proposition would carry through, with an appropriate shift of the debt limit by d_P.

[20] This point has been made by a number of authors. Gottlieb (2008) provides a detailed analysis of the effects of competition in markets with time inconsistent consumers.

4.2 The Effects of Distortions

In the environment considered up to now, debt was not directly distortionary: the distortions originated only from the effect of debt on individuals' private commitments.

We now consider the case in which government debt can be directly distortionary. There are a number of ways in which this can happen. For instance, debt could interfere with optimal smoothing of tax distortions, or because the small open economy assumption is violated, and debt has general equilibrium effects, or because the rate at which resources can be borrowed from abroad is high relative to citizens' discount rate.

In this initial analysis we assume a simple distortion: for every dollar raised in period 3 to transfer resources to period 2, a fraction η is destroyed. Thus, a per-capita debt of d to be paid in period 3 yields $d(1-\eta)$ in period 2. This is a reduced form way to capture distortions that could come from a variety of sources as mentioned above.[21]

Given savings from period 1 of s_{12} and s_{13}, a voter of type β in period 2 would choose debt to maximize $u(s_{12}+d(1-\eta))+\beta u(s_{13}-d)$. The first order condition is $u'(c_2)(1-\eta)=\beta u'(c_3)$. In contrast, the analogous first order condition evaluated in period 1 is $u'(s_{12})=u'(s_{13})$. It follows that for any individual with preference parameter $\beta<(1-\eta)$, period-2 self still wants to transfer resources from the third to the second period at the commitment solution.[22]

The definition of optimal consumption levels now involves a subtlety that was absent in the case of no distortions: while in the previous analysis the optimal consumption sequences with and without commitment were independent of government debt (as long as debt was relatively small), this is no longer the case when there are distortions, because debt destroys wealth. Let $c_1^*(\beta;d), c_2^*(\beta;d)$, and $c_3^*(\beta;d)$ be the commitment sequence of consumption given debt d. Namely, $c_1^*(\beta;d), c_2^*(\beta;d)$, and $c_3^*(\beta;d)$ is the solution of the following problem:

$$\begin{aligned} & \max\left\{u(c_1)+\beta\left(u(c_2)+u(c_3)\right)\right\} \\ s.t.\quad c_1+c_2+c_3 &= k-\eta d \end{aligned}$$

Analogously, let $c_1^U(\beta;d), c_2^U(\beta;d)$, and $c_3^U(\beta;d)$ be the corresponding quantities without commitment.

[21] Of course, there is no particular reason to expect these distortions to be proportional. This is assumed mainly for convenience. The qualitative analysis of this section does not depend on this assumption. We will consider convex distortions later and show that some of the main features remain unchanged.

[22] Note that we are implicitly assuming here that, with no government debt, consumers still face a rate of return of 1 on both the liquid and the illiquid assets. Our analysis is qualitatively unchanged if we assume that the rate a consumer at time $t=2$ faces on borrowing from private markets is $\frac{1}{1-\eta}$, that is, equal to the rate faced by the government when issuing public debt; See the analysis below.

For expositional simplicity, we assume that utilities are such that both the commitment and the no-commitment consumption sequences are continuous in the debt level d. By construction, they are all decreasing functions of d.

As in the case of no distortions, the behavior of equilibrium debt and consumption is divided into three regions depending on the debt limit. In order to determine the limits of these regions we need to define two values of debt that we call d^* and d^{**}.

Define d^* as the solution of $c_2^*(\beta; d^*) = d^*$

We now introduce an artificial constrained-maximization problem for a voter of preference parameter $\beta < 1 - \eta$.

$$\begin{aligned} & \max u(c_1) + \beta \left[u(c_2) + u(c_3) \right] \\ s.t. \;\; u'(c_2) &= \frac{\beta}{1-\eta} u'(c_3), \\ c_1 + c_2 + c_3 &= k - d\eta. \end{aligned} \tag{2}$$

The first constraint is a "relaxed" commitment constraint, where resources transferred between period 3 and 2 suffer a unit loss of η. This will be the relevant constraint in determining debt in the second period. The smaller the distortion η, the tighter this constraint. The second constraint reflects the loss of resources due to the distortion. Denote by $V^u(\beta, \eta)$ the value of this problem and by $(c_1^\eta(\beta, d), c_2^\eta(\beta, d), c_3^\eta(\beta, d))$ the consumption sequence that solves the problem . We now define d^{**} to be the solution of $d^{**} = c_2^\eta(\beta, d^{**})$.[23]

It is easy to show that $d^* < d^{**}$.

Proposition 2 (Distortionary Equilibrium Debt) *(i) If $\beta > 1 - \eta$, then in equilibrium there is no debt and consumption is given by $(c_1^*(\beta), c_2^*(\beta), c_3^*(\beta))$.*

ii) Assume that $\beta < 1 - \eta$. If $\overline{d} \leq d^*$, then equilibrium debt is given by $\overline{d}$ and consumption is given by $\left(c_1^*\left(\beta; \overline{d}\right), c_2^*\left(\beta; \overline{d}\right), c_3^*\left(\beta; \overline{d}\right)\right)$. If $d^* < \overline{d} \leq d^{**}$, then equilibrium debt is given by $\overline{d}$ and period 2 consumption is given by $c_2 = \overline{d}$. If $\overline{d} > d^{**}$, then debt is given by d^{**} and period 2 consumption is given by $c_2 = d^{**}$.

Proof. (i) We first show that there is an equilibrium with zero debt. Given an expected second-period debt of zero, in period 1 voters choose the mix of liquid and illiquid assets $s_{12} = c_2^*(\beta)$ and $s_{13} = c_3^*(\beta)$ that implements the commitment consumption sequence $(c_1^*(\beta), c_2^*(\beta), c_3^*(\beta))$. Given this mix of savings, $u'(c_2^*(\beta)) = u'(c_3^*(\beta))$. Thus, if $\beta > 1 - \eta$, $u'(c_2^*(\beta)) < \frac{\beta}{1-\eta} u'(c_3^*(\beta))$ and no voter has an incentive to vote for positive

[23] Note that the intermediate value theorem assures that such d^* and d^{**} always exist since $c_2^*(\beta^*; 0), c_2^U(\beta^*; 0) > 0$, and both $c_2^*(\beta^*; 0)$ and $c_2^U(\beta^*; 0)$ are continuous and bounded.

debt. Consider now any level of expected debt d. The mix of savings has to be such that $u'(s_{12}+d) \leq u'(s_{13}+s_{23}-d)$. But then $u'(s_{12}+d) < \frac{\beta}{1-\eta}u'(s_{13}+s_{23}-d)$, inducing all voters to vote to reduce debt.

(ii) Consider now the case in which $\beta < 1-\eta$. Given any $\overline{d} < d^*$ and any expected $d \leq \overline{d}$, optimal savings in period 2 are given by $s_{23}=0$ and s_{12}, s_{13} are such that $u'(s_{12}+d) = u'(s_{13}-d)$. Thus, $u'(s_{12}+d) > \frac{\beta}{1-\eta}u'(s_{13}-d)$ and voters would vote to increase debt. Thus, in this scenario equilibrium debt must be $\overline{d}$ and consumption must be given by $\left(c_1^*\left(\beta;\overline{d}\right), c_2^*\left(\beta;\overline{d}\right), c_3^*\left(\beta;\overline{d}\right)\right)$. If $d^* < \overline{d} \leq d^{**}$, then, by the same reasoning, equilibrium debt must be at least d^*. But then, by the definition of d^*, debt is higher than second-period commitment consumption, and optimal savings are at a corner: $s_{12}=s_{23}=0$, implying that $c_2 = d$. Because $d < d^{**}$, we then have that $\frac{\beta}{1-\eta}u'(c_3) < u'(c_2) < u'(c_3)$. This implies that voters vote for higher debt unless $d=\overline{d}$. Finally, If $\overline{d} \geq d > d^{**}$, then by the definition of d^{**}, $u'(d) < \frac{\beta}{1-\eta}u'(c_3)$, so voters would vote to reduce debt. This proves that, for any $\overline{d} \geq d^{**}$ equilibrium debt is given by d^{**}. ■

This result says that debt accumulation can result in very large distortions in a world where voters are time inconsistent. The intuition is fairly similar to the one that we described for the case of no distortions, and a similar iteration of steps can be illustrated for this case. Because debt is determined by voters' collective choices, individual saving decisions in prior periods have no impact on debt: voters have an incentive to try to undo expected second period debt by optimizing their mix of liquid-illiquid assets by saving less for period 2 and more for period 3. But, when the debt ceiling is not too low, this individual optimization will, *in the aggregate*, generate demand for transfers in the second period, leading to voting for a positive debt. Thus, savings decisions in period 1 generate their own demand for debt in the second period, even if this is distortionary.

5 Institutions, Welfare, and Policy

We now evaluate how welfare in the equilibrium allocation presented in Proposition 2 compares with several alternative benchmarks/policies. We consider: 1. Private debt incurred via market intermediaries; 2. Social planner without commitment; 3. Banning of illiquid assets, thereby eliminating commitment possibilities; and 4. Tighter debt limits.

In order to evaluate these scenarios, it is useful to understand the welfare consequences of distortions. The following result provides a comparison of equilibrium welfare with and without distortions.

Proposition 3 (Welfare Effects of Distortions) *For any homogeneous population of preference parameter β, the equilibrium with positive distortions $0 < \eta < 1 - \beta$ leads to lower first period welfare than the equilibrium corresponding to no distortions, $\eta = 0$. If $\eta > 1 - \beta$, then first period welfare is higher than that induced by any $\eta < 1 - \beta$.*

The proof of this proposition is in the Appendix. As mentioned above, there are two contrasting effects of positive distortions. On the negative side, given that there is debt in equilibrium, the presence of distortions causes wealth destruction. On the positive side, distortions relax the commitment constraint in the artificial maximization that determines equilibrium debt. In fact, when η is very high ($\eta > 1 - \beta$), distortions serve as a full commitment device since, in equilibrium, voters do not vote for positive debt in the second period. The proposition shows that the negative effect dominates.

5.1 Private Debt versus Public Debt

Consider now an environment in which there is no government. However, individuals can borrow on the private market from intermediaries such as credit card companies. The model is otherwise the same as in Section 3. For the purposes of comparison with our analysis of government debt, assume that credit card companies charge a proportional fee η for every dollar borrowed in the second period. This could be due to markups in an imperfectly competitive credit market or to costs born by credit card companies. We do not claim that this is a realistic model of credit card debt with or without time inconsistency.[24] The point of this stark model is to draw an important contrast between private and public debt.

The agent solves the following problem:

$$\begin{aligned} & \max_{s_{12}, s_{13}} u(c_1) + \beta \left[u(s_{12} + (1-\eta)\, d(s_{12}, s_{13})) + u(s_{13} - d(s_{12}, s_{13})) \right] \\ s.t. \quad c_1 + c_2 + c_3 &= k - d(s_{12}, s_{13})\, \eta \end{aligned} \tag{3}$$

where debt $d(s_{12}, s_{13})$ is determined in the second period and must satisfy the following condition:

$$u'(s_{12} + (1-\eta)\, d(s_{12}, s_{13})) = \frac{\beta}{1-\eta} u'(s_{13} - d(s_{12}, s_{13})) \text{ for any } s_{12}, s_{13}.$$

Proposition 4 (Equilibrium with Credit Cards) *For any $\eta > 0$, agents make portfolio decisions in period 1 that ensure that no transactions take place with credit card companies.*

[24] See Angeletos et al. (2001) for a model with coexistence of credit card debt and investment in illiquid assets.

For any β, the equilibrium consumption sequence is given by $\left(c_1^{\eta}(\beta,0), c_2^{\eta}(\beta,0), c_3^{\eta}(\beta,0)\right)$. In equilibrium first period welfare is increasing in η.

Proof. Suppose s_{12}^*, s_{13}^*, and $d\left(s_{12}^*, s_{13}^*\right) > 0$ constitute part of an equilibrium. The agent can choose to set $s_{12} = s_{12}^* + (1-\eta)\, d\left(s_{12}^*, s_{13}^*\right)$, $s_{13} = s_{13}^* - d\left(s_{12}^*, s_{13}^*\right)$. She would then satisfy $u'\left(s_{12} + (1-\eta)\, d\left(s_{12}, s_{13}\right)\right) = \frac{\beta}{1-\eta} u'\left(s_{13} - d\left(s_{12}, s_{13}\right)\right)$ with $d\left(s_{12}, s_{13}\right) = 0$. But then she will have saved $\eta d\left(s_{12}^*, s_{13}^*\right)$ which she can consume in period 1. This is a profitable deviation.

$\left(c_1^{\eta}(\beta,0), c_2^{\eta}(\beta,0), c_3^{\eta}(\beta,0)\right)$ clearly solves the above problem. ■

The logic of this result is the following. The availability of credit in the second period limits the commitment possibilities for time-inconsistent agents. However, sophisticated agents anticipate this issue and take appropriate steps to counteract this temptation. Every consumption profile that is attainable via positive debt with credit cards is also attainable with an appropriate mix of liquid-illiquid assets. Thus, with positive distortions it cannot be the to ever end up with positive credit car debt. Agents internalize the commitment constraint in period 2 and 'give up on commitment' just enough that they do not waste resources by dealing with credit card companies. Clearly first period welfare is increasing in η because higher η relaxes the commitment constraint.

This result provides a stark contrast with Proposition 3, and the key difference is the fact that public debt is a result of collective action, so individuals have a private incentive to undo public debt.

We have discussed private debt and public debt separately, assuming that debt is either public or private but not both. One can easily examine a model with coexistence of private and public debt. In our model with linear distortions, the coexistence yields uninteresting results. Let η_G be the distortion associated with public debt and η_P the distortion (markup) associated with private debt. Then, if $\eta_G < \eta_P$, it is possible to show that only public debt matters. If $\eta_G > \eta_P$, then only private debt matters. It is not clear what assumption is more reasonable (e.g., interest rates on credit card debt often exceed 20%).[25] However, this result hinges on the linearity of the distortions. It can also be shown that, if distortions are convex, then private debt and public debt coexist. However, the model is much more complicated in this case.

[25] Notice that when private and public debt are both available, there can be a multiplicity of equilibria. Indeed, if no one takes on private debt at the outset, there is no demand for public debt later on (and any agent putting themselves into private debt at the beginning will not be able to repay). However, if everyone takes on private debt, there is a collective demand for public debt later on, which sustains the initial individual private demands for debt.

5.2 Social planner without commitment

In the environment we study, voters are time inconsistent while politicians simply pursue office in each period. As an alternative, consider a situation in which a time-inconsistent social planner, sharing the population preference parameter β, determines consumption allocations. Notice that this would correspond to the decision process emerging in a citizen-candidate version of our model. As for the case of private debt, it is easy to see that the allocation determined by such a social planner is given by $(c_1^\eta(\beta,0), c_2^\eta(\beta,0), c_3^\eta(\beta,0))$, namely by the solution of the maximization problem given in (2). Therefore, first-period welfare is increasing in η since higher distortions lessen the commitment constraint.[26] This is clearly in stark contrast with the result in Proposition 3. Thus, in our setting, there is an interesting non-monotonicity in the effect of government intervention: moderate government intervention in the form of democratically elected politicians offering debt-financed transfers leads to worse outcomes than either decentralized allocations or fully centralized allocations. This is in principle informative about the debate over libertarian paternalism.

5.3 Period 1 Financial Structure

We now consider the socially optimal mix of liquid and illiquid assets when government is fiscally irresponsible. A common argument in the behavioral literature is that in environments with time-inconsistent agents, an efficiency enhancing paternalistic policy is to subsidize or otherwise promote the existence of illiquid commitment assets.

Our results suggest that, in evaluating such policies, it is important to consider how this affects the political economy of debt.

When agents have no access to illiquid assets they have no commitment power. This can arise whenever, say, agents have access to personal credit cards (with a rate of return of 1) that allow them to undo any commitment plan they have entered in earlier periods. Alternatively, whenever agents have access to illiquid assets and debt is non-distortionary and has no limit, agents are effectively tied to uncommitted consumption paths. The comparison with such environments is less straightforward since it presents a trade-off. On the one hand, debt allows for some level of commitment when illiquid assets are available. On the other hand, it entails a wealth loss. Specifically, in our model, an implication of Proposition 3

[26] Krusell et al. (2002) show that in an economy with capital accumulation there is an additional issue in contrasting a decentralized economy and a social planner without commitment. Specifically, the social planner takes into account the fact that, while individuals take the returns to savings as given, the social planner takes into account the fact that, with decreasing returns, increased aggregate savings reduce returns to capital accumulation. This leads to even worse undersaving when a social planner is present.

is that welfare is higher for all selves when illiquid assets are banned or taxed, rather than subsidized.

Proposition 5 (Banning Illiquid Assets) *Assume that $\beta < 1 - \eta$. Then, first period welfare is higher if illiquid assets are banned.*

Of course, this result should be evaluated with caution since there may be many reasons why the personal benefits of commitment are not offset by subsequent increases of government debt. However, it provides a useful additional effect to be aware of when evaluating the appropriate asset mix. The result easily extends to allow for some heterogeneity in β. As long as the heterogeneity is not too large, all selves of all agents are made better off by eliminating illiquid assets. We discuss the effects of heterogeneity in more detail below.

5.4 Debt Limits

It is clear that in our context, tighter debt limits operate by eliminating access to distortionary debt while maintaining agents' access to illiquid assets. Thus, this should increase welfare since it allows agents to maintain their full commitment consumption patterns without destroying wealth through debt distortions. The analysis is made more complex when we consider heterogeneous agents, and intermediate debt limits, as well as constitutional means that would work to impose such debt limits without being overturned in the future.[27]

Suppose now that debt limits are finite. For sufficiently low debt limits ($\overline{d} \leq d^*$), agents are able to implement their full commitment consumption plan with a wealth reduced by the frictions due to the debt implemented. Denote the resulting (indirect) utility of each agent with commitment and no debt by $V^C(\beta)$, with no commitment and no debt by $V^U(\beta)$, and with commitment and debt level d by $V^C(\beta; d)$. Notice that $V^C(\beta; d)$ is decreasing in d as increases in d are tantamount to decreases in wealth. Furthermore, $V^C(\beta; 0) = V^C(\beta) > V^U(\beta)$.

For intermediate levels of debt, $d^* < d \leq d^{**}$, agents can implement 'partial commitment'. In this region, as the debt limit increases there are two effects on agents' equilibrium utility. The direct effect is that present for lower levels of debt as well, lowering the effective wealth of the agent. The indirect effect pertains to the agents' decline in commitment ability. Since both these effects work in tandem, the indirect utility decreases in this region as well (with greater sloe around d^*).

[27]For instance, the U.S. federal government does have a debt limit, which was, however, increased 74 times since 1962, and 10 times during the last decade.

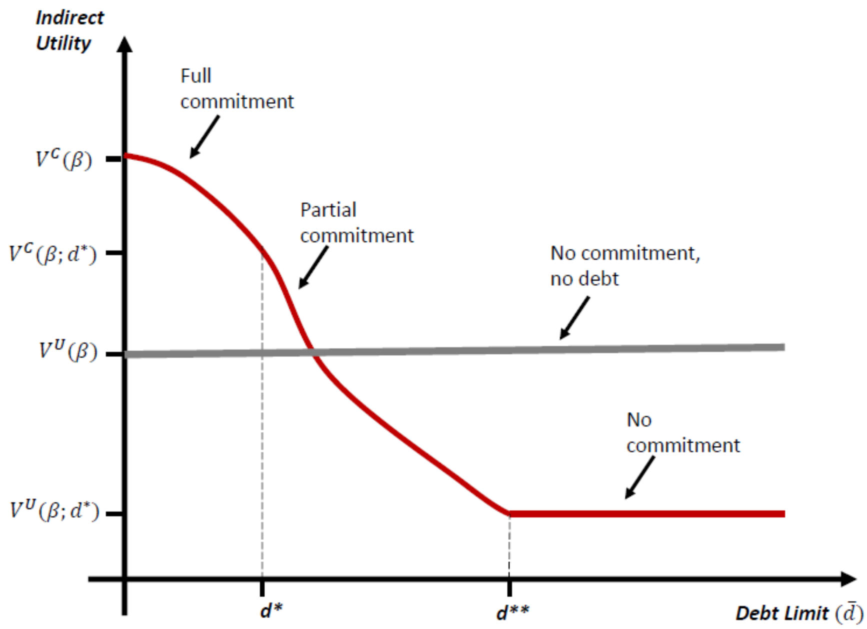

Figure 1: Welfare as a Function of Debt Limits

Finally, for large levels of debt limits, $d > d^{**}$, equilibrium debt is fixed at d^{**} and agents cannot commit. The resulting (indirect) utility is given by $V^U(\beta; d^{**})$. Since the debt in this region implies a loss of wealth without enabling agents to commit, $V^U(\beta; d^{**}) > V^U(\beta)$.

Figure 2 summarizes this discussion. As a corollary, it follows that for sufficiently small debt limits, welfare is higher in our setting relative to the no commitment case, while for sufficiently high debt limits, welfare is lower in our setting. That is:

Corollary *There exists $\tilde{d} \in (0, d^{**})$ such that for all $\overline{d} < \tilde{d}$, equilibrium welfare exceeds that generated in an economy without illiquid assets. For all $\overline{d} > \tilde{d}$, welfare in our setting is lower than that generated in an economy without illiquid assets.*

5.5 Heterogeneity

We now consider what happens when agents are heterogeneous in their present-bias parameter β.

We now assume that second period consumption $c_2^{\eta}(\beta, d)$ increases monotonically in β. This holds when the utility function has sufficient curvature. However, there are many

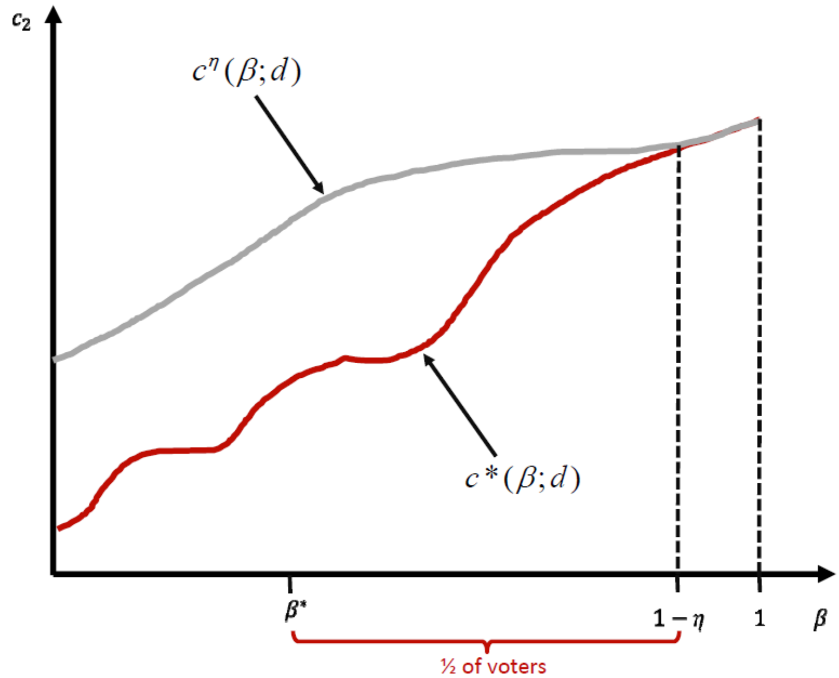

Figure 2: Consumption Patterns for a Given Debt Level

preferences for which this does not hold. For instance, with log utility, consumption is not monotonic: it reaches a maximum and then decreases. However, even in such a case our initial discussion will be valid for a fairly wide class of distributions over β. We discuss the more general case below.

Let β^* be such that $G(1-\eta) - (\beta^*) = 1/2$. That is, half the population has preferences that are between β^* and $1-\eta$. Figure 1 depicts the shape of the commitment and the no-commitment consumption levels in period 2 as a function of preferences for a particular debt level.

The agent of type β^* turns out to be the pivotal agent for determining debt in this environment. We can now define $d^*(\beta^*)$ and $d^{**}(\beta^*)$ as the solutions of $d^* = c_2^*(\beta^*, d^*)$ and $d^{**} = c_2^{\eta}(\beta^*, d^{**})$.

Proposition 6 *(i) If $\beta_M > 1-\eta$, then in equilibrium there is no debt, and consumption is given by $c_1^*(\beta), c_2^*(\beta), c_3^*(\beta)$.*

ii) Assume that $\beta_M < 1-\eta$. If $\overline{d} \leq d^{**}(\beta^*)$, then equilibrium debt is given by $\overline{d}$. If $\overline{d} > d^{**}(\beta^*)$, then debt is given by $d^{**}(\beta^*)$.

For any equilibrium debt level d, individual consumption for an agent of preference parameter β, period-2 consumption level in equilibrium is given by:

$$c_2(\beta; d) = \begin{cases} c_2^U(\beta; d) & \beta \leq \beta_L(d) \\ d & \beta_L(d) \leq \beta < \beta_H(d) \\ c_2^*(\beta; d) & \beta \geq \beta_H(d) \end{cases}.$$

With respect to the distribution of preferences, notice that a shift in distribution changes the debt structure in the economy only when it modifies the preferences β^* of the 'pivotal agent'. As β^* increases, $c_2^*(\beta^*; d)$ and $c_2^U(\beta^*; d)$ increase for all d, and therefore both d^* and d^{**} increase.

We say G' is a *median preserving spread* of G if both share the same median β_M and for any $\beta < \beta_M$, $G'(\beta) \geq G(\beta)$, while for any $\beta > \beta_M$, $G'(\beta) \leq G(\beta)$. Intuitively, this implies that, under G', more weight is put on more extreme values of β (see Malamud and Trojani (2009) for applications to a variety of other economic phenomena).

The above discussion then implies the following corollary.

Corollary 1 (Distributional Shifts)

1. *Assume $G(1-\eta) = G'(1-\eta)$. If G' First Order Stochastically Dominates G, and the corresponding medians $\beta_M, \beta'_M < 1-\eta$, then equilibrium debt under G' is (weakly) higher than that under G.*

2. *If G' is a Median Preserving Spread of G, then equilibrium debt under G' is (weakly) lower than that under G.*

Part 1 of this corollary says that, as the population becomes more "virtuous" or less subject to self control problems, equilibrium debt increases. This is potentially surprising but is a natural consequence of the logic of our model. There are two ways to see the logic of this result. The more mechanical one is to recall that equilibrium debt is equal to second period consumption. As the β^* increases, so does the desired second period consumption of the pivotal agent β^*. Thus, equilibrium debt increases. Alternatively one can notice that in our model debt arises because of the desire by the pivotal agent to constrain his future self, and the subsequent response of the political system undoing this commitment. The more virtuous the pivotal agent, the higher the level of debt that is required to prevent this agent from attempting to commit at an even higher level.

We now discuss the more general case where second period consumption may not be increasing in β. For any η, denote by d^p the debt level such that:

$$G\left\{\beta \mid c_2^{\eta}(\beta; d^p) < d^p\right\} = \frac{1}{2}.$$

Proposition 6 can now be restated with d^p playing the role of $d^{**}(\beta^*)$. If second period consumption is *decreasing* in β, then d^p will correspond to $c_2^{\eta}(\beta_M; d^{**})$: the median voter will be pivotal. Otherwise, there may be multiple pivotal voters.

For a given equilibrium level of debt $d > 0$, for sufficiently low preference parameters β, d is too large to allow for any level of commitment. These agents lose wealth due to the debt distortions, but gain no commitment value. In particular, their indirect utility is lower relative to the world in which no illiquid assets are available. At the other extreme, consider agents who are nearly time consistent, with preference parameter β close to 1. These agents suffer very minor commitment issues. Therefore, the loss of commitment is preferable to them then the loss of wealth due to distortionary debt. Formally, we get the following proposition:

Proposition 7 (Preference Parameters and Welfare) *For any debt level $d > 0$, there exist $\beta_L, \beta_H \in (0, 1)$ such that all agents with preference parameters $[0, \beta_L] \cup [\beta_H, 1]$ are weakly worse off by the introduction of illiquid assets.*

Note that the proposition implies a spread of the distribution of preferences that maintains the pivotal preference parameter β^*, and therefore the equilibrium level of debt, reduces welfare relative to the world in which no illiquid assets are available.

Throughout the paper we assumed that voters act under a pre-determined debt ceiling. Nonetheless, the debt limit itself is conceivably determined through a political process much like the one we study. Suppose then that in period 1 agents vote on the *debt limit* that would affect the debt imposed in period 2 as in the model studied thus far. In such a setting, all agents would favor low debt limits in period 1. In fact, since illiquid assets allow agents to commit without experiencing the loss of wealth that results from distortionary debt, equilibrium would entail a debt limit fixed at zero. Of course, if agents could vote again on the debt limit in period 2 (prior to determining the debt level itself, as in the model studied thus far), they would collectively choose a positive debt limit and consumption would be distorted (relative to the commitment paths). This suggests the importance of timing in constitutional reform. Since most amendments take a substantial amount of time to pass, changes in debt limits are likely to occur a significant time prior to the 'temptation' of consumption. Even if multiple elections occurred over such amendments, it would be difficult to achieve a super-majority to agree *over time* on an increase on the debt limit itself (as pointed above, early in the process, one would expect voters to reject debt limit increases).

6 Extensions

6.1 Elections in period 1

The model studied so far allowed for government actions and elections in periods 2 and 3. We now extend the model to consider elections in period 1 as well. The objective of this extension is to evaluate whether collective action in period 1 could effectively satisfy the demand for commitment agents display in period 1 or at least limit the distortions associated with debt accumulation in period 2.

The economic environment is the same as the one assumed in previous sections. There are two candidates running for office, both in period 1 and in period 2. The candidates are office motivated. The policy space is extended to allow candidates to offer a transfer y_1 and a lump-sum tax t_1 in period 1, as well as a transfer y_2 and tax t_2 in period 2 (elections in period 3 are redundant as before). debt financing is allowed. Tax collection in any period carries distortions of a unit loss $\eta > 0$ for every unit collected.

For this robustness check, we focus on the case of high debt limit, $\overline{d} \geq d^{**}$, so that equilibrium debt is given by d^{**}. Notice that when debt is high, more agents could be expected to suffer from an inability to commit, so potentially period 1 elections could be useful.

By taxing themselves in period 1 and investing the proceeds in the liquid asset agents can effectively commit resources for consumption in period 2 and hence reduce debt accumulation. On the other hand, if the proceeds of taxes carried to period 2 are smaller than d^{**}, in per-capita terms, a strict majority of agents in period 2 will support a positive debt level so as to increase consumption in period 2. Let $t = t_1 - y_1$ denote per-capita taxes in period 1 and $d = y_2 - t_2$ denote debt in period 2. It turns out that even though by taxing themselves in period 1 agents can indeed limit debt accumulation in period 2, this strategy simply shifts some of the repayment of debt from period 3 to period 1, but does not alter total distortions and has no ultimate effects on consumption profiles.

Proposition 8 *In the economy with elections in every period and with high debt limit,* $\overline{d} \geq d^{**}$*, the set of equilibria is characterized by pairs of period 1 taxes and period 2 debt of the form* (t, d) *such that* $t \in [0, d^{**}]$ *and* $d = d^{**} - t$*; total distortions and consumption profiles* *are unchanged for all agents relative to the case in which elections take place only in period 2.*[28]

It is easy to show that debt limits would be the only way to reduce distortions even in the model with period 1 elections. The reason is that any limit on surpluses would just shift the financing to higher debt in the second period.

6.2 Arbitrary number of periods

We now study an economy that may last for $T > 3$ periods. We are interested in the case in which T is arbitrarily large and later consider the limit case as $T \to \infty$. As in the analysis of the previous sections, we assume that voters can choose to invest in liquid or illiquid assets. An illiquid asset with maturity m, acquired in period t, pays off in period $t+m$ and cannot be sold before then. We assume that in any period $t = 1, ..., T-2$ illiquid assets are available with any maturity m between 2 and $T-t-1$. A liquid asset is instead, by definition, an asset with maturity 1. We assume that liquid assets are available in any period $t = 1, ..., T-1$.

We will discuss the consequences of extending the horizon T of this economy. It is natural to allow the aggregate endowment of the economy to grow at the same rate as the length of the horizon so that consumption does not become infinitesimal in every period.

Absent government intervention, by appropriate choice of the mix of liquid and illiquid assets with different maturities, a voter can commit to any desired consumption stream. We study, however, an economy in which elections occur in any period $t \geq 2$ (though period T elections are vacuous).Let D_t denote accumulated debt at t, while d_t denotes the deficit at t; that is $D_t = \sum_{\tau=2}^{t} d_t$. Note that debt accumulation begins in period 2, when the first elections are held.

Consider first the economy with linear distortions. In this case, it is straightforward to extend the analysis of Section 4.2 to show that at equilibrium debt is accumulated until period $T-1$, being repaid completely only in the last period T. With linear distortions agents have no incentive to smooth debt repayment. Furthermore, at each time $t < T$ agents consume exclusively off of debt (the equilibrium is at a corner), repaying the debt at time T, when they consume off of time 1 savings:

$$c_t = d_t, \quad \text{for any } t < T; \;\; c_T = s_{1T} - D_{T-1}.$$

Thus, with linear distortions the analysis easily extends to a long horizon model, and debt explodes as the number of periods increases. However, one could worry that this is an

[28] Only if the distortion on taxes at $t = 1$ were smaller than the distortion on taxes at $t = 3$ election in period 1 would help reducing debt in period 2 and hence distortions in period 3.

artificial result of linear distortions. Moreover, the extreme nature of this result may be worrisome. We therefore want to allow for an environment in which distortions are strictly convex, so that there is a motive to smooth distortions over time. We show that, even with convex distortions, debt accumulation can be very large when voters are time inconsistent and the political system does not include debt limits. Indeed, we will extend our analysis to an arbitrarily large number of periods and show that at the limit debt will grow without bounds.

It is convenient to start the analysis of convex distortions in the three period model studied in the previous section. As before, we assume that for every dollar of debt d incurred in period 2, $1+\eta$ dollars need be raised as taxes in period 3. We assume however that η is now a function of d, $\eta(d)$. In particular, we assume that the tax distortion are non-negative, strictly increasing and strictly convex:

$$\eta(d) \geq 0, \text{and } \eta'(d) > 0, \eta''(d) > 0, \text{ for any } d \geq 0.$$[29]

We consider the case where all agents are homogeneous, they have the same β, and no debt limits are imposed on debt accumulation. At time 2, the electoral process (the representative voter) takes as given savings from period 1, s_{12} and s_{13}, and choose debt d to solve:

$$\max_{d \geq 0} u\left(s_{12}+d\right)+\beta u\left(s_{13}-d\left(1+\eta(d)\right)\right) \tag{4}$$

At time 1 any agent takes debt d as given, and solves

$$\begin{aligned} &\max\nolimits_{c_1, s_{12}, s_{13} \geq 0} \; u\left(c_1\right)+\beta\left[u\left(s_{12}+d\right)+u\left(s_{13}-d\left(1+\eta(d)\right)\right)\right] \\ &\text{s.t.} \\ &c_1+s_{12}+s_{13}=k \end{aligned} \tag{5}$$

Even with convex distortions, the equilibrium level of debt d at time $t=2$ is always strictly positive. Indeed, suppose by way of contradiction that $d=0$. This implies $(1+\eta(d)+d\eta'(d))=1$. Thus, a corner solution at $d=0$ obtains if

$$u'\left(s_{12}\right)<\beta u'\left(s_{13}\right).$$

But consider then a solution to problem (5) with $d=0$: it would require $s_{12}=s_{13}>0$ by the Inada conditions and optimization, a contradiction with $u'\left(s_{12}\right)<\beta u'\left(s_{13}\right)$.

In contrast with the model with linear distortions, in the case of convex distortions the equilibrium level of debt d could be either at a corner, as in the case of linear distortions, or else it could be determined at an interior.

[29] We also require some smooth pasting conditions at $d=0$: $\eta(0)=\eta'(0)=\eta''(0)=0$; they guarantee smoothness of the electoral decision problem with $T>3$ period. See the Appendix.

The choice of d, by the electoral process at time 2, trades-off consumption at time 2 (which is valued as $u'(c_2)$ at the margin) and at time 3 (which is instead valued as $\beta u'(c_3)$ at the margin). Other things being equal, an increase in d increases consumption at time 2 but decreases consumption at time 3 more than one-for-one. In fact, the distortions introduce increasing marginal costs for any unit of debt repaid at time 3, which provides a wedge at the margin equal to $(1+\eta(d)+d\eta'(d))$. The larger the wedge, the higher the marginal cost of an increase in d.

When the equilibrium is interior, the choice of s_{12} and s_{13} at time 1, by any agent, will equalize at the margin the value of consumption at times 2 and 3, for any given d. It is easy to see that in this case the equilibrium is characterized by

$$u'(c_1) = \beta u'(c_2) = \beta u'(c_3)$$

$$\beta(1+\eta(d)+d\eta'(d)) = 1.$$

When, on the other hand,

$$u'(d) < u'(s_{13} - d\eta(d)),$$

only negative savings s_{12} could equalize at the margin the value of consumption at times 2 and 3, and a corner solution with $s_{12} = 0$ will hold in equilibrium.[30]

We now consider the full dynamic economy with arbitrary finite horizon $T \geq 3$. Let q_t denote repayment at time t.[31] We can then conveniently write distortions in terms of repayments:

$$A(q) = q(1+\eta(q)).$$

We continue to assume that, for $q > 0$, $A(q) = q(1+\eta(q))$ is strictly convex in q.

We will show that the dynamics of debt has an accumulation phase followed by a repayment phase. In the first phase a relatively large level of debt is accumulated at equilibrium, in the sense that agents consume exclusively off of government spending (the equilibrium deficit is determined by corner conditions), as in the linear economy. Only in the last accumulation date deficit is determined by interior conditions. On the other hand, agents smooth the repayment of debt over time.

[30]It is straightforward but tedious to obtain conditions on utility functions $u(c)$ and on distortions $\eta(d)$ which distinguish the two cases.

[31]That is, negative deficit: $q_t = -d_t$. The notation is therefore redundant but convenient.

Proposition 9 *There exists a $\tilde{t} \geq 2$ such that the government accumulates debt up to $t = \tilde{t}$ and repays subsequently for $t > \tilde{t}$.*

(i) At each time $2 \leq t \leq \tilde{t} - 1$, agents consume exclusively off of deficit-financed spending: $c_t = d_t$;[32]

(ii) For $t \geq \tilde{t}$, the consumption sequence satisfies

$$u'(c_t) = u'(c_{t+1}), \text{ for } \tilde{t} \leq t \leq T - 1.$$

(iii) For $t > \tilde{t}$ the repayment sequence q_t satisfies $q_t > 0$; $A(q_t)$ is in the strict interior of cohull $\left([A'(q_\tau)]_{t+1}^T\right)$; finally,

$$A'(q_t) = \beta \left[\sum_{\tau=t+1}^{T} A'(q_\tau) \frac{\partial q_\tau}{\partial D_t} \right], \text{ where } \sum_{\tau=t+1}^{T} \frac{\partial q_\tau}{\partial D_t} = 1. \tag{6}$$

This result also establishes that the equilibrium sequence of consumption is divided into two phases: an initial phase when consumption is exclusively financed by government transfers in which consumption decreases over time and satisfies an Euler equation that is analogous to what would hold absent commitment; a subsequent repayment phase where consumption is constant and satisfies the commitment Euler equation.

It may be surprising that, even in the case of convex distortions, in the accumulation phase agents consume exclusively off of deficit (the equilibrium deficit is determined by a corner condition for savings). The intuition for this result is the following: the marginal condition that characterizes the voting equilibrium at $\tilde{t}$ essentially determines the maximal level of debt $D_{\tilde{t}}$ that is accumulated in the economy. Other things being equal, this condition trades off the marginal cost of future distortions due to an infinitesimal increase in debt and the marginal benefit of an increase in consumption at $\tilde{t}$. At every time $t < \tilde{t}$, therefore, an increase in debt has a positive marginal effect on current consumption without affecting the level of debt at $\tilde{t}$ (since consumption in period $\tilde{t}$ falls by the same amount), and hence without affecting the future cost of distortions at the margin.

The following results are simple corollaries of the main proposition.

Corollary 10 *For any $t > \tilde{t}$, the sequence of repayments q_t is strictly increasing.*

[32] This statement is empty if $\tilde{t} = 2$; our analysis of the three period economy in this section should clarify this case.

The proof follows directly from equation (6), solving it backwards from $t = T$. For instance, at time $t = T - 1$ condition (6) implies

$$A'(q_{T-1}) = \beta A'(q_T). \tag{7}$$

and hence $q_{T-1} < q_T$. At time $t = T - 2$ condition (6) implies instead

$$A'(q_{T-1}) = \beta \left[A'(q_T) \left(\beta \frac{\partial q_{T-1}}{\partial D_{T-2}} + \frac{\partial q_T}{\partial D_{T-2}} \right) \right]. \tag{8}$$

By the convex hull representation above, $\frac{\partial q_{T-1}}{\partial D_{T-2}}, \frac{\partial q_T}{\partial D_{T-2}} > 0$, and $\frac{\partial q_{T-1}}{\partial D_{T-2}} + \frac{\partial q_T}{\partial D_{T-2}} = 1$. Therefore, we have that

$$\beta < \beta \frac{\partial q_{T-1}}{\partial D_{T-2}} + \frac{\partial q_T}{\partial D_{T-2}}$$

and hence

$$q_{T-2} < q_{T-1} < q_T.$$ [33]

We now ask whether infinite-debt can be supported at equilibrium in the infinite horizon limit.

Corollary 11 *In equilibrium the level of debt $D_{\tilde{t}}$ grows without bound as T goes to infinity.*

The proof proceeds by showing that, for any $t > \tilde{t}$, $A'(q_t) \to 0$ as $T \to \infty$. In other words, spreading the repayment of any finite amount of debt D_t over a large number of future periods induces smaller and smaller marginal distortions at t that converge to 0 as T goes to infinity, thereby allowing for debt accumulation which can grow without bounds as the number of future periods grows to infinity.

For simplicity, we provide here only the heuristic arguments for the case in which $\frac{\partial q_\tau}{\partial D_t}$ is constant, for any $\tau > t > \tilde{t}$. It can be shown that this holds if $A(q) = e^q$. The general case is discussed in the Appendix.

In this case we can write the condition (6) recursively as follows:

$$\begin{aligned}
A'(q_{T-1}) &= \beta A'(q_T), \\
A'(q_{T-2}) &= \frac{\beta}{2} (1 + \beta) A'(q_T), \\
A'(q_{T-3}) &= \frac{\beta}{3} (1 + \beta) \left(1 + \frac{\beta}{2} \right) A'(q_T), \\
&\cdots \\
A'(q_t) &= \frac{\beta}{T - t} \prod_{\tau = t+1}^{T+t-1} \left(1 + \frac{\beta}{\tau} \right) A'(q_T).
\end{aligned}$$

[33] As noticed in the Appendix, the argument proceeds recursively backwards until $t = \tilde{t} + 1$.

It can, in fact, be shown by the ratio convergence test that the series converges to zero (taking $t = 0$ without loss of generality):

$$\lim_{T\to\infty} \frac{\beta}{T} \prod_{\tau=1}^{T-1} \left(1 + \frac{\beta}{\tau}\right) = 0.$$

7 Concluding Remarks

We introduced a political process determining fiscal policy when voters are time inconsistent. Several messages arise from our analysis. First, absent distortions, as long as debt limits are low enough, the availability of illiquid assets makes debt irrelevant for ultimate consumption levels since agents can adjust foreseen debt income by an appropriate ex-ante allocation of liquid and illiquid assets. In particular, there is a Ricardian equivalence of sort. When debt limits are high, agents' ability to commit is impaired. That is, electorally accountable politicians ultimately choose policies that interfere with individuals' ex-ante desire to commit. When debt is distortionary some of these effects are accentuated since debt entails an effective loss of wealth. In fact, we show that there can be a substantial loss in welfare relative to the case in a world without any ability to commit and without debt. The paper highlights the importance of analyzing the political process when contemplating enlarging the menus of policies directed at enhancing the welfare of 'behavioral' electorates. Indeed, when focusing on time inconsistency, the underlying message of our paper is that governments may not be very effective in satisfying the demand for commitment.

In assessing the relevance of these results, it is important to note that agents' behavior does not require particularly complicated behavior. Agents need to forecast the time path of transfers and debts to understand their expected consumption and tax liabilities so that they can optimize their savings and portfolio behavior. Equilibrium imposes restrictions on this path of debts but the agents do not need a particularly sophisticated understanding of the economy. In particular, no more knowledge is required than in a standard neoclassical model.

8 Appendix

Proof of Lemma 1

Suppose by way of contradiction that $c_2^*(\beta) > c_2^U(\beta)$. Since $c_2^*(\beta) = c_3^*(\beta)$ and $c_2^U(\beta) > c_3^U(\beta)$, this implies that $c_3^*(\beta) > c_3^U(\beta)$. Thus, by the resource constraint, $c_1^*(\beta) < c_1^U(\beta)$. Denote by $e_2(\beta) = e_1 - c_1^U(\beta)$. For any wealth e_2 left at period 2, denote by $c_2^U(\beta; e_2)$ and $c_3^U(\beta; e_2)$ the uncommitted solution. Then, optimization requires that:

$$u'\left(c_1^U(\beta)\right) = \beta\left[u'\left(c_2^U(\beta; e_2)\right)\frac{\partial c_2^U(\beta; e_2)}{\partial e_2} + u'\left(c_3^U(\beta; e_2)\right)\frac{\partial c_3^U(\beta; e_2)}{\partial e_2}\right].$$

Since $c_3^U(\beta; e_2) = e_2 - c_2^U(\beta; e_2)$, we then have:

$$u'\left(c_1^U(\beta)\right) = \beta\left[u'\left(c_2^U(\beta; e_2)\right)\frac{\partial c_2^U(\beta; e_2)}{\partial e_2} + u'\left(c_3^U(\beta; e_2)\right)\left(1 - \frac{\partial c_2^U(\beta; e_2)}{\partial e_2}\right)\right].$$

From the constraint of period 2 self,

$$u'(c_2^U(\beta; e_2)) = \beta u'(e_2 - c_2^U(\beta; e_2)).$$

Using the Implicit Function Theorem,

$$\frac{\partial c_2^U(\beta; e_2)}{\partial e_2} = \frac{\beta u''(e_2 - c_2^U(\beta; e_2))}{u''(c_2^U(\beta; e_2)) + \beta u''(e_2 - c_2^U(\beta; e_2))} < 1.$$

Since $c_2^U(\beta; e_2) \geq c_3^U(\beta; e_2)$, we get that $u'\left(c_1^U(\beta)\right) > \beta u'\left(c_2^U(\beta)\right)$. Optimization with commitment implies that $u'(c_1^*(\beta)) = \beta u'(c_2^*(\beta))$. But these conditions cannot hold simultaneously if $c_1^*(\beta) < c_1^U(\beta)$ and $c_2^*(\beta) > c_2^U(\beta)$.■

Proof of Proposition 3

Consider the following maximization problem:

$$\begin{aligned} &\max u(c_1) + \beta\left[u(c_2) + u(c_3)\right] \\ &s.t.\ \ u'(c_2) = \tfrac{\beta}{1-\eta}u'(c_3) \\ &c_1 + c_2 + c_3 = k - \eta c_2. \end{aligned} \tag{9}$$

This is an artificial problem corresponding to an agent who chooses the debt level and her consumption plan in tandem but consuming c_2 destroys resources just as debt does. In particular, this problem generates a higher overall utility (from period 1's perspective) than that experienced by an agent who consumes $c_1^\eta(\beta, d^{**}), c_2^\eta(\beta, d^{**}), c_3^\eta(\beta, d^{**})$ because such an agent takes the equilibrium level of debt as given and cannot alter it unilaterally. The latter generates the equilibrium level of welfare for distortions η. Furthermore, the two coincide when $\eta = 0$. We now show that the maximized objective of problem (9) is decreasing in η. Indeed, suppose $\eta_1 > \eta_2$. Denote the solution of (9) for distortions η_1 by (c_1, c_2, c_3). We now approximate a policy under distortions η_2 small enough that satisfies the constraints and generates a strictly higher value for the objective.

For η_2 close enough to η_1, there exists $\varepsilon > 0, \varepsilon < c_3$ such that

$$u'(c_2) = \frac{\beta}{1-\eta_2} u'(c_3 - \varepsilon).$$

Therefore,

$$u'(c_2) = \frac{\beta}{1-\eta_2} \left[u'(c_3) - \varepsilon u''(c_3) + O(\varepsilon^2) \right].$$

Since (c_1, c_2, c_3) is a solution to the problem with distortions η_1, $u'(c_2) = \frac{\beta}{1-\eta_1} u'(c_3)$. It follows that:

$$\varepsilon = \frac{(\eta_2 - \eta_1)\, u'(c_2)}{\beta u''(c_3)} + O(\varepsilon^2).$$

Consider then the policy $(c_1 + \varepsilon + (\eta_1 - \eta_2)\, c_2, c_2, c_3 - \varepsilon)$ when the distortions are η_2. Notice that, by construction, this policy satisfies the two constraints in problem (9). The difference between the generated objective and the maximal value of the objective under distortions η_1 is then:

$$\Delta = [u(c_1 + \varepsilon + (\eta_1 - \eta_2)\, c_2) - u(c_1)] + \beta\, [u(c_3 - \varepsilon) - u(c_3)].$$

Using a first order approximation,

$$\begin{aligned}
\Delta &= (\varepsilon + (\eta_1 - \eta_2)\, c_2)\, u'(c_1) - \beta \varepsilon u'(c_3) = \\
&= (\eta_1 - \eta_2)\, c_2 u'(c_1) + \frac{(\eta_2 - \eta_1)\, u'(c_2) u'(c_1)}{\beta u''(c_3)} - \frac{(\eta_2 - \eta_1)\, u'(c_2) u'(c_3)}{u''(c_3)} + O(\varepsilon^2) \\
&= (\eta_1 - \eta_2)\, u'(c_2) \left[\frac{u'(c_1) c_2}{u'(c_2)} - \frac{u'(c_1) - \beta u'(c_3)}{\beta u''(c_3)} \right] + O(\varepsilon^2).
\end{aligned}$$

Notice that the solution to problem (9) with distortions η_1 must satisfy $u'(c_1) = \beta\, [u'(c_2) + u'(c_3)]$ and so:

$$\Delta = (\eta_1 - \eta_2)\, u'(c_2) \left[\frac{u'(c_1) c_2}{u'(c_2)} - \frac{u'(c_2)}{u''(c_3)} \right] + O(\varepsilon^2),$$

which from concavity of the instantaneous utility u, is positive whenever η_1 and η_2 are close enough. In particular, the optimal solution for problem (9) with distortions η_2 must generate a strictly higher level of the objective function than the solution with distortions

η_1. It follows that welfare in our distortion economy is lower under any $\eta > 0$ relative to the case of $\eta = 0$. ■

Proof of Proposition 6 Similar arguments to those of Lemma 1 imply that for any level of debt d, $c_2^*(\beta; d)$ and $c_2^U(\beta; d)$ are increasing in β.

Assume first that $\overline{d} \le d^*$. Suppose equilibrium debt is $d < \overline{d}$. Notice that monotonicity implies that $c_2^*(\beta; d) \ge c_2^*(\beta^*; d) \ge d$ for all $\beta \ge \beta^*$. Furthermore, by definition of d^* and continuity of $c_2^*(\beta; d)$, for sufficiently small $\varepsilon > 0$, $c_2^*(\beta; d) \ge d$, for all $\beta \ge \beta^* - \varepsilon$. It follows that all agents with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ best respond by investing in illiquid assets leaving them with period 2 wealth of $c_2^*(\beta; d) - d$. However, in period 2, these agents are keen to shift resources from period 3 to period 2, and would therefore prefer a slightly higher debt level. From the definition of β^*, there would therefore be a strict majority support for higher debt. In particular, the only candidate for equilibrium debt is $\overline{d}$. Now, when debt is expected to be $\overline{d}$, for sufficiently small $\varepsilon > 0$, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond in period 1 by investing in illiquid assets so that $\max\{0, c_2^*(\beta; d) - d\}$ is left for the period 2 self. These agents, forming a strict majority, would oppose any ex-post reduction of debt in period 2. It follows that $\overline{d}$ constitutes the equilibrium debt level and the commitment consumption stream is implemented for the agent with preferences β^*.

Consider now the case $d^* < \overline{d} \le d^{**}$. As above, for any $d < d^*$, when voters use best responses, there would be a strict majority support for an increase in debt in period 2. Suppose, then, that in equilibrium the debt is d, $d^* \le d < \overline{d}$. For all $\beta \ge \beta^*$, monotonicity implies that $c_2^U(\beta; d) \ge c_2^U(\beta^*; d) \ge d$. Furthermore, by definition of d^{**} and continuity of $c_2^U(\beta; d)$, for sufficiently small $\varepsilon > 0, c_2^U(\beta; d) \ge d$, for all $\beta \ge \beta^* - \varepsilon$. Therefore, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond by investing in illiquid assets so that $\max\{0, c_2^*(\beta; d) - d\}$ is left for the period 2 self. In particular, all agents with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$, constituting a strict majority of agents, would support a higher debt in period 2. It follows that $\overline{d}$ is the only candidate for equilibrium debt. In fact, the above arguments suggest that whenever debt is expected to be $\overline{d}$, a strict majority of voters would oppose any ex-post reduction of debt in period 2. Therefore, in equilibrium, debt is given by $\overline{d}$, coinciding with the consumption level in period 2 of the agent with preference parameter β^*.

Finally, suppose that $\overline{d} > d^{**}$. As before, for any $d < d^{**}$, whenever voters best respond, there would be a strict majority support for an increase in debt in period 2. Assume equilibrium debt is $d > d^{**}$. As before, from monotonicity and continuity of $c_2^U(\beta; d)$, and the definition of d^{**}, there exists $\varepsilon > 0$ such that for all $\beta \leq \beta^* + \varepsilon, c_2^U(\beta; d) < d$. Since distortions imply that, in period 2, agents never desire a debt level that exceeds their intended consumption, it follows that all agents with preference parameters $[0, \beta^* + \varepsilon]$, a strict majority, would desire a lower debt level. Debt d^{**} constitutes part of an equilibrium. Indeed, continuity and monotonicity imply that any debt $d > d^{**}$ would exceed the period-2 uncommitted consumption level for agents with preference parameter $[0, \beta^* + \varepsilon]$ for some $\varepsilon > 0$, and therefore be opposed by a strict majority. Similarly, any debt $d < d^{**}$ would imply that a strict majority of agents with preference parameter $[\beta^* - \varepsilon, 1 - \eta)$, for some $\varepsilon > 0$, that have used a best response, cannot afford their uncommitted consumption level in period 2, and therefore oppose a deviation from d^{**} to d. The proposition claim then follows. ■

Proof of Proposition 7

For any debt level d, let β^U be the maximal parameter β such that uncommitted consumption level in period 2 with wealth $k - \eta d$ is lower than d. Since period 2 uncommitted consumption is monotonic in β for all levels of wealth, all agents with parameter $\beta \leq \beta^U$ consume the uncommitted consumption profile with debt d, that generates lower utility than the uncommitted consumption profile absent distortionary debt. For agents with preference parameter $\beta = 1$, the uncommitted consumption path coincides with the full-commitment consumption path. In particular, for these agents, the introduction of distortionary debt entails a welfare reduction due to the effective loss of wealth. Continuity implies the claim of the proposition. ■

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