Repeated Games

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`http://www.ssc.upenn.edu/~gmailath/book.html`

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Introduction

Repeated games with perfect and public monitoring are (thought to be) well understood.

Repeated games with private monitoring are more complicated, and until recently, little was known.

Now, we know something, and this has shed light on games with perfect and public monitoring.

Since Abreu, Pearce, and Stacchetti (1990), the analysis of public monitoring games has tended to emphasize characterizing the set of equilibrium payoffs, rather than the structure of behavior. Earlier analyses of perfect monitoring games did also focus on structure—optimal penal codes for example (Abreu, 1988), and the complexity literature (Rubinstein (1986), Kalai and Stanford (1988), Abreu and Rubinstein (1988)).

The theoretical reputation literature has also focused on the payoff bounds, rather than on the structure of the equilibria.

Interesting things can be learnt from focusing on the structure of behavior.

Prisoners' Dilemma

Partnership $A_i = \{E, S\}$, imperfect monitoring $Y = \{\underline{y}, \bar{y}\}$.

$$\Pr\{\bar{y}|a\} = \rho(\bar{y}|a) = \begin{cases} p, & \text{if } a = EE, \\ q, & \text{if } a = ES \text{ or } SE, \\ r, & \text{if } a = SS. \end{cases}$$

		$\underline{y}$	$\bar{y}$
ex post	E	$\frac{3-2q-p}{p-q}$	$\frac{-p-2q}{p-q}$
payoffs	S	$\frac{3(1-r)}{q-r}$	$\frac{-3r}{q-r}$

PD		E	S
ex ante	E	$2, 2$	$-1, 3$
payoffs	S	$3, -1$	$0, 0$

Two periods, payoffs added. Second period stage game:

	G	B
G	3,3	0,0
B	0,0	1,1

Trigger profile: EE in first period, GG in the second after $\bar{y}$, and BB after $\underline{y}$.

A PPE if $2(p-q) \geq 1$.

Private Monitoring

Player i observes $y_i \in Y_i \equiv \{\underline{y}_i, \bar{y}_i\}$. Joint distribution over signal vector $(y_1, y_2) \in Y_1 \times Y_2$ given by $\pi(y_1 y_2 | a)$.

Marginal distribution, $\pi_i(y_i | a)$.

ex post payoffs: $u_i^*(y_i, a_i)$

ex ante payoffs: $u_i(a) = \sum_{y_i \in Y_i} u_i^*(y, a_i) \pi_i(y_i | a)$.

Almost-public private monitoring: $\rho(y|a) > 0$ and for all a,

$$|\rho(y|a) - \pi(yy|a)| < \varepsilon.$$

For ε sufficiently small, under almost-public monitoring, players signals are highly correlated.

$a_1 a_2$	$\underline{y}_2$	$\bar{y}_2$
$\underline{y}_1$	$(1-\alpha)(1-2\varepsilon)$	ε
$\bar{y}_1$	ε	$\alpha(1-2\varepsilon)$

Conditionally-independent private monitoring: for all a,

$$\pi(y_1 y_2|a) = \pi_1(y_1|a)\pi_2(y_2|a).$$

For example,

$$\pi_i(y_i|a) = \begin{cases} 1-\varepsilon, & \text{if } y_i = \bar{y}_i \text{ and } a_j = E, \text{ or} \\ & \qquad y_i = \underline{y}_i \text{ and } a_j = S,\ j \neq i, \\ \varepsilon, & \text{otherwise,} \end{cases}$$

EE	$\underline{y}_2$	$\bar{y}_2$
$\underline{y}_1$	ε^2	$(1-\varepsilon)\varepsilon$
$\bar{y}_1$	$(1-\varepsilon)\varepsilon$	$(1-\varepsilon)^2$

SE	$\underline{y}_2$	$\bar{y}_2$
$\underline{y}_1$	$(1-\varepsilon)\varepsilon$	ε^2
$\bar{y}_1$	$(1-\varepsilon)^2$	$(1-\varepsilon)\varepsilon$

For ε small, this is almost-perfect private monitoring.

More generally, a private-monitoring game with private monitoring distribution (Ω, π) has almost-perfect monitoring if, for all players i, there is a partition of Ω_i, $\{\Omega_i(a)\}_{a \in A}$, such that for all action profiles $a \in A$,

$$\sum_{\omega_i \in \Omega_i(a)} \pi_i(\omega_i | a) > 1 - \eta.$$

Almost-perfect private monitoring does not make any assumptions about the correlation structure: both almost-public and conditionally-independent private monitoring distributions can be almost-perfect.

Equilibria when Almost-Public Monitoring

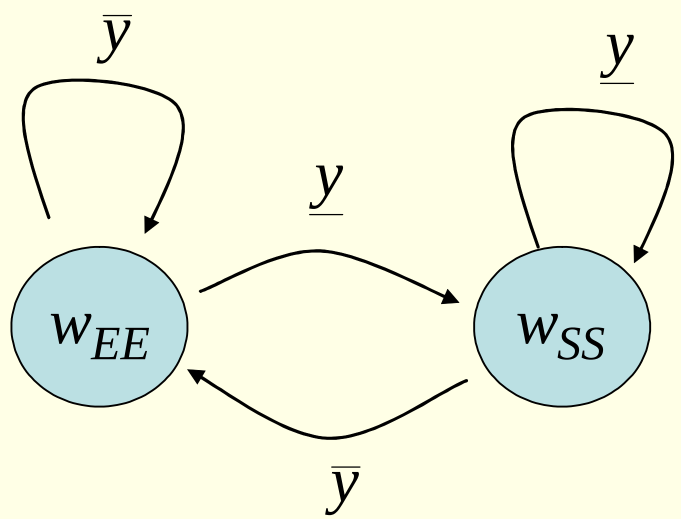
(Mailath and Morris, 2002, 2005)

Induced behavior by trigger eq: both play E in first period, in second period, player i plays G after $\bar{y}_i$ and B after $\underline{y}_i$.

For π close to ρ, this is an eq:

- $\Pr(y_1 = y_2 | a) \approx 1$, so BB or GG in second period with probability close to ρ.
- first period incentives are close to first period incentives under ρ.

Infinitely repeated games.
A forgiving profile



strict PPE if

$$\frac{1}{(3p-2q-r)} < \delta < \frac{1}{(p+2q-3r)}.$$

This forgiving profile has bounded recall: last period's signal completely determines current state.

Behavior induced by public forgiving profile in private monitoring game: $(\mathcal{W}, w^0, f_i, \tau_i)$, where $\mathcal{W} = \{w_{EE}, w_{SS}\}$ is set of states, $w^0 = w_{EE}$ is the initial state (common to both players), $f_i(w_a) = a_i$ is the decision rule, and $\tau_i : \mathcal{W} \times Y_i \to \mathcal{W}$ is the private transition function.

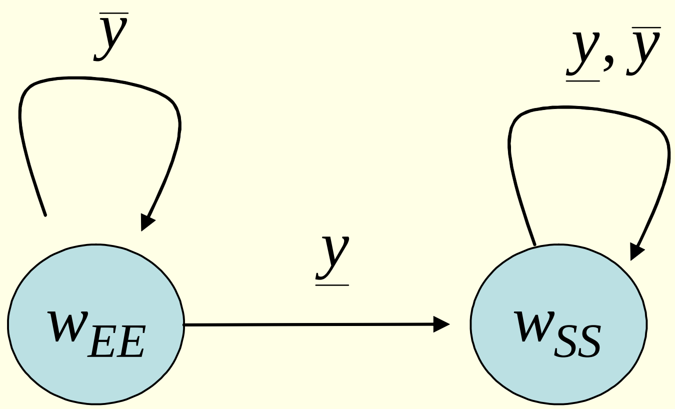
After private history $h_i^t = (y_i^0, a_i^0; \ldots, y_i^{t-1}, a_i^{t-1})$, player i has beliefs $\beta_i(\cdot|h_i^t) \in \Delta(\mathcal{W}_j)$ over player j's current private state.

Private history also implies a current private state for i, $w_i^t = \tau_i(w^0, h_i^t)$. Eg., $w_i^2 = \tau(\tau(w_i^0, y_i^0), y_i^1)$.

In forgiving profile, after h_i^t,

$$\beta_i(w_i^t|h_i^t) = \Pr(w_j^t = w_i^t|h_i^t) = \Pr(y_j^{t-1} = y_i^{t-1}|h_i^t) \approx 1.$$

Grim trigger



strict PPE if

$$\delta > \frac{1}{(3p - 2q)}.$$

Profile has unbounded recall.

- If $q > r$, implied private profile is not a Nash equilibrium in any close-by game with full-support private monitoring.

- If $r \geq q$, implied private profile is a Nash equilibrium in every close-by game with full-support private monitoring. (And so there is a sequential equilibrium with the same outcome as that of grim trigger.)

- For all r, q, implied private profile is not a sequential equilibrium in any close-by game with full-support private monitoring.

$q > r$: Grim trigger is not Nash.

S is **not** optimal after long histories of the form $\left(E, \underline{y}_1; S, \bar{y}_1; S, \bar{y}_1; S, \bar{y}_1; \cdots\right)$:

Immediately after $\underline{y}_1$, 1 assigns prob very close to 0 to 2 being in w_{EE} (because with prob close to 1, player 2 also observed $\underline{y}_2$). Thus, playing S in the subsequent period is optimal.

But π has full support $\Longrightarrow$ 1 does **not** know that 2 is in w_{SS}.

$\rho(\bar{y}|SE) = q > r = \rho(\bar{y}|SS) \Longrightarrow \bar{y}_1$ after playing S is an indication that player 2 had played E.

$q \le r$: Grim trigger is Nash.

S is optimal after $\left(E, \underline{y}_1; S, \bar{y}_1; S, \bar{y}_1; S, \bar{y}_1; \cdots\right)$:

Immediately after $\underline{y}_1$, 1 assigns prob very close to 0 to 2 being in w_{EE} (because with prob close to 1, player 2 also observed $\underline{y}_2$). Thus, playing S in the subsequent period is optimal.

$\rho(\bar{y}|SE) = q \le r = \rho(\bar{y}|SS) \implies \bar{y}_1$ after playing S is an indication that player 2 had played S (if $q = r$, $\bar{y}_1$ is uninformative).

Observing $\underline{y}_1$ is signal that 2 had played E, but if 2 had also observed $\underline{y}_2$, then 2 transits to w_{SS}.

E is optimal after $(E, \bar{y}_1; E, \bar{y}_1; E, \bar{y}_1; E, \bar{y}_1; \cdots)$:

Posterior that 2 is in state w_{EE} cannot fall very far.

π full support $\Longrightarrow$ Pr$\{2$ in $w_{SS} | 1$ in $w_{EE}\} > 0$.

But $\bar{y}_1$ is signal that 2 had played E.

For all r, q: Grim trigger is not sequential.

S is not optimal after long histories of the form $\left(E\underline{y}_1; E\bar{y}_1; E\bar{y}_1; E\bar{y}_1; \cdots\right)$:

Immediately after $\underline{y}_1$, 1 assigns prob very close to 0 to 2 being in w_{EE} (because with prob close to 1, player 2 also observed $\underline{y}_2$). Thus, playing S in the subsequent period is optimal.

But π has full support $\Longrightarrow$ 1 is not sure that 2 is in w_{SS}.

$\rho(\bar{y}|EE) = p > q = \rho(\bar{y}|ES) \Longrightarrow \bar{y}_1$ after playing E is an indication that player 2 had played E.

Important to understand the structure of equilibrium behavior.

Mailath and Morris (2002) obtain folk thm for almost-perfect almost-public monitoring. Folk theorem for perfect monitoring can be proved using profiles with bounded recall.

Unknown if folk theorem for public monitoring (Fudenberg, Levine, and Maskin, 1994) can be proved using bounded recall strategies.

However, for some repeated prisoners' dilemmas, the restriction to strongly symmetric bounded recall PPE results in a dramatic collapse of the set of equilibrium payoffs (Cole and Kocherlakota, forthcoming).

Essentially, only bounded recall strict PPE are robust to sufficiently rich almost-public private monitoring (Mailath and Morris, 2005).

Equilibria with Conditionally-Independent Monitoring

(Bhaskar and van Damme, 2002)

In every pure strategy equilibrium of the two period game, SS is played in the first period (no matter how close to perfect the monitoring is).

Consider putative equilibrium with EE in the first period. To support this, player i should play G after $\bar{y}_i$ and B after $\underline{y}_i$.

But, i's beliefs over the signals observed by j are independent of the signal he observes, and so are his best replies. For ε small, these are strict, and so sequentially rational play must ignore the signal.

Different situation with mixing. Consider symmetric profile with probability μ on E in the first period.

Implies a type space for i, $T_i = \{E, S\} \times \{\underline{y}_i, \bar{y}_i\}$, with joint dsn:

	$E\bar{y}_2$	$E\underline{y}_2$	$S\bar{y}_2$	$S\underline{y}_2$
$E\bar{y}_1$	$\mu^2(1-\varepsilon)^2$	$\mu^2\varepsilon(1-\varepsilon)$	$\mu(1-\mu)\varepsilon(1-\varepsilon)$	$\mu(1-\mu)\varepsilon^2$
$E\underline{y}_1$	$\mu^2\varepsilon(1-\varepsilon)$	$\mu^2\varepsilon^2$	$\mu(1-\mu)(1-\varepsilon)^2$	$\mu(1-\mu)\varepsilon(1-\varepsilon)$
$S\bar{y}_1$	$\mu(1-\mu)\varepsilon(1-\varepsilon)$	$\mu(1-\mu)(1-\varepsilon)^2$	$(1-\mu)^2\varepsilon^2$	$(1-\mu)^2\varepsilon(1-\varepsilon)$
$S\underline{y}_1$	$\mu(1-\mu)\varepsilon^2$	$\mu(1-\mu)\varepsilon(1-\varepsilon)$	$(1-\mu)^2\varepsilon(1-\varepsilon)$	$(1-\mu)^2(1-\varepsilon)^2$

Mixing generates needed correlation between information of different players. Taking $\varepsilon \longrightarrow 0$ gives:

	$E\bar{y}_2$	$E\underline{y}_2$	$S\bar{y}_2$	$S\underline{y}_2$
$E\bar{y}_1$	μ^2	0	0	0
$E\underline{y}_1$	0	0	$\mu(1-\mu)$	0
$S\bar{y}_1$	0	$\mu(1-\mu)$	0	0
$S\underline{y}_1$	0	0	0	$(1-\mu)^2$

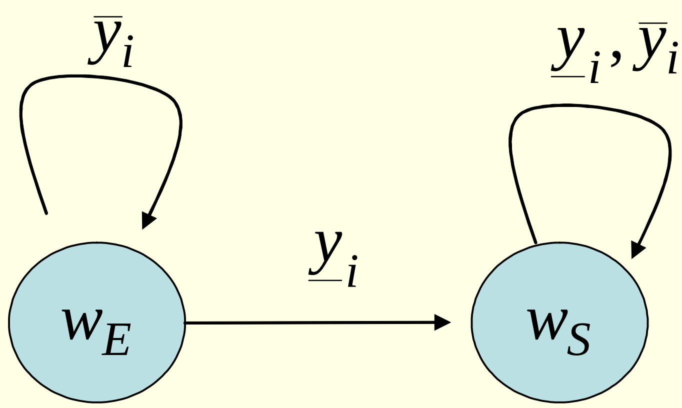
and so can specify G after $E\bar{y}_i$ and B after $E\underline{y}_i$, $S\bar{y}_i$, and $S\underline{y}_i$.

Using public correlation to introduce the possibility of GG after $\{E\underline{y}_i, S\bar{y}_i, S\underline{y}_i\}$, can achieve EE with arbitrarily high prob ($\mu \approx 1$) for ε sufficiently small.

Infinitely repeated games.

(Sekiguchi, 1997)

Initial private state is determined randomly, with probability ξ on W_E and $1 - \xi$ on w_S.



Can achieve efficiency using public correlation to restart game (Bhaskar and Obara, 2002), who also obtain a partial folk theorem).

Belief-free Equilibria

So far, discussed belief-based analysis of behavior in games with private monitoring. Return to two periods with conditionally-independent private monitoring, but now suppose second period stage game is:

	R	P
R	$10, 10$	$0, 10$
P	$10, 0$	$0, 0$

Strategy for i: Play E in first period, play P with probability

$$\alpha = 1 - \frac{1}{10(1 - 2\varepsilon)}$$

after $E\bar{y}_i$, and for sure otherwise. i's best reply is belief-free.

Infinitely repeated games.

Belief-free equilibria in repeated PD (Piccione, 2002; Ely and Välimäki, 2002). Illustrate using long-lived and short-lived players:

	h	ℓ
H	$2,3$	$0,2$
L	$3,0$	$1,1$

Player 1 (row player) is long-lived, and player 2 is short-lived. Suppose game has perfect monitoring.

A one-dimensional family of eq, when $\delta \geq \frac{1}{2}$:

$$\mathcal{W} = \{w^L, w^H\}, \quad w^0 = w^H,$$

$$f_1(w) = \frac{1}{2} \circ H + \frac{1}{2} \circ L, \quad \forall w, \quad \text{and}$$

$$f_2(w) = \begin{cases} \alpha' \circ h + (1 - \alpha') \circ \ell, & \text{if } w = w^H, \\ \alpha'' \circ h + (1 - \alpha'') \circ \ell, & \text{if } w = w^L, \end{cases}$$

where $\alpha' - \alpha'' = 1/2\delta$, and transitions,

$$\tau(w, a) = \begin{cases} w^H, & \text{if } a_1 = H, \\ w^L, & \text{if } a_1 = L. \end{cases}$$

Note that 1 is indifferent between H and L in both w^L and w^H.

These are not equilibria in which histories coordinate future play!

Game with almost-perfect private monitoring, with player 1's private signal space $\Omega_1 = \{\hat{h}, \hat{\ell}\}$, and player 2's private signal space $\Omega_2 = \{\hat{H}, \hat{L}\}$.

Consider the profile in which player 1 randomizes in every period with probability $\frac{1}{2}$ on H. Player 2's behavior is described by $\mathcal{W}_2 = \{w^H, w^L\}$, $w_2^0 = w^H$,

$$f_2(w_2) = \begin{cases} \alpha' \circ h + (1-\alpha') \circ \ell, & \text{if } w_2 = w^H, \\ \alpha'' \circ h + (1-\alpha'') \circ \ell, & \text{if } w_2 = w^L, \end{cases}$$

and

$$\tau_2(w_2, \omega_2, a_1) = \begin{cases} w^H, & \text{if } \omega_2 = \hat{H}, \\ w^L, & \text{if } \omega_2 = \hat{L}. \end{cases}$$

2's incentives are trivially satisfied.

Let $V_1(a_1; w_2)$ be the value to player 1 from the action a_1 when player 2 has current private state w_2.

Player 1's payoff from a_1 after private history h_1^t is

$$\sum_{w_2} V_1(a_1; w_2)\beta_1(w_2|h_1^t).$$

Belief-free equilibrium if

$$V_1(H; w^H) = V_1(L; w^H) \quad \text{and}$$
$$V_1(H; w^L) = V_1(L; w^L).$$

Solving gives a one-dimensional family of equilibria:

$$\alpha' - \alpha'' = \frac{1}{2\delta(1-2\varepsilon)}.$$

Piccione (2002) and Ely and Välimäki (2002) prove a folk theorem for the repeated PD using belief-free strategies for almost-perfect monitoring.

Ely, Hörner, and Olszewski (2005) provide a recursive description of belief-free eq (strong self-generation), characterize the set of belief-free eq payoffs in two player games with almost-perfect private monitoring. In general, these payoffs are bounded away from the feasible and IR set.

Hörner and Olszewski (2005) use belief-free as a building block to prove the folk theorem for almost-perfect ($\Omega_i = A_i$) private-monitoring games.

Matsushima (2004)proves a folk theorem for a class of repeated PD's with conditionally-independent, but not almost-perfect or almost-public monitoring. Proof combines elements of review phases (ala Radner (1985)) and belief-free eq.

Alternative route to constructing nontrivial equilibria and resurrecting recursive structure in games with private monitoring is to allow for communication (Compte, 1998; Kandori and Matsushima, 1998). More recent contributions are Fudenberg and Levine (2004)and McLean, Obara, and Postlewaite (2002).

Private Strategies in Public Monitoring

In public-monitoring games, attention is typically restricted to public perfect equilibria, because of tractability (they are "recursive"), and there is a folk theorem using PPE.

But we have just seen that it is possible to handle private histories (and belief-free eq have a recursive structure).

Focusing on public strategies (and associated PPE) can be restrictive: efficiency can sometimes be achieved when the PPE folk thm does not apply, and even when it does apply, for a fixed high discount factor, there may be private equilibria with higher payoffs.

Return to the last two period example with public monitoring, $Y = \{\underline{y}, \bar{y}\}$.

first period

	E	S
E	$2, 2$	$-1, 3$
S	$3, -1$	$0, 0$

second period

	R	P
R	$10, 10$	$0, 10$
P	$10, 0$	$0, 0$

$$\Pr\{\bar{y}|a\} = \rho(\bar{y}|a) = \begin{cases} p, & \text{if } a = EE, \\ q, & \text{if } a = ES \text{ or } SE, \\ r, & \text{if } a = SS. \end{cases}$$

Suppose $p = 0.9$, $q = 0.8$, $r = 0.2$, and $\delta = 2/3$.

Best symmetric equilibrium in pure (realization equivalent to pure public) strategies: Play EE in first period, play RR after $\bar{y}$, and play PP after $\underline{y}$.

Payoff is 20/3.

Since first period incentives are strict, can use public correlation to play RR and PP with equal probability after $\underline{y}$ to increase payoff to 7.

Public mixed strategy: Play E with prob α in first period, and play RR after $\bar{y}$, and play $\phi \circ PP + (1 - \phi) \circ RR$ after $\underline{y}$.

The best such equilibrium has $\alpha = 0.969$ and a value of 7.0048.

Mixing implies improved informativeness of public signal about behavior.

But profile requires positive probability on PP even when players had played E, i.e., when signal is relatively uninformative.

Consider private strategies, where P is only played after S:

Play E with prob α in first period, and play RR after E and $S\bar{y}$, and play $\phi \circ PP + (1 - \phi) \circ RR$ after $S\underline{y}$.

Best such equilibrium has $\phi = 0$, $\alpha = 11/12$, and a payoff of $7.14 > 7.0048$.

Second stage is nongeneric: each player is indifferent between R and P, for all beliefs over the play of opponent.

In a repeated PD game, same property can be obtained using belief-free strategies (Kandori and Obara, 2003). Other finite horizon examples in Mailath, Matthews, and Sekiguchi (2002).

Idiosyncratic Small Players

Ex ante payoffs	h	ℓ
H	$2,3$	$0,2$
L	$3,0$	$1,1$

Player 1 (row player) is long-lived.
Continuum of player 2's.
Player 1 observes distribution of 2's behavior, so each player 2 behaves myopically.

Each player 2_i observes a private signal of 1 action: $\Omega_i = \{\underline{y}_i, \bar{y}_i\}$ (where $0 < q < p < 1$),

$$\Pr(\bar{y}_i|a) = \begin{cases} p, & \text{if } a_1 = H, \\ q, & \text{if } a_1 = L. \end{cases}$$

There are many belief-free eq (as above): 1 always randomizes with prob $\frac{1}{2}$ on H, a player 2_i observing $\bar{y}_i$ plays h for sure, and after $\underline{y}_i$ randomizes with prob $\frac{1}{2\delta(p-q)}$ on ℓ.

In period 1, 1's payoff is

$$2 - \frac{(1-p)}{\delta(p-q)},$$

while continuation payoffs are lower (after first period, at least a fraction $(1-p)$ observe $\underline{y}_i$).

Is this a plausible description of behavior? Note that the structure of this equilibrium is the same as for public monitoring.

In this setting, surely $L\ell$ is more plausible. Only Harsanyi purifiable outcome (with additively separable payoff shocks)?

Conclusion

Interesting things can be learnt from focusing on the structure of behavior.

Interesting results on games in continuous time (Sannikov, 2004; Sannikov and Skrzypacz, 2005; Faingold, 2005; Faingold and Sannikov, 2005).

Complexity.

Structure of interactions, multimarket interactions (behavior can be described independently of the game).

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